Physikalisches Institut
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Theoretische Physik

Exercises 3
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Exercises on Theoretical Particle Physics II

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In this exercise sheet we learn how to construct SUSY invariant actions. Starting with a left-chiral superfield ϕ, we can write down the most general SUSY invariant action as

$$S = \int d^4x \left[\int d^2\theta d^2\bar{\theta} K(\phi, \phi^\dagger) + \int d^2\theta W(\phi) + \text{h.c.} \right]. \tag{1}$$

The function K is the *Kähler potential* and W is the *superpotential*. The hermitian conjugate is added in order to make the expression real.

3.1 The Superpotential (*5 credits*)

In this exercise we check that the superpotential is indeed SUSY invariant and investigate its structure. From sheet 1 we know that the expansion of a field in θ is finite. The left-chiral superfield ϕ is commonly expanded as:

$$\phi_L(x, \theta) = \varphi(x) + \sqrt{2}\theta\psi(x) + F(x)\theta\theta, \tag{2}$$

where φ and F are bosonic fields and ψ is fermionic.

(a) Check that an arbitrary function of left-chiral superfields is again a left-chiral superfield. In particular, $W(\phi)$ as a holomorphic function of ϕ is a left-chiral superfield. (*1 credit*)

(b) Show that the part containing the superpotential in (1) is indeed SUSY invariant up to a total derivative (why doesn't this matter in the action?). To show this, use the infinitesimal SUSY trafo $\delta_{(\epsilon,\bar{\epsilon})} = \epsilon Q + \bar{Q}\bar{\epsilon}$, with Q and $\bar{Q}$ in left-chiral representation. (*1 credit*)

(c) Let us take the superpotential

$$W = m\phi^2 + \lambda\phi^3. \tag{3}$$

Calculate all occuring terms in the left-chiral representation using (2).(*3 credits*)

3.2 The Kähler potential *(8 credits)*

In this exercise we look at the simplest Kähler potential $K = \phi\phi^\dagger$ and investigate its properties.

(a) Show that the part containing K in (1) is also SUSY invariant up to a total derivative. *(1 credit)*

(b) Show that $[\phi_L(x,\theta)]^\dagger$ transforms in the right-chiral representation of the SUSY algebra. Argue that $V = \phi\phi^\dagger$ is a vector superfield. *(1 credit)*

(c) Use the relations between S, S_L, and S_R to determine the relations between ϕ, ϕ_L, and ϕ_R. *(3 credits)*

(d) Calculate all terms coming from $K = \phi\phi^\dagger$. Use again the left-chiral representation. *(3 credits)*

3.3 The Wess-Zumino model *(8 credits)*

By combining the results of the previous two exercises we can build the Wess-Zumino model:

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\phi\phi^\dagger + \int d^2\theta \left[m\phi^2 + \lambda\phi^3\right] + \text{h.c.}. \tag{4}$$

Let us investigate its properties:

(a) In the first two exercises, we used left-chiral fields, while the Lagrangean density (4) is written in terms of non-chiral fields. Argue that the shift which connects the two representations (derived in 3.2)c)) does not change the Lagrange density. *(1 credit)*

(b) The F-field in (2) is a so-called auxiliary field, i.e. its equation of motion (EOM) is purely algebraic. Calculate its EOM and use the result to eliminate F from the Lagrange density given in (4). *(2 credits)*

(c) Show that the scalar potential $V(\phi)$ is obtained from the superpotential via

$$V(\phi) = \left|\frac{\partial W(\varphi)}{\partial\varphi}\right|^2, \tag{5}$$

where $W(\varphi)$ means to take the $\theta = 0$ part of the superpotential $W(\phi)$.*(1 credit)*

(d) Calculate the equations of motions from the Lagrangean density (4) for φ and ψ. Read off the masses of the fields. What do you observe? *(4 credits)*