Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercises 4
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SS 2014

Exercises on Theoretical Particle Physics II

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8. The Wess-Zumino model (*8 credits*)

By combining the results from the last exercise sheet we can build the Wess-Zumino model:

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\phi\phi^\dagger + \int d^2\theta \left[m\phi^2 + \lambda\phi^3\right] + \text{h.c.}. \tag{1}$$

Let us investigate its properties:

(a) We used left-chiral fields, while the Lagrange density (1) is written in terms of non-chiral fields. Argue that the shift which connects the two representations does not change the Lagrange density. (*2 credits*)

(b) The F-field is a so-called auxiliary field, i.e. its equation of motion (EOM) is purely algebraic. Calculate its EOM and use the result to eliminate F from the Lagrange density given in (1). (*2 credits*)

(c) Calculate the equations of motions from the Lagrange density (1) for φ and ψ. Read off the masses of the fields. What do you observe? (*4 credits*)

9. Super Yang-Mills theory and coupling to matter (*12 credits*)

The way matter couplings are realized in supersymmetric actions will guide us to the **non-Abelian** generalization of the gauge invariant action we already studied. Consider a chiral superfield Φ transforming under a global symmetry

$$\Phi \mapsto \Phi' = e^{-\mathrm{i}\lambda^a T_a}\Phi, \qquad \lambda^a \in \mathbb{R}, \quad a = 1, \ldots, \dim(\mathfrak{g}), \tag{2}$$

of a Lie algebra $\mathfrak{g}$ with generators T_a. In order to gauge this symmetry consistently the transformed superfield Φ' has to remain chiral.

1. Check that (2) respects the chirality of Φ for $\lambda \in \mathbb{R}$ constant and for $\lambda = \Lambda(x, \theta)$ a complete chiral superfield. Although $W(\Phi)$ can be arranged to be gauge invariant, $\Phi^\dagger\Phi$ cannot. Determine its transformation behaviour. (*3 credits*)

2. In order for this to be gauge invariant introduce a minimal coupling of the vector superfield to the matter contained in the chiral superfield of the form

$$\mathcal{L}_{\text{matter}} \supset \Phi^\dagger e^V \Phi\big|_{\theta^2\bar{\theta}^2}, \qquad V = V^a T_a, \quad a = 1, \ldots, \dim(\mathfrak{g}). \tag{3}$$

   Determine the right transformation property of e^V for gauge invariance. What is the first order transformation of V? (*3 credits*)

3. Rewrite (3) in the left-chiral represenation by shifting x^μ. This yields $e^{V-2\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu}$. Why do you expect the covariant derivative to appear?

   Calculate the D-term ($\theta^2\bar{\theta}^2$-term) of (3) in the Wess-Zumino (WZ) gauge which means $(V_{\text{WZ}})^n = 0$ for $n \geq 3$ in the left-chiral representation. Identify the covariant derivatives. (*3 credits*)

4. The non-Abelian field strength is defined by

$$W_\alpha = -\tfrac{1}{4}\bar{D}\bar{D}\left(e^{-V}D_\alpha e^{V}\right), \qquad \bar{W}_{\dot{\alpha}} = \tfrac{1}{4}DD\left(e^{V}\bar{D}_{\dot{\alpha}}e^{-V}\right). \tag{4}$$

How does W_α transform under a gauge transformation of e^V? Insert $e^{V_{\mathrm{WZ}}} = 1 + V_{\mathrm{WZ}} + \frac{1}{2}V_{\mathrm{WZ}}^2$ to deduce

$$W_\alpha = -\tfrac{1}{4}\bar{D}\bar{D}D_\alpha V + \tfrac{1}{8}\bar{D}\bar{D}\left[V, D_\alpha V\right].$$

Calculate W_α explicitly. *(3 credits)*