Cosmology with Extra Dimensions

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Motivation and Introduction

- Motivation
- From 10D to 4D: Dimensional Reduction
- Common Approach

Inflation and Extra Dimensions

- Alternative Approach
- Example
- Instabilities and New Dynamics

Dark Energy and the CMB

- Dark Energy vs. Cosmological Constant
- Dark Energy and the CMB
- Holography and the CMB
Outline

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Early Universe

Inflation

- de Sitter Geometry
- $H$: Expansion rate
- $ds^2 = -dt^2 + e^{2Ht} dx^2$

Flatness
Homogeneity
Monopoles
Origin of structure

Embedding of Inflation in theory with ED

Late-time Universe

String Dilaton/Moduli fields
LSP/KK particles

Cosmological Perturbations

Universe: homogeneous and isotropic on large scales and early times

- All structure expressed as Cosmological Perturbations
- Powerful tool for Exploration of the Dynamics

Exploration of the “Dark Side” with EDs
**Motivation and Introduction**

**Inflation and Extra Dimensions**

**Dark Energy and the CMB**

**Summary**

**Motivation**

**From 10D to 4D: Dimensional Reduction**

**Common Approach**

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**Dimensional Reduction**

**General (Kaluza–Klein) Procedure:**

- **Metric:**
  
  \[ g_{MN} \sim \begin{pmatrix} \gamma_{\mu\nu} & A_{\mu} \\ A_{\mu} & e^{\sigma} \end{pmatrix} \]
  
  \[ R_5 \rightarrow R_4 - (\nabla_{\mu}\sigma)^2 - e^{\sigma} F_{\mu\nu}^2 \]

- **Matter field fluctuations:**
  
  \[ \phi(x, y) \sim \sum_n \phi_n(x) \sin\left(\frac{2\pi n}{R} y\right) \]
  
  \[ (\nabla_M \phi)^2 \rightarrow (\nabla_{\mu} \phi)^2 + m_n^2 \phi^2 \quad , \quad m_n \sim \frac{n}{R} \]

**Important!**

- Small extra dimensions \(\rightarrow n = 0\) states only (consistent truncation)
- Many additional light (moduli) fields \(\leftrightarrow\) Size, shape, topology of EDs
- Moduli fields \(\rightarrow\) Effective couplings, charges, and masses
- \(\mathcal{N} = 1\) SUSY \(\rightarrow\) Kähler and **Super potential** for moduli fields
$\mathcal{N} = 1$ Compactifications (Engineering)

**Comp. on CY:**
- $\mathcal{O}(100)$ moduli fields
- unstabilized, undetermined (*flat directions*)
- $\rightarrow$ disaster for phenomenology

**Comp. on CY + Fluxes:**
- Some moduli fields stabilized
- SUSY vacua $\Omega_\Lambda < 0$

**Comp. on CY + Fluxes + non-perturb. Effects:**
- All moduli stabilized
- Vacua with broken SUSY, $\Omega_\Lambda > 0$

**Inflaton Potentials:**
- Dasgupta, Hsu, Kallosh, Linde, Zagermann ’04

**Dark Energy Potentials:**
- Kallosh, Kachru, Linde, Trivedi ’03
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Braneworlds
Kaluza–Klein Compactification

Generic Background:

\[ ds^2 = e^{2B(y)} \left\{ -dt^2 + e^{2Ht} dx^2 + g_{mn}(y) dy^m dy^n \right\} \]

\[ \gamma_{\mu\nu}(x) dx^\mu dx^\nu \]
Flux Compactification

Action:

$$S = \int d^4x d^4y \sqrt{|G|} \left\{ \frac{1}{2} R - \frac{1}{2q!} F_q^2 - \Lambda \right\}$$

Ansatz:

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2 + \rho^2 d\Omega_q^2$$

$$F_{m_1 \cdots m_q} = c \varepsilon_{m_1 \cdots m_q}$$

Perturbations:

<table>
<thead>
<tr>
<th>Metric:</th>
<th>Scalars</th>
<th>$\delta g_{mn}$</th>
<th>$a_{m_2 \cdots m_q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux:</td>
<td>Vectors</td>
<td>$\delta g_{\mu n}$</td>
<td>$a_{\mu m_3 \cdots m_q}$</td>
</tr>
<tr>
<td></td>
<td>Tensors</td>
<td>$\delta g_{\mu \nu}$</td>
<td></td>
</tr>
</tbody>
</table>

Dynamics for the Perturbations:

$$\delta G_{MN} = \delta T_{MN}, \quad \delta \left( F^{M_1 \cdots M_q}_{M_1} ; M_1 \right) = 0$$
Universal Gravitational Instability

\[ m_{\text{rad}}^2 = - \frac{12q}{2 + q} H^2 + \cdots \]

- Universal appearance in models with extra dimensions \( q \) and cosmological spacetimes \( H \)
- Known asymptotics
- Can be stabilized
- Zero mode of the volume modulus
Non-perturbative Results

Braneworlds

\[ S = \int d^5x \sqrt{-g} \left\{ \frac{1}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right\} \]

\[ - \sum_{i=1,2} \int d^4x \sqrt{-\gamma} \left\{ [K] + U_i(\phi) \right\} \]


- Brane collision
  - Potentials \( V, U \) unimportant
  - Anisotropic Kasner-like asymptotics
  - Collapse of extra dimension

- Brane separation

- Transition
  - towards stable solution
  - with smaller \( H \)

Compactifications \( dS_p \times S^q \)

\[ S = \int d^{p+q}x \sqrt{-g} \left\{ \frac{1}{2} R - \Lambda - \frac{1}{2q!} F^2_q \right\} \]

\[ F_{m_1 \ldots m_q} = c(t) \varepsilon_{m_1 \ldots m_q} \]

\[ ds^2 = -dt^2 + a(t)^2 d\bar{x}^2 + b(t)^2 d\Omega^2_q \]

Phasespace Analysis: \((a_0 = e^{Ht}, b_0)\)

CONTALDI, KOFMAN & PELOSO (2004)

- Collapse of internal space
  - Anisotropic Kasner geometry

- Decompactification
  - 11 dimensional de Sitter geometry

KRISHNAN, PABAN & ŻANIĆ (2005)

- Transitions
  - de Sitter with smaller \( H \)
  - Minkowski or \( AdS_4 \)
Effective Potential for the Volume modulus

- Collaps of ED  \rightarrow \text{ Decompactification of ED}

\[ V_{\text{eff}}(\sigma) \]

- Inflation + ED  \rightarrow \text{ Instabilities}
- Limits on the scale of Inflation
- Interesting Dynamics? \text{ Schwindt, Wetterich '05}
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Idea: One of the moduli fields ↔ Dark energy

Fine-tuning problem: Scales of dark energy potential ↔ $\Omega_\Lambda \sim H_0$

Dynamics

$$\ddot{\phi} + 3H_0\dot{\phi} + V'(\phi) = 0 \quad w = \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)}$$

$$\delta\ddot{\phi} + 3H_0\delta\dot{\phi} + \left[ V''(\phi) + \frac{k^2}{a^2} \right] \delta\phi = \left[ 4\dot{\phi} - 2V'(\phi) \right] \Phi$$

$V'(\phi), V''(\phi) \gg H_0$:
- $w \sim -1$, $w_1 \sim 0$

$V'(\phi), V''(\phi) \sim H_0$:
- $w > -1$, $w_1 \neq 0$

$V'(\phi), V''(\phi) \ll H_0$:
- $w \sim -1$, $w_1 \sim 0$
Work in Progress

Numerics

- Time evolution of background+perturbations $(\gamma, \nu, \Lambda_{\text{CDM}}, b, \delta \phi, \Phi)$
- Hydrodynamical equations
- Collisionless Boltzmann equation $\frac{\Delta T}{T}$

Dark energy potentials

- $w < -1$?
- Anti-correlated isocurvature perturbations
  - Hu, Gordon '04
Holography and the CMB

Enqvist, Hannestad, Sloth ’05

$$\Omega_\Lambda > 0 \quad \Longrightarrow \text{effective de Sitter geometry}$$

$$\quad \Longrightarrow \text{Event Horizon:}$$

$$r_H = \frac{a(t)}{H} \left( 1 - e^{-Ht} \right)$$

$$\Longrightarrow \text{Finite Universe!}$$

$$\Longrightarrow \text{Boundary conditions?}$$

**Periodic BC**
- familiar from spatially compact topologies
- unobserved symmetries and multiple images
- highly constrained

**Dirichlet/Neumann BC**
- Horizon $\Rightarrow$ IR cutoff
- Dirichlet: Quantum fields vanish at cutoff scale
- Neumann: No current fbw beyond the cutoff
Summary

1. **Inflation and Extra Dimensions**
   - Instabilities in cosmological compactifications
   - Limit on the scale of inflation from moduli stabilization
   - Dynamical transitions to stable configurations possible
   - Dynamical mechanism for lowering the CC?

2. **Dark Energy and the CMB**
   - Besides SN and WL — CMB additional probe for exploration of DE
   - CMB particular sensitive to dynamical properties of DE
   - CMB — data on largest scales vs. cosmic variance