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Example sheet 2 (v2)
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Elementary Particle Physics II

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1. One finite element of the group of SUSY transformations is

$$S(a^\mu, \alpha, \bar{\alpha}) \equiv \exp\left[\alpha Q + \bar{Q}\bar{\alpha} - i a^\mu P_\mu\right] \; . \qquad (1)$$

   Show that $S(a^\mu, \alpha, \bar{\alpha}) \circ S(b^\mu, \beta, \bar{\beta})$ is again a group element.

2. Starting with

$$S(a^\mu, \alpha, \bar{\alpha}) \left[\Phi(x^\mu, \theta, \bar{\theta})\right] = \Phi(x^\mu + a^\mu - i\,\alpha\sigma^\mu\bar{\theta} + i\,\theta\sigma^\mu\bar{\alpha},\, \theta + \alpha,\, \bar{\theta} + \bar{\alpha})\,, \qquad (2)$$

   show that one linear representation of the SUSY algebra is

$$\begin{aligned} P_\mu &= i\frac{\partial}{\partial x_\mu} \equiv i\partial_\mu\,, & (3)\\ Q_\alpha &= \partial_\alpha - i\,\sigma^\mu_{\alpha\dot{\beta}}\,\bar{\theta}^{\dot{\beta}}\,\partial_\mu\,, & (4)\\ \bar{Q}_{\dot{\beta}} &= -\bar{\partial}_{\dot{\beta}} + i\,\theta^\alpha\sigma^\mu_{\alpha\dot{\beta}}\,\partial_\mu\,. & (5) \end{aligned}$$

3. Check that the following derivatives commute with an infinitesimal SUSY transformation ($D_\alpha(\delta_S[\Phi]) = \delta_S[D_\alpha\Phi]$), and are therefore covariant:

$$\begin{aligned} D_\alpha &\equiv \partial_\alpha + i\,\sigma^\mu_{\alpha\dot{\beta}}\,\bar{\theta}^{\dot{\beta}}\,\partial_\mu\,, & (6)\\ \bar{D}_{\dot{\beta}} &\equiv -\bar{\partial}_{\dot{\beta}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\beta}}\,\partial_\mu\,. & (7) \end{aligned}$$

4. Different possible representations for the SUSY transformations are

$$S_L(a^\mu, \alpha, \bar{\alpha}) \equiv \exp\left[\alpha Q - i a^\mu P_\mu\right] \cdot \exp\left[\bar{Q}\bar{\alpha}\right] \,, \qquad (8)$$

$$S_R(a^\mu, \alpha, \bar{\alpha}) \equiv \exp\left[\bar{Q}\bar{\alpha} - i a^\mu P_\mu\right] \cdot \exp\left[\alpha Q\right] \,. \qquad (9)$$

   (a) How do S_L and S_R relate to S from equation (1)?

   (b) Show that $\Phi(x^\mu, \theta, \bar{\theta}) = \Phi_L(x^\mu + i\,\theta\sigma^\mu\bar{\theta}, \theta, \bar{\theta}) = \Phi_R(x^\mu - i\,\theta\sigma^\mu\bar{\theta}, \theta, \bar{\theta})$.

(c) (*) You may use these relations in (d). Derive them if you like.

$$S(a^\mu, \alpha, \bar{\alpha}) \left[\Phi_L(x^\mu, \theta, \bar{\theta})\right] = \Phi_L(x^\mu + a^\mu + 2i\,\theta\sigma^\mu\bar{\alpha} + i\,\alpha\sigma^\mu\bar{\alpha},\ \theta+\alpha,\ \bar{\theta}+\bar{\alpha})\,, \quad (10)$$

$$S(a^\mu, \alpha, \bar{\alpha}) \left[\Phi_R(x^\mu, \theta, \bar{\theta})\right] = \Phi_R(x^\mu + a^\mu - 2i\,\alpha\sigma^\mu\bar{\theta} - i\,\alpha\sigma^\mu\bar{\alpha},\ \theta+\alpha,\ \bar{\theta}+\bar{\alpha})\,. \quad (11)$$

(d) What form do Q_α, $\bar{Q}_{\dot{\alpha}}$ and P_μ take for the S_L representation?
(Proceed as in question 2, use (8). Optionally, do the same for S_R.)

(e) (*) For completeness, here are the covariant derivatives. You could check that they commute with an infinitesimal SUSY transformation.

$$D_{(L)\alpha} = \partial_\alpha + 2i\,\sigma^\mu_{\alpha\dot{\beta}}\,\bar{\theta}^{\dot{\beta}}\,\partial_\mu\,, \qquad \bar{D}_{(L)\dot{\beta}} = -\bar{\partial}_{\dot{\beta}}\,, \quad (12)$$

$$D_{(R)\alpha} = \partial_\alpha\,, \qquad \bar{D}_{(R)\dot{\beta}} = -\bar{\partial}_{\dot{\beta}} - 2i\theta^\alpha\sigma^\mu_{\alpha\dot{\beta}}\,\partial_\mu\,. \quad (13)$$

5. (a) Calculate the effect of an infinitesimal pure SUSY transformation $S(0, \alpha, \bar{\alpha})$ on the component fields $(\varphi, \psi, \bar{\chi}, M, N, \ldots)$ of the general scalar superfield Φ (see below) to obtain the transformation rules for each of them.

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) \;=\;& \varphi(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) \\ & + \theta\theta\, M(x) + \bar{\theta}\bar{\theta}\, N(x) + \theta\sigma^\mu\bar{\theta}\, V_\mu(x) \\ & + \theta\theta\,\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\,\theta\xi(x) \\ & + \theta\theta\,\bar{\theta}\bar{\theta}\, D(x)\,. \end{aligned} \quad (14)$$

(A written solution will be provided next week to compare all the signs, etc.)

(b) The general scalar superfield is a reducible representation of the SUSY algebra. Making a choice of $\bar{D}_{\dot{\alpha}}\Phi = 0$, we find the following irreducible representation, the *chiral superfield*, which we can write in the form corresponding to S_L as

$$\Phi_L(x, \theta) = \varphi(x) + \theta\psi(x) + \theta\theta\, F(x)\,. \quad (15)$$

How do its component fields transform?

(c) Show that the product of two left chiral superfields is again a left chiral superfield. Write out the product of three left chiral superfields in the component notation of (15).