Exercises on Elementary Particle Physics

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1. *Supersymmetric Gaugetheory*

   (a) The supersymmetric generalization of the electromagnetic fieldstrength is given by the chiral spinorsuperfield

   $$W_\alpha = \bar{\mathscr{D}}^2 \mathscr{D}_\alpha V \ ,$$

   where V is the vectorsuperfield of exercise 10.3 and $\mathscr{D}, \bar{\mathscr{D}}$ are the supercovariant derivatives. Show that W_α is a lefthanded chiral superfield.

   (b) Show: W_α is invariant under supergaugetransformations

   $$V \to V + i(\Lambda - \Lambda^\dagger) \ , \tag{1}$$

   with Λ a lefthanded superfield.

   (c) Compute the transformations of the componentfields of V and fix the gauge by choosing $C = \chi = M = N = 0$ (*Wess-Zumino* gauge). What are the residual symmetries?

   (d) Compute the explicit form of W_α.

   (e) Compute the F-component (the term proportional to Θ^4) of $W^\alpha W_\alpha$ by integrating over the whole superspace and give a supersymmetric Lagrange density.
   Hint: Computation quite lengthy again. You might find the idenitiy

   $$(\sigma_\nu)^{\alpha\dot{\beta}}(\sigma_\mu)_{\beta\dot{\beta}}(\sigma_\rho)_{\alpha\dot{\gamma}}(\sigma_\sigma)^{\beta\dot{\gamma}} = 2(\eta_{\mu\nu}\eta_{\rho\sigma} + \eta_{\nu\rho}\eta_{\mu\sigma} - \eta_{\nu\sigma}\eta_{\mu\rho} + i\epsilon_{\nu\mu\sigma\rho})$$

   useful. If so, you don't have to derive it!

2. *Gaugeinvariant Coupling*

   (a) Couple the chiral superfields of exercise 10.2 to the vectorsuperfield. Start by generalizing the known $U(1)$ transformatinons from electrodynamics to

   $$\phi \to e^{-iq\Lambda}\phi \ ,$$

   where ϕ and Λ are lefthanded superfields. Is the kinetic term $\phi^\dagger\phi$ gaugeinvariant? Substitute this term by

   $$\phi^\dagger e^{qV}\phi \ ,$$

   where V is a vectorsuperfield. How must V transform in order to make this term gaugeinvariant? Compare your result to (1).

   (b) Compute

   $$\int d^2\theta d^2\bar{\theta}\phi^\dagger e^{qV}\phi$$

   in *Wess-Zumino* gauge.

3. *Supersymmetric Electrodynamics (SQED)*

   (a) Let ϕ_+ and ϕ_- be chiral superfields which transform under $U(1)$ as

   $$\phi_+ \to e^{-ie\Lambda}\phi_+ \ , \quad \phi_- \to e^{ie\Lambda}\phi_- \ .$$

   Why is the *Lagrange* density

   $$\mathscr{L} = \left[\int d^2\theta(\frac{1}{64}W^\alpha W_\alpha + m\phi_+\phi_-) + h.c.\right] + \int d^2\theta d^2\bar{\theta}(\phi_+^\dagger e^{eV}\phi_+ + \phi_-^\dagger e^{-eV}\phi_-)$$

   gaugeinvariant and supersymmetric?

   (b) Compute the components of the given *Lagrange* density in *Wess-Zumino* gauge.