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Exercise 7
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SS 08

Exercises on Elementary Particle Physics II

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1. *The SUSY invariant action and the Wess-Zumino Model*

   The most general form of a SUSY-invariant action involving one chiral multiplet encoded in the left-chiral[1] superfield ϕ is given by

   $$S = \int \mathrm{d}^4x \left[\int d^2\theta d^2\bar{\theta} K(\phi, \phi^\dagger) + \int \mathrm{d}^2\theta \; W(\phi) + \text{h.c.} \right], \tag{1}$$

   where K as well as W are arbitrary functions up to now. The integration ensures that only the highest components of the superfield occurs in the action. Therefore, the first term is called the **D-term** and the second (and third) one the **F-term**. Note that $K(\phi, \phi^\dagger)$ is no more chiral as it is assumed to be a real **vector superfield**. First of all let us analyze the general action (1) for a while.

   (a) Why is $W(\phi)$ again a left-chiral superfield assuming ϕ is? Show that (1) is SUSY-invariant by applying an infinitesimal SUSY-transformation $\delta_{(\epsilon,\bar{\epsilon})} = \epsilon Q + \bar{Q}\bar{\epsilon}$.
   Hint: Use the explicit form for Q, $\bar{Q}$ of Ex. 6.1, (c) and (g) as well as the integration rules for Grassmann variables of Ex. 5.2. The Lagrangian in (1) is invariant up to a total derivative.

   (b) Switch to the left-chiral representation $\phi_L(x,\theta) = \varphi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x)$ of the left-chiral superfield ϕ. Determine the Taylor expansion of $W(\phi_L)$ in θ about the scalar component φ of ϕ_L. Which terms occur in the action and how do you have to constrain W in order to obtain a remormalizable action?

   Thus, W contains only potential terms of the Lagrangian in (1). Consequently, it is called the **superpotential**. The kinetic terms are contained in K. The simplest choice is $K(\phi, \phi^\dagger) = \phi\phi^\dagger$.

   (c) Assuming a left-chiral ϕ show that $\phi^\dagger$ is right-chiral. Choosing $\phi \equiv \phi_L$ check that $[\phi_L(x,\theta)]^\dagger$ transforms in the right-chiral representation.

   (d) Using the relations between S and $S_{L,R}$ of in Ex. 6.1, (f)) determine the relations between ϕ, ϕ_L and ϕ_R?
   Hint: First, try to relate the expressions for Q, $\bar{Q}$ in the various representations by shifting the variable $x^\mu \mapsto x^\mu - i\theta\sigma^\mu\bar{\theta}$ in ϕ_L. Then use Ex. 6.1, (f).

[1] It is a common terminology to call a left-(/right-)chiral superfield a (anti-)chiral superfields.

(e) Evaluate $\phi\phi^\dagger$ in the left-chiral representation. Note that this is not chiral, thus, you have to use (d) to convert representations of superfields into each other.

(f) Combine the results to obtain the complete action S of (1). Why does S not change under the shift of x^μ necessary to convert from the left-chiral to the normal representation S? Solve the purely algebraic equation of motion of the **auxiliary field** F and insert the result into S. Why is the scalar potential

$$V = \sum_i \left| \frac{\partial W}{\partial \varphi_i} \right|^2 , \tag{2}$$

where we generalized slightly by considering several superfields ϕ_i, $i = 1, \ldots, n$? In this case we have to consider $K(\phi_i, \phi_i^\dagger) = \sum_i \phi_i \phi_i^\dagger$ as well as $W \equiv W(\phi_i)$.

Now, we are prepared to consider the simplest example, the **Wess-Zumino model**, defined by

$$W = \tfrac{1}{2} m \phi^2 + \tfrac{1}{3} g \phi^3, \qquad K(\phi, \phi^\dagger) = \phi\phi^\dagger. \tag{3}$$

(g) As an additional practice calculate S directly using the left-chiral representation. Solve the equation of motion for F explicitly. Compare your results to (f). Determine the scalar potential $V(\varphi)$.

(h) Compare the masses of the fermions and bosons as well as the Yukawa coupling constant with the bosonic coupling constant. Compare to the results in the context of the Higgs mass correction in Ex. 5.2.
Hint: Take care of the canonical normalization for the kinetic terms and consider the equations of motion to fix the right normalization of the mass terms in case you are uncertain.