Physikalisches Institut
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Theoretische Physik

Exercise 01
18. October 2011
WS 11/12

Exercises on String Theory I

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–Home Exercises–
Due 25. November 2011

Exercise 1.1: Solutions to classical string e.o.m. *10 Credits*

1. Start with the fixed-metric Polyakov action

$$S = \int \mathrm{d}^2\sigma \left(\dot{X}_\mu \dot{X}^\mu - X'_\mu X'^\mu \right) ,$$

to derive the string equation of motion $\partial_+\partial_- X_\mu = 0$ with $\partial_\pm = \partial/\partial\sigma_\pm$, $\sigma_\pm = \tau \pm \sigma$.
(*1 credit*)

2. Show that $X^\mu(\sigma,\tau) = X^\mu_L(\sigma_+) + X^\mu_R(\sigma_-)$ with

$$X^\mu_L(\sigma_+) = \frac{1}{2}x^\mu + \frac{1}{2}l_\mathrm{S}^2\, p^\mu \sigma_+ + \frac{\mathrm{i}}{2} l_\mathrm{S} \sum_{n\neq 0} \frac{1}{n}\tilde{\alpha}^\mu_n e^{-2\mathrm{i}n\sigma_+}$$
$$X^\mu_R(\sigma_-) = \frac{1}{2}x^\mu + \frac{1}{2}l_\mathrm{S}^2\, p^\mu \sigma_- + \frac{\mathrm{i}}{2} l_\mathrm{S} \sum_{n\neq 0} \frac{1}{n}\alpha^\mu_n e^{-2\mathrm{i}n\sigma_-}$$

is indeed the general solution to $\partial_+\partial_- X^\mu = 0$ and the boundary conditions $X^\mu(\sigma+\pi,\tau) = X^\mu(\sigma,\tau)$. *Hint: Show that the general solution splits into left– and right–mover. Fourier transform their derivatives. Then integrate and use boundary conditions.*
(*3 credits*)

3. Find the mode expansion for twisted closed strings, which are defined by the boundary condition

$$X^\mu(\sigma+\pi,\tau) = -X^\mu(\sigma,\tau) .$$

(*3 credits*)

4. Find the mode expansion for Neumann–Dirichlet open strings with boundary conditions

$$X^\mu(0,\tau) = 0 \qquad \text{Dirichlet at } \sigma = 0 ,$$
$$\partial_\sigma X^\mu(\sigma,\tau)\Big|_{\sigma=\pi} = 0 \qquad \text{Neumann at } \sigma = \pi .$$

(*3 credits*)

Exercise 1.2: String Quantization *10 Credits*

Verify that the canonical Poisson brackets

$$[P^\mu(\sigma,\tau)\,, X^\nu\,(\sigma',\tau)]_{\mathrm{PB}} = \eta^{\mu\nu}\delta(\sigma-\sigma')\ , \qquad [P^\mu, P^\nu]_{\mathrm{PB}} = [X^\mu, X^\nu]_{\mathrm{PB}} = 0$$

lead to the algebra of the α's,

$$[\alpha^\mu_m, \alpha^\nu_n]_{\mathrm{PB}} = [\tilde{\alpha}^\mu_m, \tilde{\alpha}^\nu_n]_{\mathrm{PB}} = \mathrm{i}m\eta^{\mu\nu}\delta_{m+n,0}\,, \qquad [\alpha^\mu_m, \tilde{\alpha}^\nu_n]_{\mathrm{PB}} = 0\,.$$

Hint: What is the oscillator expansion of the canonical momentum $P^\mu := \delta S/\delta\dot{X}_\mu$? Express α^μ_n, $\tilde{\alpha}^\mu_n$ as linear combinations of $X^\mu(\tau,\sigma)$ and $P^\mu(\tau,\sigma')$ for fixed τ.