Symmetry breaking: Pion as a Nambu-Goldstone boson

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General theory of spontaneous symmetry breaking

What it is
Spontaneous breaking of a global continuous symmetry (classical)
Spontaneous breaking of a global continuous symmetry (QFT)
Quantum fluctuations

Pion as a Nambu-Goldstone Boson
Weak decays
Pion as a Nambu-Goldstone boson

Summary
A symmetry is said to be **spontaneously broken**, if the Lagrangian exhibits a symmetry but the ground state/vacuum does not have this symmetry.
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**Goldstone-theorem**

If the Lagrangian is invariant under a **continuous global** symmetry operation $g \in G$ and the vacuum is invariant under a subgroup $H \subset G$, then there exist $n(G/H) = n(G) - n(H)$ **massless spinless particles**.

With $n(G)$ the number of generators of the group $G$. 
Spontaneous breaking of a global continuous symmetry (classical)

\[ \mathcal{L}(\Phi_i, \partial \Phi_i) = \frac{1}{2} (\partial_\mu \Phi_i)^2 - V(\Phi_i) \]

with \( V(\Phi_i) \) invariant under a **global continuous** symmetry group \( G \)

i.e.

\[ V(\Phi_i) = V(\rho(g)\Phi_i) \quad \text{with} \quad g \in G \]

\( \Phi_i \mapsto \Phi'_i = \rho(g)\Phi_i \approx \Phi_i + \delta \Phi_i = \Phi_i + i\epsilon_a T^a \Phi_i \)
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We demand that the vacuum has a **minimal energy**

\[ \Rightarrow \text{minimize } V \]

\[ \Phi_{i,\text{min}} = \langle 0 | \Phi_i | 0 \rangle = \text{const.} \quad \chi_i := \Phi_i - \Phi_{i,\text{min}} \]

\[ V(\Phi_i) = V(\Phi_{i,\text{min}}) + \frac{\partial V}{\partial \Phi_i}(\Phi_{i,\text{min}}) \chi_i + \frac{1}{2} \frac{\partial^2 V}{\partial \Phi_i \partial \Phi_j} \chi_i \chi_j + \ldots \]
Spontaneous breaking of a global continuous symmetry (classical)

\[ V(\Phi_{i,\text{min}}) \] is a minimum, therefore

\[
\frac{\partial V}{\partial \Phi_i}(\Phi_{i,\text{min}}) = 0
\]

\[
\frac{\partial^2 V}{\partial \Phi_i \partial \Phi_j} : = M_{ij}^2 \]

is semi-positive definite
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\[ V(\Phi_{i, \text{min}}) \] is a minimum, therefore

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Now use the symmetry:

\[ V(\Phi_{i, \text{min}}) = V(\rho(g)\Phi_{i, \text{min}}) = V(\Phi_{i, \text{min}}) + \frac{1}{2} M_{ij}^2 (\delta \Phi_{i, \text{min}})(\delta \Phi_{j, \text{min}}) \]

\[ \Rightarrow 0 = M_{ij}^2 \delta \Phi_{j, \text{min}} \quad \forall i \]

\[ \Rightarrow 0 = M_{ij}^2 T^a \Phi_{j, \text{min}} \quad \forall i \]
Spontaneous breaking of a global continuous symmetry (classical)

Important result

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Spontaneous breaking of a global continuous symmetry (classical)

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\[ T^a \in H \subset G \quad \text{with} \]
\[ T^a \Phi_{j,\text{min}} = 0 \]

Every generator of H leaves the vacuum invariant.
Spontaneous breaking of a global continuous symmetry (classical)

**Important result**

\[ 0 = M_{ij}^2 T^a \Phi_{j,\min} \quad \forall i \]

1. \( T^a \in H \subset G \) with \( T^a \Phi_{j,\min} = 0 \)

Every generator of \( H \) leaves the vacuum invariant.

2. \( T^a \notin H \Rightarrow T^a \Phi_{j,\min} \neq 0 \)

\( T^a \Phi_{j,\min} \) is an eigenstate of \( M^2 \) with 0 eigenvalue

\( T^a \Phi_{j,\min} \) is a Nambu-Goldstone boson

The number of Nambu-Goldstone bosons is \( n(G)-n(H)=n(G/H) \).
\[ L = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} \mu^2 \phi^2 - \frac{\lambda}{4!} \phi^4 \]

with \( \phi_{\text{min}} = \sqrt{\frac{6}{\lambda}} \mu^2 \)

\[ \phi = \phi_{\text{min}} + \sigma \]

\[ L = \frac{1}{2} (\partial_\mu \sigma)^2 - \frac{1}{2} (2 \mu^2) \sigma^2 - \sqrt{\frac{\lambda}{6}} \mu \sigma^3 - \frac{\lambda}{4!} \sigma^4 \]
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\[
\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} \mu^2 \phi^2 - \frac{\lambda}{4!} \phi^4
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**Massive** modes correspond to fluctuations in **radial** direction.

**Massless** modes correspond to fluctuations in **angular** direction.
Spontaneous breaking of a global continuous symmetry (QFT)

In QFT every **continuous symmetry** implies the existence of a conserved current $J^{\mu}(x)$:

$$\partial_{\mu}J^{\mu}(x) = 0 \quad \rightarrow \quad Q = \int d^Dx J^0(\vec{x}, t) \quad \text{is conserved, i.e.}$$

$$[H, Q] = 0$$
In QFT every **continuous symmetry** implies the existence of a conserved current $J^\mu(x)$:

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- $Q|0> = 0$, for every energy-eigenstate $|\Phi \rangle = \Phi|0>$ define

  $$|\Phi' \rangle := e^{i\Theta Q} \Phi e^{-i\Theta Q}|0>$$

  $$H|\Phi' \rangle = He^{i\Theta Q} \Phi e^{-i\Theta Q}|0> = He^{i\Theta Q} \Phi|0>$$

  $$= e^{i\Theta Q} H\Phi|0> = e^{i\Theta Q} E\Phi|0> = E|\Phi' \rangle$$

Energy spectrum is organized in **multiplets** of degenerate states.
Spontaneous breaking of a global continuous symmetry (QFT)

- \( Q|0 \neq 0 \) Problem, \( Q \) is not defined

\[
<0|Q^2(t)|0> = \int d^D x <0|J^0(\vec{x}, t)Q(t)|0>
\]

\[
= \int d^D x <0|e^{i\vec{P} \cdot \vec{x}} J^0(\vec{0}, 0)e^{-i\vec{P} \cdot \vec{x}} Q(t)|0>
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Spontaneous breaking of a global continuous symmetry (QFT)

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$$= \int d^Dx < 0|J^0(\vec{0}, t)Q(t)|0 >$$

But the **commutator with $Q$** may be well defined. Current conservation implies:

$$0 = \int d^Dx [\partial^\mu J_\mu(\vec{x}, t), \Phi(0)] = \partial^0 \int d^D [J^0(\vec{x}, t), \Phi(0)]$$

$$+ \int d\vec{S} \cdot [\vec{J}(\vec{x}, t), \Phi(0)]$$
Spontaneous breaking of a global continuous symmetry (QFT)

\[ \Rightarrow \frac{d}{dt} [Q(t), \Phi(0)] = 0 \]

The **symmetry is broken**, if there is an operator \( \Phi \) s.t.:

\[ <0|[Q(t), \Phi(0)]|0> = \eta \neq 0 \]
Spontaneous breaking of a global continuous symmetry (QFT)

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The symmetry is broken, if there is an operator \( \Phi \) s.t.:

\[ < 0 | [Q(t), \Phi(0)] | 0 >= \eta \neq 0 \]

then:

\[ \eta = \sum_n \int d^Dx \{ < 0 | J_0(x) | n > < n | \Phi(0) | 0 > \]

\[ - < 0 | \Phi(0) | n > < n | J_0(x) | 0 > \}

\[ = \sum_n (2\pi)^D \delta^D(\vec{p}_n) \{ < 0 | J_0(0) | n > < n | \Phi(0) | 0 > e^{-iE_nt} \]

\[ - < 0 | \Phi(0) | n > < n | J_0(0) | 0 > e^{iE_nt} \} \]
Since this is time independent, there has to be a state with:

### Nambu-Goldstone Boson

\[
E_n = 0 \quad \text{for} \quad \vec{p}_n = 0 \quad \Rightarrow \quad m_n = 0
\]

\[
\langle n | \Phi(0) | 0 \rangle \neq 0
\]

\[
\langle 0 | J_0(0) | n \rangle \neq 0
\]
Since this is time independent, there has to be a state with:

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E_n = 0 \quad \text{for} \quad \vec{p}_n = 0 \quad \Rightarrow \quad m_n = 0
\]

\[
\langle n | \Phi(0) | 0 \rangle \neq 0
\]

\[
\langle 0 | J_0(0) | n \rangle \neq 0
\]

So the Nambu-Goldstone boson has to carry all the quantum numbers of the conserved current.

In general there will be \( n(G) \) currents \( J^a_\mu \). Of these \( n(H) \) generate the symmetry group of vacuum. So again there are \( n(G/H) = n(G) - n(H) \) Nambu-Goldstone bosons.
Quantum fluctuations

Can the massless fields wander away from their ground state? Calculate mean square fluctuation:

\[ \langle b(0)^2 \rangle = \frac{1}{Z} \int Dbe^{iS[b]} b(0)b(0) \]
\[ = \lim_{x \to 0} \frac{1}{Z} \int Dse^{iS(b)} b(x)b(0) \]
\[ = \lim_{x \to 0} \int \frac{d^d k}{(2\pi)^d} \frac{e^{ik \cdot x}}{k^2} \]

The UV divergencies can be regulated by a cutoff.

But for \( d \leq 2 \) there is an IR divergence.
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\[ = \lim_{x \to 0} \int d^dk \frac{e^{ik \cdot x}}{(2\pi)^d k^2} \]

The UV divergencies can be regulated by a cutoff.

But for \( d \leq 2 \) there is an \textbf{IR divergence}.

Coleman-Mermin-Wagner theorem

Spontaneous symmetry breaking is impossible in \( d \leq 2 \).
How do pions enter the game?

We want to calculate decay rates of semileptonic decays like:

\[ n \rightarrow p + e^- + \bar{\nu} \]
\[ \pi^- \rightarrow \pi^0 + e^- + \bar{\nu} \]

Use an effective Lagrangian

\[ \mathcal{L} = G [\bar{e} \gamma^\mu (1 - \gamma^5) \nu] (J_\mu - J_{\mu 5}) \]

Where \( J_\mu \) and \( J_{\mu 5} \) are hadronic currents which include strong interaction effects.

We have to calculate \( < p | J_\mu(x) - J_{\mu 5}(x) | n > \) or \( < 0 | J_\mu(0) - J_{\mu 5}(0) | \pi^- > \)
Nucleon $\beta$-decay

$$< p(k')|J_{\mu 5}(x)|n(k) > = < k'|e^{iP\cdot x}J_{\mu 5}(0)e^{-iP\cdot x}|k >$$

$$= < k'|J_{\mu 5}(0)|k > e^{i(k'-k)\cdot x}$$

Use Lorentz invariance to simplify $< k'|J_{\mu 5}(0)|k >$ with $q := k' - k$

$$< k'|J_{\mu 5}(0)|k > = \bar{u}_p(k')[-i\gamma_\mu\gamma_5 F'(q^2) + q_\mu\gamma_5 G(q^2)$$

$$+ (k'_\mu + k_\mu)\gamma_5 F_2(q^2) + i[\gamma_\mu, \gamma_\nu]\gamma_5 q^\nu F'_3(q^2)]u_n(k)$$
Nucleon $\beta$-decay

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\[
< k'|J_{\mu 5}(0)|k > = \bar{u}_p(k')[-i\gamma_\mu \gamma_5 F'(q^2) + q_\mu \gamma_5 G(q^2) + (k'_\mu + k_\mu)\gamma_5 F'_2(q^2) + i[\gamma_\mu, \gamma_\nu]\gamma_5 q'\nu F'_3(q^2)]u_n(k)
\]

And use the Gordan-Identity:

\[
\bar{u}_p(k')[i\frac{1}{2}[\gamma_\mu, \gamma_\nu]\gamma_5 q'\nu]u_n(k) = \bar{u}_p(k')[i(k'_\mu + k_\mu)\gamma_5 + (m_p - m_n)\gamma_\mu \gamma_5]u_n(k)
\]
So if we define:

\[ F_3(q^2) = F'_3(q^2) + \frac{1}{2i} F'_2(q^2) \]
\[ F(q^2) = F'(q^2) - (m_p - m_n) F'_2(q^2) \]
Nucleon $\beta$-decay

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$$F(q^2) = F'(q^2) - (m_p - m_n) F'_2(q^2)$$

we obtain:

**Decomposition of $< k' | J_{\mu 5}(0) | k >$ into Form factors**

$$< k' | J_{\mu 5}(0) | k > = u_p(k') \left[ -i \gamma_\mu \gamma_5 F(q^2) + q_\mu \gamma_5 G(q^2) 
+ i [\gamma_\mu, \gamma_\nu] \gamma_5 q^\nu F_3(q^2) \right] u_n(k)$$

And similarly we obtain:

$$< 0 | J_{5\mu}^{\mu}(0) | \pi (k) > = i F_\pi k^\mu$$
Problems with strongly interacting particles

In general a perturbation theory is **not well defined.**
Breaking of chiral symmetry

\[ M_\pi \approx 139 \text{MeV} \ll 938 \text{MeV} \approx m_N \Rightarrow M_\pi \approx 0 \]

Is it possible that the pion is a Nambu-Goldstone Boson?
Breaking of chiral symmetry

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Is it possible that the pion is a Nambu-Goldstone Boson?

Suppose \( \partial_\mu J_5^{\mu}(x) = 0 \) i.e. chiral symmetry holds then:

\[ <0|J_5^{\mu}(0)|\pi(k)> = iF_\pi k^\mu \iff <0|J_0(0)|n> \neq 0 \]
\[ <\pi|\pi(0)|0> = 1 \iff <n|\Phi(0)|0> \neq 0 \]
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Suppose \( \partial_\mu J_5^\mu (x) = 0 \) i.e. chiral symmetry holds then:

\[ < 0 | J_5^\mu (0) | \pi (k) >= iF_\pi k^\mu \quad \iff \quad < 0 | J_0 (0) | n > \neq 0 \]

\[ < \pi | \pi (0) | 0 >= 1 \quad \iff \quad < n | \Phi (0) | 0 > \neq 0 \]

**Pion as a Nambu-Goldstone Boson**

The Pion is a Nambu-Goldstone Boson in a world in which chiral symmetry holds.
Is the Pion massless?

If chiral symmetry holds then:

\[-ik_{\mu} < 0|J_{5}^{\mu}(0)|\pi(k) > \exp(-ik \cdot x) = \partial_{\mu} < 0|J_{5}^{\mu}(0)|\pi(k) > \exp(-ik \cdot x) = 0\]
Is the Pion massless?

If chiral symmetry holds then:

\[-i k_\mu < 0 |J_5^{\mu}(0)|\pi(k)\rangle \exp(-i k \cdot x) = \partial_\mu < 0 |J_5^{\mu}(0)|\pi(k)\rangle \exp(-i k \cdot x) \]
\[= \langle 0 |\partial_\mu J_5^{\mu}(x)|\pi(k)\rangle = 0\]

\[k^{\mu} < 0 |J_5^{\mu}(0)|\pi(k)\rangle = iF_\pi k^{\mu} k_\mu = iF_\pi M_\pi^2\]

Thus $\partial_\mu J_5^{\mu}(x) = 0$ implies that $M_\pi^2 = 0$
Goldberger-Treiman relation

\[ q^\mu < k' | J_{\mu 5}(0) | k > = -i < k' | \partial^\mu J_{\mu 5}(x) | k > \exp(-iq \cdot x) = 0 \]

\[ q^\mu < k' | J_{\mu 5}(0) | k > = q^\mu \bar{u}_\nu(k')[-i\gamma_\mu \gamma_5 F(q^2) + q_\mu \gamma_5 G(q^2) + i[\gamma_\mu, \gamma_\nu] \gamma_5 q^\nu F_3(q^2)]u_n(k) \]

\[ = \bar{u}_N(k')(k'^\mu (-i)\gamma_\mu \gamma_5 F(q^2) - k^\mu (-i)\gamma_\mu \gamma_5 F(q^2) + q^2 \gamma_5 G(q^2) + i[\gamma_\mu, \gamma_\nu] \gamma_5 q^\nu q^\mu F_3(q^2)]u_N(k) \]

\[ = \gamma_5 [2m_N F(q^2) + q^2 G(q^2)] = 0 \]

by using the Dirac equation:

\[ \bar{u}_N(k')(k'_\mu \gamma^\mu - im_N) = (k_\mu \gamma^\mu - im_N)u_N(k) = 0 \]

But if \( q \to 0 \) this implies \( m_N = 0 \) !?!
Goldberger-Treiman relation

\[
\text{this diagram gives a contribution}
- iF_\pi q^\mu \frac{i}{q^2} G_{\pi NN} \bar{u}_N(k') \gamma_5 u_N(k)
\]

Note that there are still \textbf{infinitely many diagrams} which have a pole at \( q^2 = 0 \)

\[
G(q^2) \sim F_\pi \frac{1}{q^2} G_{\pi NN} \quad \text{for} \quad q \to 0
\]

Goldberger-Treiman relation

\[
G_{\pi NN} = \frac{2m_N g_A}{F_\pi} \quad \text{with} \quad g_A = -F(0)
\]
Experimental test of Goldberger-Treiman relation

\[ m_N = \frac{(m_p + m_n)}{2} = 939.9\text{MeV} \]
\[ g_A = 1.257 \]
\[ F_\pi = 93\text{MeV} \]
\[ G_{\pi NN}^{GTR} \approx 25.4 \]
\[ G_{\pi NN} = 27.0 \]

In good agreement with the experiment.
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Please note that this result is quite **general**.
No assumptions about the broken symmetry have been made.
More about chiral symmetry

The Lagrangian of strong interaction:

\[ \mathcal{L} = -\bar{u} \gamma^{\mu} D_\mu u - \bar{d} \gamma^{\mu} D_\mu d + m_u (\bar{u}_R u_L + \bar{u}_L u_R) + m_d (\bar{d}_L d_R + \bar{d}_R d_L) + \ldots \]

has a \( SU(2)_V \otimes SU(2)_A \) symmetry in the case \( m_u = m_d = 0 \).

\[ q \mapsto q' = \exp(i \vec{\Theta}_V \cdot \vec{\tau} + i \gamma_5 \vec{\Theta}_A \cdot \vec{\tau}) q \]

with \( \vec{\tau} \) the isospin (Pauli) matrices.
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And the conserved currents:

\[ \vec{J}^\mu = i\bar{q} \gamma^\mu \vec{\tau} q \quad \text{and} \quad \vec{J}_5^\mu = i\bar{q} \gamma_\mu \gamma_5 \vec{\tau} q \]

with charges:

\[ \vec{Q}_V = \int d^3x \vec{J}^0(\vec{x}, t) \]
\[ \vec{Q}_A = \int d^3x \vec{J}_5^0(\vec{x}, t) \]
More about chiral symmetry

They act in the following way on $q$:

$$[\tilde{Q}_V, q] = -\tau q$$
$$[\tilde{Q}_A, q] = -\gamma_5 \tau q$$

So if the symmetry is unbroken $\tilde{Q}_A$ transforms a state $|h>\)$ into a state of $\tilde{Q}_A|h>$ of opposite parity.

No such parity doubling is observed in the hadron spectrum. Conclude that $SU(2)_V \otimes SU(2)_A$ is broken to $SU(2)_V$ isospin group.

The three Pions are the three Nambu-Goldstone Bosons of the three currents $\tilde{J}_5^\mu$. 
Summary/Remarks

- Breaking of a continuous global symmetry leads to **Nambu-Goldstone Bosons**.
- No information about how the breaking occurs is needed.
- Natural explanation for **smallness** of **Pion mass**.
- Found a technique to relate infinitly many diagramms.
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- Natural explanation for **smallness** of **Pion mass**.
- Found a technique to **relate** infinitly many diagramms.
- From the assumption that Pions and even Kaons and the Eta are Goldstone bosons one can construct **effective field theories** like non-linear sigma model or chiral pertubation theory.
- The QCD Lagrangian for massless Quarks has two other **global symmetries**. One, $U(1)_V$ implies **Baryon-number conservation**. The other one, $U(1)_A$ will be treated **next week**.
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- Found a technique to **relate** infinitely many diagrams.
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Thank you
Sources

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