Exercises on Theoretical Particle Physics II

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22. Reduction of 11 dimensional SUGRA (*20 credits*)

(a) Use the 11 dimensional vielbein ansatz

$$E^A{}_M = \begin{pmatrix} e^{\beta\phi} e^a{}_\mu & 0 \\ e^{\alpha\phi} A_\mu & e^{\alpha\phi} \end{pmatrix}$$

where A and M run from 0 to 10. Further a and μ run from 0 to 9. M and μ are indices for curved coordinates and A and a for flat coordinates. $e^a{}_\mu$ is a 10 dimensional vielbein, ϕ is a scalar field and A_a a 10 dimensional abelian gauge field. Calculate the inverse vielbein $E^M{}_A$. Determine the metric with the help of

$$G_{MN} = E_M{}^A \eta_{AB} E^B{}_N$$

where η_{AB} is the 11 dimensional Minkowski metric. This result is also true for the 10 dimensional vielbeins.

(*2 credits*)

(b) Use the definition of one-forms

$$E^A = E^A{}_M dx^M, \qquad e^a = e^a{}_\mu dx^\mu, \qquad A = A_\mu dx^\mu$$

to find

$$E^{10} = e^{\alpha\phi}(dx^{10} + A), \qquad E^a = e^{\beta\phi} e^a.$$

Use this result to read of the components of the two-form ω from

$$dE^M = -\omega^M{}_N \wedge E^N, \qquad de^\mu = -\hat{\omega}^\mu{}_\nu \wedge e^\nu$$

which is valid if torsion is vanishing. Your result should be

$$\omega^{10}{}_\mu = \alpha \partial_\mu \phi e^{-\beta\phi} E^{10} + \frac{1}{2} e^{(\alpha - 2\beta)\phi} F_{\mu\nu} E^\nu,$$
$$\omega^\mu{}_\nu = \hat{\omega}^\mu{}_\nu - \beta e^{-\beta\phi} (\partial^\mu \phi E_\nu - \partial_\nu \phi E^\mu) - \frac{1}{2} e^{(\alpha - 2\beta)\phi} F^\mu{}_\nu E^{10}$$

with

$$dA = F = \frac{1}{2} F_{\mu\nu} e^\mu \wedge e^\nu.$$

(*4 credits*)

(c) Use your result from part (b) together with

$$R^M{}_N = d\omega^M{}_N + \omega^M{}_P \wedge \omega^P{}_N, \qquad r^\mu{}_\nu = d\hat{\omega}^\mu{}_\nu + \hat{\omega}^\nu{}_\sigma \wedge \hat{\omega}^\sigma{}_\nu$$

to calculate $R^{10}{}_\mu$ and $R^\mu{}_\nu$.

(*4 credits*)

(d) Use

$$R^M{}_N = \frac{1}{2} R^M{}_{NOP} E^O \wedge E^P$$

and your result from part (c) to read of $R = 2R^{10}{}_{\mu 10}{}^\mu + R^\mu{}_{\nu\mu}{}^\nu$. Show that the term which involves the connection ω vanishes if $\alpha = -9\beta$.

(*4 credits*)

(e) Enter the result from part (d) into

$$S_{11} \supset -\frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-G} R$$

and integrate to obtain the 10 dimensional action. Reduce also $\sqrt{-G}$ to the 10 dimensional determinant.

(*2 credits*)

(f) The full bosonic part of the 11 dimensional SUGRA action can be written as

$$S_{11} = -\frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-G} R - \frac{1}{2\kappa_{11}^2} \int F_4 \wedge *F_4 + \frac{1}{12\kappa_{11}^2} \int A_3 \wedge F_4 \wedge F_4.$$

Try to reduce also the two last terms to 10 dimensions. Maybe it is a good idea to check the literature to find suitable tricks. The full result is the bosonic part of the IIA SUGRA action in 10 dimensions.

(*4 credits*)