Physikalisches Institut
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Theoretische Physik

Exercises 11
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Exercises on Theoretical Particle Physics I

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20. More about electroweak interactions (*6 credits*)

(a) Use the definitions from part (a) of exercise 9 and the covariant derivative from part (h) of exercise 9 to rewrite the kinetic terms

$$\mathscr{L}_{\text{Leptons}} = \bar{R}(i\gamma^\mu D_\mu)R + \bar{L}(i\gamma^\mu D_\mu)L.$$

(*2 credits*)

(b) Use part (a) to verify the vertex Feynman rule for Z bosons as given in part (a) of exercise 14. What is the result for the coupling to neutrinos?

(*2 credits*)

(c) Let us introduce Standard Model quarks like

	$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$D = d_R$	$U = u_R$
Hypercharge Y	$1/3$	$-2/3$	$4/3$
$\mathrm{SU}(2)_L$ rep.	**2**	**1**	**1**
Lorentz rep.	$(1/2, 0)$	$(0, 1/2)$	$(0, 1/2)$

Repeat the analysis of part (a) for the quarks using the kinetic terms

$$\mathscr{L}_{\text{Quarks}} = \overline{Q}(i\gamma^\mu D_\mu)Q + \overline{D}(i\gamma^\mu D_\mu)D + \overline{U}(i\gamma^\mu D_\mu)U.$$

Use your result to determine the electric charges of the up and down quarks.

(*2 credits*)

21. Flavor in the Standard Model *(14 credits)*

(a) The Standard Model consists of three families of quarks and leptons. This means we have to promote the introduced fields with a flavor index, for example like $L \to L_i$. The Yukawa couplings for the leptons look now like

$$\mathscr{L}_{\text{Yukawa}} \supset -G_e^{ij} \bar{L}_i \Phi R_j - \text{h.c.}$$

where we call G_e^{ij} Yukawa matrix. Family indices are contracted with a Kronecker delta. One can diagonalize the Yukawa matrix by a biunitary transformation

$$e_L^i \to U_e^{ij} e_L^j, \qquad e_R^i \to V_e^{ij} e_R^j.$$

Use the unitary gauge to identify the mass term of the electron after spontaneous symmetry breaking.

(2 credits)

(b) Use your result from part (a) of exercise 20 and introduce flavor indices with diagonal structure. Perform a biunitary transformation of the electron on the interaction terms. Show that the interactions with A_μ and Z_μ stay diagonal. Verify that with a transformation of the neutrinos also the interaction with $W_\mu^\pm$ stays diagonal. Why is this transformation on the neutrinos allowed?

(2 credits)

(c) Show that the Yukawa couplings for the down quarks

$$\mathscr{L}_{\text{Yukawa}} \supset -G_d^{ij} \bar{Q}_i \Phi D_j - \text{h.c.}$$

are gauge invariant.

(2 credits)

(d) Show that $\tilde{\Phi} = i\sigma_2 \Phi^*$ transforms as **2** under $\text{SU}(2)_L$. What is the hypercharge of $\tilde{\Phi}$?

(2 credits)

(e) Use part (d) to show that the Yukawa couplings for the up quarks

$$\mathscr{L}_{\text{Yukawa}} \supset -G_u^{ij} \bar{Q}_i \tilde{\Phi} U_j - \text{h.c.}$$

are gauge invariant. Use the unitary gauge to show that these terms result in masses for the up quarks.

(2 credits)

(f) Use part (c) of exercise 20 and show that biunitary transformations of the up and down quarks, similar to the calculation in part (b) for leptons, leave the interactions with A_μ and Z_μ diagonal. What does this imply?

(2 credits)

(g) Check the behaviour of the interaction between quarks and $W_\mu^\pm$ under biunitary transformation. Determine the mixing matrix which is called Cabibbo-Kobayashi-Maskawa matrix V_{CKM}.

(2 credits)