Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 4
November 6^{th}, 2002
WS 2002/2003

Exercises on *Theoretical Elementary Particle Physics*

Prof. H. Dreiner

1. *A few remarks on groups, representations and all that*

   (a) An orthogonal matrix R leaves a skalar product invariant. Which condition does this impose on R, the metric being arbitrary?

   (b) Focussing on a Euclidean metric, writing $R \equiv e^M$, which condition does one get for M?

   (c) Hence, how many free parameters does one have in N dimensions?

   (d) Now consider three dimensions. Show that the generators τ^i can be written as $[\tau^i]^{jk} = -\varepsilon^{ijk}$.

   (e) Using $\varepsilon^{kil}\varepsilon^{kjm} = \delta^{ij}\delta^{lm} - \delta^{im}\delta^{lj}$, show that one has

   $$\left[\tau^i, \tau^j\right] = \varepsilon^{ijk}\ \tau^k. \tag{1}$$

   Thus the particular representation of $SO(3)$ we are using is defined by its own structure constants, the so-called *adjoint* representation:

   $$[\tau^i]^{jk} = f^{ijk}.$$

   How does (1) compare to the Lie-Algebra of the Pauli matrices?[1]

   (f) Recall your knowledge about irreducible representations (irreps).

   (g) Discuss the particle contents of $SU(2), SU(3), SU(5)$ in the fundamental and adjoint representations. To do so, first determine how many generators one has for $SU(N)$.

   (h) Determine the Gell-Mann matrices, i.e. the generators of $SU(3)$. Discuss how one can see from their shape that $SU(3)$ has the subgroup $SU(2) \times U(1)$; an analogous consideration shows that $SU(5)$ unites $SU(3) \times SU(2) \times U(1)$.

[1] For more on this read *Kalka/Soff, Supersymmetrie, Teubnerverlag*, chapters 5 and 7. Very good is also *Cheng/Li, Gauge theory of elementary particle physics, Oxford University Press*; they also treat Young tableaux.

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1. Several Young tableau gymnastics, given in the exercise.

2. For $e^-\mu^-$ scattering the absolute value squared and averaged matrix element is given as

$$|\overline{\mathcal{M}}|^2 = 2e^4\,\frac{s^2+u^2}{t^2}.$$

   Using crossing symmetry, calculate $|\overline{\mathcal{M}}|^2$ for $e^-e^+ \to \mu^+\mu^-$, also expressed in terms of Mandelstam variables.