
Exercises on Theoretical Astroparticle Physics

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1. Nucleosynthesis

The temperature of the universe $10^{-4}s$ after the big bang was about $100MeV$. At this moment, apart from electrons, neutrinos and photons, already bound quark states existed in form of protons and neutrons. Due to the high temperature protons and neutrons were in thermal equilibrium with electrons and electron neutrinos. The number densities of protons and neutrons in the non-relativistic case is given by the following

$$n_{p,n} = 2 \left(\frac{m_{p,n}}{2\pi T} \right)^{3/2} \exp \left(-\frac{m_{p,n}}{T} \right). \quad (1)$$

- How high was the temperature in Kelvin?
- What are the reactions which convert p into n and vice versa?
- If the reaction rate drops below the Hubble parameter, the assumption of thermal equilibrium does not hold anymore. For the reaction $\nu_e + n \leftrightarrow p + e^-$, this happened about $1s$ after the big bang at a temperature of approximately $0.8MeV$. Compute the neutron-proton ratio at this moment.
- About $300s$ after the big bang the temperature dropped below the critical temperature for deuteron photo-disintegration ($d + \gamma \rightarrow p + n$) such that stable helium nuclei could form. What was the neutron-proton ratio at this point in time?
- Compute the expected amount of helium with respect to the total mass of visible matter in the universe under the assumption that the predominant part of the neutrons were bound in helium. What would have been the consequences for the further development of the universe if the lifetime of the free neutron would be much smaller than $100s$?

2. Baryogenesis

In nucleosynthesis a finite baryon number density is assumed to exist.

- Estimate the baryon number density at $T \ll m_p$ if the universe contained equal numbers of baryons and antibaryons. *Hint: You can assume a constant baryon antibaryon annihilation cross section is about $3 \cdot 10^{-27}cm^2 (= 30mb)$.*
- Evidently this baryon density is much too small to explain BBN (not to mention the fact that we don't see any antibaryons around us). We conclude that the (visible part of the) universe must contain many more baryons than antibaryons. How large would the baryon asymmetry

$$A_b = \frac{n_b - n_{\bar{b}}}{n_b + n_{\bar{b}}} \quad (2)$$

have been at $T \geq m_p$? *Hint: Recall that the baryon-to-photon ration at BBN was around 10^{-9} .*

- (c) The result in (b) makes imposing the finite A_b as a boundary condition on the very early universe quite unattractive. Besides, this cannot be reconciled with the ideas of inflation. Andrei Sakharov showed in 1967 that any baryon asymmetry generating baryogenesis must satisfy three conditions: (1) It must violate baryon number: (2) It must violate both charge conjugation and the product of charge and parity symmetry: (3) It must have occurred out of thermal equilibrium. Show that these conditions are indeed necessary.