Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 5
November 13^{th}, 2002
WS 2002/2003

Exercises on *Theoretical Elementary Particle Physics*

Prof. H. Dreiner

1. Young tableaux II
   (a) Using Young tableaux, calculate $\overline{\mathbf{3}} \times \mathbf{3}$ for $SU(3)$, and $\mathbf{2} \times \mathbf{2}$ for $SU(2)$.
   (b) Starting from $SU(3)$'s $\mathbf{3}, \mathbf{6}, \mathbf{8}, \mathbf{10}$ determine the shape of the Young tableaux of $\overline{\mathbf{3}}, \overline{\mathbf{6}}, \overline{\mathbf{8}}, \overline{\mathbf{10}}$.
   (c) In order to have a challenge, tackle $SU(3)$'s $\mathbf{8} \times \mathbf{8}$. Does it contain a $\mathbf{1}$, so that thus $\mathbf{8} = \overline{\mathbf{8}}$?

2. (a) Show that for $\mathbf{M} \in SL(2, C)$ one has $\mathbf{M}^T \, \epsilon \, \mathbf{M} = \epsilon$ (*hint*: use the expression for the determinant of $\mathbf{M}$). How does this generalize to $\mathbf{M} \in SL(n, C)$? Compare also to $\mathbf{\Lambda}^T \mathbf{g} \mathbf{\Lambda} = \mathbf{g}$.
   (b) Show that $\chi^T \, i\sigma^2 \, \xi$ is a Lorentz-invariant quantity. If $\xi = \chi$, what consequences does this have for the components of the spinors?

3. Recall your knowledge about Weyl spinors, $(1/2, 0)$ vs. $(0, 1/2)$, dotted and and undotted indices, etc.

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1. Reduce $\overline{\mathbf{3}} \times \overline{\mathbf{3}}$, $\mathbf{3} \times \mathbf{3}$, $\mathbf{3} \times \mathbf{3} \times \mathbf{3}$ for $SU(3)$, likewise $\mathbf{2} \times \mathbf{2} \times \mathbf{2}$ for $SU(2)$, $\overline{\mathbf{5}} \times \mathbf{5}$, $\mathbf{5} \times \mathbf{5}$ for $SU(5)$.

2. A Lorentz transformation turns the Weyl-spinor ξ into $\mathbf{M}\xi$, with (infinitesimally) $\mathbf{M} = \mathbb{1} - \frac{i}{2}\vec{\theta} \cdot \vec{\sigma} - \frac{1}{2}\vec{\beta} \cdot \vec{\sigma}$. Show that $\mathbf{M}^{-1^T} = i\sigma^2 \mathbf{M} (i\sigma^2)^T$.

3. Show the following three identities: $\bar{\phi}\bar{\psi} = \bar{\psi}\bar{\phi} = (\phi\psi)^\dagger = (\psi\phi)^\dagger$. Using $\bar{\sigma}^{\mu T} = (-i\sigma^2)\sigma^\mu(i\sigma^2)$, show that $\sigma^{\mu A\dot{B}} = \bar{\sigma}^{\mu \dot{B} A}$; show that $\phi\sigma^\mu\bar{\psi} = -\bar{\psi}\bar{\sigma}^\mu\phi$.

4. Show that $det(p'_\mu \, \sigma^\mu) = p_\mu p^\mu$.

5. Starting from the principle of least action, derive the Euler-Lagrange equation for fields (not point particles like in theoretical mechanics).