Physikalisches Institut
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Theoretische Physik

Exercise 08
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WS 11/12

Exercises on String Theory I

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Due 20. December 2011

Exercise 8.1: D0 branes from M–Theory (*3 credits*)

On exercise sheet 6, we compactified 11 dimensional supergravity (SUGRA) on a circle to get the 10 dimensionals Type IIA supergravity description. The 11 dimensional SUGRA descends from M–Theory. On this exercise sheet, we will use the correspondence between M–Theory and Type IIA theory to investigate non–perturbative states in Type IIA, the D0 branes. Remember from exercise sheet 6 that we made the ansatz for the metric

$$G_{MN} = e^{-\frac{2}{3}\Phi} \begin{pmatrix} g_{\mu\nu} + e^{2\Phi} A_\mu A_\nu & e^{2\Phi} A_\mu \\ e^{2\Phi} A_\nu & e^{2\Phi} \end{pmatrix}, \quad G^{MN} = e^{\frac{2}{3}\Phi} \begin{pmatrix} g^{\mu\nu} & -A^\mu \\ -A^\nu & e^{-2\Phi} + A_\rho A^\rho \end{pmatrix}. \quad (1)$$

in string frame normalization.

(a) Use (1) with $e^\Phi = g_s$, the result from 6.2(b), and

$$16\pi\kappa_{11} = \frac{1}{2\pi}(2\pi\ell_P)^9, \qquad 16\pi\kappa_{10} = \frac{1}{2\pi}(2\pi\ell_s)^8 g_s^2, \quad (2)$$

(with ℓ_P the 11D Planck length) to express the compactification radius R of the circle in terms of the string length ℓ_s and the string coupling g_s. (*2 credits*)

(b) The D0 branes of Type IIA in 10D arise as the first Kaluza-Klein excitations of the massless supergravity multiplet. Calculate the mass of a D0 brane in terms of the string length and the string coupling. (*1 credit*)

Exercise 8.2: BPS states (*10 credits*)

Consider $\mathcal{N} = 2$ SUSY $(A, B = 1, 2)$ of massive particles in the presence of central charges:

$$\{Q^A_\alpha, \overline{Q}^B_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \delta^{AB}, \qquad \{Q^A_\alpha, Q^B_\beta\} = 2\varepsilon_{\alpha\beta} Z^{AB}.$$

Here, P_μ is the momentum and σ^μ are the Pauli matrices.

(a) Why is possible to write $Z^{AB} = 2\varepsilon^{AB} Z$? (*1 credit*)

(b) Show that the operator Z commutes with all the other elements of the algebra (i.e. it is a central charge). (*3 credits*)

We define

$$S^{\pm}_{\alpha} = Q^{1}_{\alpha} \pm \varepsilon_{\alpha\beta}\overline{Q}^{2}_{\beta}\,, \qquad (S^{\pm}_{\alpha})^{\dagger} = \overline{Q}^{1}_{\alpha} \pm \varepsilon_{\alpha\beta}Q^{2}_{\beta}\,,$$

(c) Calculate the non–vanishing anti–commutators of $S^{\pm}$ and $(S^{\pm})^{\dagger}$. For simplicity, you may assume $Z \geq 0$ and perform the calculation in the particle rest frame. (*4 credits*)

(d) Use the anti–commutation relations to derive a lower bound (the BPS bound) on the mass in terms of the central charge. Argue that when the BPS bound is saturated (the mass exactly saturates the BPS inequality), short multiplets arise. (*2 credits*)