Physikalisches Institut
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Exercise 5
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SS 06

Exercises on Elementary Particle Physics II

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1. *Vector superfields and gauge transformations*

   (a) Check that

$$\begin{aligned} V(x,\theta,\bar\theta) \;=\;& C(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) \\ &+\frac{1}{2}i\theta\theta\,[M(x)+iN(x)] - \frac{1}{2}i\bar\theta\bar\theta\,[M(x)-iN(x)] \\ &+\theta\sigma^\mu\bar\theta V_\mu(x) \\ &+i\theta\theta\bar\theta\left[\bar\lambda(x)+\frac{i}{2}\bar\sigma^\mu\partial_\mu\chi(x)\right] - i\bar\theta\bar\theta\theta\left[\lambda(x)+\frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right] \\ &+\frac{1}{2}\theta\theta\bar\theta\bar\theta\left[D(x)-\frac{1}{2}\partial_\mu\partial^\mu C(x)\right] \end{aligned}$$

   is a vector superfield, i.e. it satisfies $(V(x,\theta,\bar\theta))^\dagger = V(x,\theta,\bar\theta)$.
   Hint: $C(x)$, $M(x)$, $N(x)$ and $D(x)$ are real scalar fields.

   (b) Consider the left-chiral superfield $\Lambda(x,\theta)$, written in the left-chiral representation as $\Lambda_L(x,\theta)$:

$$\Lambda_L(x,\theta) = \Lambda(x) + \sqrt{2}\theta\Psi_\Lambda(x) + \theta\theta F_\Lambda(x)$$

   Show that the combination $i(\Lambda - \Lambda^\dagger)$ is a vector superfield. Evaluate the expression and identify the $\theta\sigma^\mu\bar\theta$ coefficient.
   So the vector superfield $V(x,\theta,\bar\theta)$ transforms under a U(1) gauge transformation as

$$V \to V' = V + i(\Lambda - \Lambda^\dagger).$$

   (c) By choosing the so called Wess-Zumino gauge for $V(x,\theta,\bar\theta)$, the following component fields can be eliminated: $C(x) = \chi(x) = M(x) = N(x) = 0$. Calculate V_{WZ}, V_{WZ}^2 and V_{WZ}^3.

2. *Pure supersymmetric abelian Yang-Mills*

(a) The generalization of the field strength is given by the chiral superfield:

$$W_\alpha = \bar{D}\bar{D}D_\alpha V$$

Show that W_α is a left chiral superfield and that it is gauge invariant. Write W_α in components.

Hint: Use the Wess-Zumino gauge. Bring V_{WZ} to the left-chiral representation and use the left-chiral representation also for the SUSY-covariant derivatives D_α and $\bar{D}_{\dot{\beta}}$.

(b) Show that the F-term of $W^\alpha W_\alpha$ contains the kinetic terms of both, the gauge boson and the gaugino and a $D^2(x)$ term.