Axions

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Structure

Introduction

- Instanton solutions create a new problem in QCD.
- A new symmetry is introduced, the axial $U(1)_{PQ}$ symmetry.
- This includes the introduction of a new dynamical pseudoscalar field and thus a particle, the axion $a(x)$.

Formulae

- Mapping $f : S^3 \to S^3$

- Winding number:

$$n = -\frac{1}{24\pi^2} \int d\theta_1 d\theta_2 d\theta_3 tr \left(\epsilon_{ijk} A_i A_j A_k\right)$$

where $A_i = f^{-1}(x_0, \vec{x}) \partial_i f(x_0, \vec{x})$

- Consider

$$\mathcal{L} = \frac{1}{2g^2} tr \left(F^{\mu\nu} F^{\mu\nu}\right)$$

Formulae

- $S_E = \int d^4x\mathcal{L}$ should be finite s.t. for $|\vec{x}| \to \infty$: $F_{\mu\nu}(x) \to 0$ and $A_\mu(x) \to U^{-1}\partial_\mu U$.

- Instantons correspond to Us with nontrivial winding number!

- By comparison:

$$\int d^4x \; tr(F^{\mu\nu}\tilde{F}^{\mu\nu}) = \frac{1}{2}\int d^4x \; \partial^\mu K^\mu$$

with

$$K_\mu = \frac{4}{3}\epsilon_{\mu\nu\lambda\rho} tr\left[(U^+\partial_\nu U)(U^+\partial_\lambda U)(U^+\partial_\rho U)\right]$$

Formulae

- Thus:

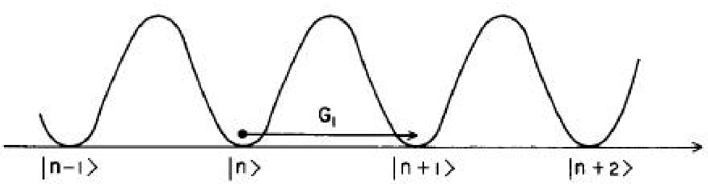
$$n = \frac{1}{16\pi^2} \int d^4x \; tr \left(F^{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

- We have multiple vacua that are characterized by their winding number n.
- Instantons can be seen as the connection between vacuum states as they have winding numbers themselves.
- Consider

$$G_1 \left| n \right\rangle = \left| n+1 \right\rangle ,$$

where G_1 is a gauge transformation of winding number 1.

Instantons - The θ-vacuum



- Transition amplitude:

$$T = \langle n | e^{-iHt} | m \rangle_J = \int [dA]_{n-m} \exp\left[-i \int (\mathcal{L} + J \cdot A) d^4x\right]$$

- Semiclassically this leads to:

$$T = \exp[-S_E] \approx \exp\left[-\frac{8\pi^2 n}{g^2}\right]$$

Instantons - The θ-vacuum

- $[G_1, H] = 0$ because of the gauge invariance.
- Then:
$$|\theta\rangle = \sum_n e^{-in\theta} |n\rangle$$
s.t.
$$G_1 |\theta\rangle = e^{i\theta} |\theta\rangle$$
- This θ-vacuum leads to an adequate formulation of the theory.

Instantons - The θ-vacuum

- Transition amplitude:

$$T = \langle \theta' | e^{-iHt} | \theta \rangle_J = \sum_{m,n} e^{im\theta'} e^{-in\theta} \langle m | e^{-iHt} | n \rangle_J$$

$$= \sum_{m,n} e^{-i(n-m)\theta} e^{im(\theta'-\theta)} \int [dA]_{n-m} \exp\left[i \int (\mathcal{L} + J \cdot A) d^4x \right]$$

- Result:

$$\mathcal{L}_{eff} = \mathcal{L} + \frac{\theta}{32\pi^2} tr \left(F^{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

The U(1) problem

- For two quark flavours and the chiral limit $m_{u,d} \to 0$ the Lagrangian possesses the symmetry:

$$SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$$

- $SU(2)_L \times SU(2)_R$ being the chiral symmetry

- $U(1)_V$ leading to baryon number conservation via the current $J_\mu^B = \overline{u}\gamma_\mu u + \overline{d}\gamma_\mu d$

- $U(1)_A$ leading to a current $J_\mu^5 = \overline{u}\gamma_\mu\gamma_5 u + \overline{d}\gamma_\mu\gamma_5 d$

The U(1) problem

- An invisible axial symmetry means that it should be broken. Thus we expect a Goldstone boson.
- But: There is no fourth pion except the η' which is much too heavy to be a Goldstone boson.

  $\rightarrow$ This is the U(1) or η-mass problem.
- Solution: The instantons can break U(1) without leading to a Goldstone boson.

The strong CP problem

- The instanton solution to the U(1) problem generates a new problem, namely the strong CP problem.
- The θ-term in the effective Lagrangian violates P.
- Experimentally: no CP violation in the strong interaction has been found.
- Comparison experiment vs. theory:

$$\theta < 10^{-9}$$

- So why should there be such a strong bound on θ?

With massless quarks

- Remember: $\mathcal{L}_{\text{eff}} = \mathcal{L} + \frac{\theta}{32\pi^2} tr \left(F^{\mu\nu} \tilde{F}^{\mu\nu} \right)$
- Consider an axial U(1)-transformation on the quark fields:

$$q \rightarrow e^{i\alpha\gamma_5} q$$

- Axial current:

$$j_\mu^5 = \sum_q \overline{q} \gamma_\mu \gamma_5 q = \sum_q [\overline{q}_R \gamma_\mu q_R - \overline{q}_L \gamma_\mu q_L]$$

- Yielding to

$$\partial^\mu j_\mu^5 = \frac{N_f g^2}{16\pi^2} tr \left(F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

with N_f being the number of massless quarks.

With massless quarks

- Thus, for

$$\alpha = -\frac{\theta}{2N_f}$$

  the θ-term of the effective Lagrangian can be removed.

- Therefore, in the massless quark case θ would be unphysical.

Without massless quarks

- Massive case:

$$m\bar{q}q \rightarrow m\bar{q}e^{-2i\alpha\gamma_5}q = m\cos(2\alpha)\bar{q}q - im\sin(2\alpha)\bar{q}\gamma_5 q$$

- The second part clearly violates P and T invariance and the mass term for quarks can be written in the following way:

$$-\mathcal{L}_{\mathrm{m}} = \bar{q}_{\mathrm{L}i} M_{ij} q_{\mathrm{R}j} + \bar{q}_{\mathrm{R}i} M^{+}_{ij} q_{\mathrm{L}j}$$

- Under U(1), M no longer is hermitean:

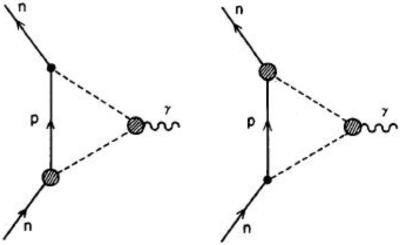
$$M \rightarrow e^{-2i\alpha} M \;\; \text{and} \;\; M^{+} \rightarrow e^{2i\alpha} M^{+}$$

Without massless quarks

- Thus: $\arg\det M \to \arg\det M + 2\alpha N$
- Define $\overline{\theta} = \theta + 2\alpha N$ as it is the quantity which is invariant under the transformation.
- $\overline{\theta}$ still has to be zero to be CP non-violating.

Neutron dipole moment

- In $\mathcal{L}$, there is a contribution to the neutron dipole moment coming from the following graphs:



Neutron dipole moment

- Pion nucleon interaction:

$$\mathcal{L}_{\pi NN} = g^{\theta}_{\pi NN} \overline{N} \tau^a N \pi^a$$

- Experiment: $d_n < 12 \cdot 10^{-26}\ e\ cm$
  Calculation: $d_n = 5.2 \cdot 10^{-16}\ \theta\ e\ cm$

- Therefore: $\theta < 2 \cdot 10^{-10}$

Introduction of the axion

- Solution proposed by Peccei and Quinn in 1977: make $\overline{\theta}$ a dynamical field.
- Later on, a new pseudoscalar boson, the axion $a(x)$, was added by Weinberg.
- Choose $\overline{\theta} = a/f_a$ with f_a being the axion decay constant.
- $$\mathcal{L} = -\frac{1}{4g^2} tr\left(F_{\mu\nu}F^{\mu\nu}\right) + \frac{1}{2}\partial_\mu a \partial^\mu a +$$
$$+ \sum_i \overline{q}_i (i\not{D} - m_i) q_i + \frac{a}{32\pi^2 f_a} tr\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right) + \frac{\overline{\theta}}{32\pi^2} tr\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

Introduction of the axion

- Still to be shown: $\overline{\theta} = 0$
- The $\overline{\theta}$ vaccum is proportional to $(1 - \cos\overline{\theta})$. Thus $\overline{\theta} = 0$ would be the correct vacuum.
- Integrating out the effect of quarks and gluons in Euclidean space yields to:

$$\int d^4xV\left[0\right] \leq \int d^4xV\left[a\right]$$

- There must be a minimum at $a = 0$ or $\overline{\theta} = 0$
- Thus the choice of $\overline{\theta} = 0$ is justified.

The axion as Goldstone boson

- Notation: - $Q_{f\mathrm{L}} = (u_{f\mathrm{L}}, d_{f\mathrm{L}})$ are the SU(2) doublets
  - $U_{f\mathrm{R}}, D_{f\mathrm{R}}$ denote the SU(2) singlets
  - φ is the doublet of the Higgs scalar fields

  and f labels the generations.

- The mass term comes from:

$$-\mathcal{L}_{\mathrm{Y}} = \overline{Q}_{f\mathrm{L}} X_{fg} \varphi D_{g\mathrm{R}} + \overline{Q}_{f\mathrm{L}} Y_{fg} \psi U_{g\mathrm{R}} + h.c.$$

with $\psi = i\tau^2\varphi^*$.

The axion as Goldstone boson

- $\langle 0|\,\varphi\,|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 e^{i\delta_1} \end{pmatrix}$ for down-like quarks
- $\langle 0|\,\psi\,|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_2 e^{i\delta_2} \\ 0 \end{pmatrix}$ for up-like quarks
- Conditions in the SM: $v_1 = v_2$ and $\delta_1 = -\delta_2$
- $$\begin{aligned} \det M &= \det\left(\frac{1}{\sqrt{2}} v_1 e^{i\delta_1} X\right) \det\left(\frac{1}{\sqrt{2}} v_2 e^{i\delta_2} Y\right) \\ &= e^{3i(\delta_1+\delta_2)} \frac{1}{8} (v_1 v_2)^3 \det(XY) \end{aligned}$$

The axion as Goldstone boson

- Thus, we extend the SM and define two independent scalar doublets φ and ψ and get:

$$\arg\det M = 3(\delta_1 + \delta_2) + \arg\det(XY)$$

- $\overline{\theta} \equiv \theta + \arg\det M = 0$

- This is exactly what we needed.

- To still ensure the symmetry, suppose:

$$\varphi \to e^{i\alpha\Gamma_1}\varphi, \psi \to e^{i\alpha\Gamma_2}\psi$$

  where Γ_1, Γ_2 are the Peccei-Quinn charges of φ, ψ

The axion as Goldstone boson

- Further transformations:

$$Q_\mathrm{L} \to e^{i\alpha\Gamma_Q}, U_\mathrm{L} \to e^{i\alpha\Gamma_u}, D_\mathrm{L} \to e^{i\alpha\Gamma_d}$$

- Invariance of $\mathcal{L}_\mathrm{Y}$ is given if:

  $\Gamma_1 + \Gamma_d = \Gamma_Q, \Gamma_2 + \Gamma_u = \Gamma_Q$, meaning
  $\Gamma_1 + \Gamma_2 = 2\Gamma_Q - \Gamma_u - \Gamma_d \neq 0$

- For φ and ψ acquiring the former vevs, the SSB yields a Goldstone boson which is associated with the phase angle of these fields.

The axion as Goldstone boson

- Look at the neutral components of the scalar doublets:

$$\varphi^0 = \frac{1}{\sqrt{2}} \left(v_1 + \rho_1(x)\right) e^{i\theta_1(x)/v_1}$$

$$\psi^0 = \frac{1}{\sqrt{2}} \left(v_2 + \rho_2(x)\right) e^{i\theta_2(x)/v_2}$$

- Thus

$$a(x) = \left[v_2\theta_1(x) + v_1\theta_2(x)\right]/v$$

where $v \equiv \left(v_1^2 + v_2^2\right)^{1/2}$

The axion as Goldstone boson

- The orthogonal second combination gets ’eaten’ and generates the Z-boson mass:

$$\chi(x) = \left[-v_1\theta_1(x) + v_2\theta_2(x)\right]/v$$

- This finally yields to the axion-quark Yukawa coupling:

$$-\mathcal{L}_{\mathrm{Y}}^{a,q} = \frac{ia(x)}{v}\left[\frac{v_2}{v_1}m_d\overline{d}\gamma_5 d + \frac{v_1}{v_2}m_u\overline{u}\gamma_5 u\right]$$

The axion as Goldstone boson

- $v \simeq 246$ GeV from weak interaction data
- Axion mass:

$$m_a = \left(\frac{v_1}{v_2} + \frac{v_2}{v_1}\right) 74 \text{ keV}$$

- Although many experiments tried to find this axion it was not found in any reaction so far. But this is not the end of the story.

The invisible axion

- Two models:

  i) There is the heavy quark theory. (KSVZ axion)
  ii) The PQ symmetry breaking scale can be separated from the electroweak breaking scale. (DFSZ axion)

- Breaking scale:

$$10^9 \text{ GeV} \leq f_{PQ} \leq 10^{12} \text{ GeV}$$

- This yield to 'invisibility'.

The invisible axion

- Possible decay processes:

  For $m_a > 2\ m_e$:

$$m_a \to e^- e^+$$

  For $m_a < 2\ m_e$:

$$m_a \to \gamma\ \gamma$$

- This is a possible detection method.

Conclusion

- The axion is a very promising idea to solve the strong CP-violation problem.
- Even if it has not been detected this does not mean that it cannot exist as an invisible axion.
- Could be a candidate for the dark matter in the universe.
- Further experiments are conducted.
- e.g. the CAST-experiment at CERN is searching for solar axions.

Literature

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