Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercises 7
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SS 2014

# Exercises on Theoretical Particle Physics II

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## 13. More about the MSSM Higgs sector (*12 credits*)

(a) Consider the Higgs scalar potential from exercise 12,

$$V(h,\bar{h}) = m_1^2|h|^2 + m_2^2|\bar{h}|^2 + m_3^2\left(\bar{h}h + \text{h.c.}\right) + \frac{g_1^2+g_2^2}{8}\left(|h|^2 - |\bar{h}|^2\right)^2 + \frac{g_2^2}{2}|h^\dagger \bar{h}|^2$$

with the VEV assignment

$$\langle h^0\rangle = \frac{1}{\sqrt{2}}v_1, \qquad \langle \bar{h}^0\rangle = \frac{1}{\sqrt{2}}v_2, \qquad \langle h^-\rangle = \langle \bar{h}^+\rangle = 0.$$

Use

$$M_Z^2 = \frac{g_1^2+g_2^2}{4}(v_1^2+v_2^2), \qquad \tan\beta = \frac{v_2}{v_1}$$

to obtain the relations

$$\begin{aligned} -2m_3^2 &= (m_{\text{soft},1}^2 - m_{\text{soft},2}^2)\tan 2\beta + M_Z^2 \sin 2\beta \\ \mu^2 &= \frac{m_{\text{soft},2}^2 \sin^2\beta - m_{\text{soft},1}^2\cos^2\beta}{\cos 2\beta} - \frac{1}{2}M_Z^2 \end{aligned}$$

in the minimum of the scalar potential.

(*4 credits*)

(b) In exercise 12 you calculated the mass matrix for the charged Higgs fields

$$m_{h^-,\bar{h}^+}^2 = \begin{pmatrix} m_1^2 + \frac{g_1^2+g_2^2}{4}(\langle h^0\rangle^2 - \langle\bar{h}^0\rangle^2) + \frac{g_2^2}{2}\langle\bar{h}^0\rangle^2 & m_3^2 + \frac{g_2^2}{2}\langle h^0\rangle\langle\bar{h}^0\rangle \\ m_3^2 + \frac{g_2^2}{2}\langle h^0\rangle\langle\bar{h}^0\rangle & m_2^2 - \frac{g_1^2+g_2^2}{4}(\langle h^0\rangle^2 - \langle\bar{h}^0\rangle^2) + \frac{g_2^2}{2}\langle h^0\rangle^2 \end{pmatrix}.$$

Use your results from part (a) to get

$$m_{h^-,\bar{h}^+}^2 = \left(\frac{m_3^2}{v_1v_2} + \frac{1}{4}g_2^2\right)\begin{pmatrix} v_2^2 & v_1v_2 \\ v_1v_2 & v_1^2\end{pmatrix}.$$

Calculate the eigenvalues of $m_{h^-,\bar{h}^+}^2$ and interpret the result.

(*2 credits*)

(c) Repeat the analysis from part (b) for the mass matrix for the two fields $\mathrm{Im}(h^0)$ and $\mathrm{Im}(\bar{h}^0)$. The mass matrix from exercise 12 was

$$m^2_{\mathrm{Im}(h^0),\mathrm{Im}(\bar{h}^0)} = \begin{pmatrix} m_1^2 + \frac{g_1^2+g_2^2}{4}(\langle h^0\rangle^2 - \langle \bar{h}^0\rangle^2) & m_3^2 \\ m_3^2 & m_2^2 - \frac{g_1^2+g_2^2}{4}(\langle h^0\rangle^2 - \langle \bar{h}^0\rangle^2) \end{pmatrix}.$$

Show that the mass of the pseudoscalar Higgs is given by

$$m^2_{A^0} = \frac{2m_3^2}{\sin 2\beta}.$$

(*2 credits*)

(d) Use the Higgs scalar potential and calculate the $2 \times 2$ mass matrix for the fields $\mathrm{Re}(h^0)$ and $\mathrm{Re}(\bar{h}^0)$. Insert the VEVs for the Higgs fields and calculate the eigenvalues which are $m^2_{h^0}$ and $m^2_{H^0}$. Your result can be written as

$$m^2_{H^0,h^0} = \frac{1}{2}\left(m^2_{A^0} + M_Z^2 \pm \sqrt{\left(m^2_{A^0} + M_Z^2\right)^2 - 4M_Z^2 m^2_{A^0}\cos^2 2\beta}\right).$$

(*3 credits*)

(e) Show that the lightest Higgs mass $m_{h^0}$ has an upper bound given by

$$m_{h^0} < \min(m_{A^0}, M_Z)|\cos 2\beta|$$

at tree level. Insert $M_Z = 91.19$ GeV and interpret your result.

(*1 credit*)

**14. Radiative corrections to the Higgs mass** (*8 credits*)

(a) Use the MSSM superpotential given in exercise 12 and calculate the top and the stop mass arising from $F$-terms. Neglect subleading $D$-term contributions and possible $\tilde{t}_R - \tilde{t}_L$ mixing. Add a universal soft mass $\tilde{m}$ for the stops to your result.

(*1 credit*)

(b) The one loop correction to the Higgs mass from top and stop loops can be deduced from the effective potential

$$\Delta V = \frac{1}{64\pi^2}\mathrm{str}\; m^4(h,\bar{h})\left(\log\frac{m^2(h,\bar{h})}{Q^2} - \frac{3}{2}\right).$$

Use your result from part (a) to obtain

$$\Delta V = \frac{3}{16\pi^2}\Bigg((\tilde{m}^2 + (y_U^t)^2|\bar{h}^0|^2)^2\left(\log\frac{\tilde{m}^2 + (y_U^t)^2|\bar{h}^0|^2}{Q^2} - \frac{3}{2}\right)$$
$$-((y_U^t)^2|\bar{h}^0|^2)^2\left(\log\frac{(y_U^t)^2|\bar{h}^0|^2}{Q^2} - \frac{3}{2}\right)\Bigg).$$

Why is $\Delta V = 0$ if $\tilde{m} = 0$?

(*2 credits*)

(c) Repeat the analysis from part (d) of exercise 13 and show that

$$m^2_{h^0,H^0} = \frac{1}{2}\Bigg( m^2_{A^0} + M_Z^2 + \frac{3G_F m_t^4}{\sqrt{2}\pi^2 \sin^2\beta} \log\frac{m^2_{\tilde{t}}}{m_t^2} \mp \sqrt{\left(m^2_{A^0}+M_Z^2\right)^2 \sin^2 2\beta + \left(\frac{M_Z^2 - m^2_{A^0}}{\cos^{-1} 2\beta} + \frac{3G_F m_t^4}{\sqrt{2}\pi^2 \sin^2\beta}\log\frac{m^2_{\tilde{t}}}{m_t^2}\right)^2}\,\Bigg) .$$

Use $y_U^t = \frac{m_t\sqrt{2\sqrt{2}G_F}}{\sin\beta}$ and neglect $\log(Q^2)$ terms as they can be absorbed in the renormalization. Note further that in contrast to exercise 13 the minimum for electroweak symmetry breaking gets shifted. Therefore the mass relation in the minimum gets changed to

$$m_2^2 = \frac{g_1^2+g_2^2}{8}(v_1^2 - v_2^2) + m_3^2\frac{v_1}{v_2} - \frac{6(y_U^t)^2}{16\pi^2}\left(m^2_{\tilde{t}}\left(\log\frac{m^2_{\tilde{t}}}{Q^2} - 1\right) - m_t^2\left(\log\frac{m_t^2}{Q^2}-1\right)\right) .$$

*(4 credits)*

(d) Use the result from part (c) and enter $G_F = 1.166\cdot 10^{-5}$ GeV together with the new result for the top quark mass (arXiv:1403.4427) $m_t = 173.34$ GeV. Assume further $m_{\tilde{t}} = \mathcal{O}(1)$ TeV, $m_{A^0} = \mathcal{O}(100)$ GeV and $\tan\beta = \mathcal{O}(10)$. What do you observe for $m_{h^0}$? Interpret your result.

*(1 credit)*