Exercises on Theoretical Particle Physics I

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Due 23.1.2017

22. Angles and phases in the quark sector (*6 credits*)

(a) Show that $MM^\dagger$ is Hermitian for a $N \times N$ matrix M and one can thus write

$$MM^\dagger = SM_d^2 S^\dagger$$

with M_d^2 being a diagonal matrix and S unitary. Show that the right-hand side of this equation has N more free parameters than the left-hand side. Show that this leaves the freedom to transform $S \to SF$ with $F = \mathrm{diag}(e^{i\phi_1}, \ldots, e^{i\phi_N})$.

(*1 credit*)

(b) Show that $V = H^{-1}M$ for a Hermitian matrix $H = SM_d S^\dagger$ is unitary.

(*1 credit*)

(c) Use part (b) to show that one may write $M = SM_d T^\dagger$ with $T = V^\dagger S$ unitary. Identify the number of free parameters in this relation.

(*1 credit*)

(d) The CKM matrix may be defined as $V_{\mathrm{CKM}} = U_u^\dagger U_d$ with the biunitary transformation matrices U_i and V_i for $i = u, d$ which diagonalize the Yukawa couplings. Use part (c) to show that V_{CKM} has $(N-1)^2$ physical parameters.

(*1 credit*)

(e) In the framework of $\mathrm{U}(N)$ the $(N-1)^2$ physical parameters can be interpreted as mixing angles which are the same as in $\mathrm{SO}(N)$ and complex phases. Show that there are

$$\frac{(N-1)(N-2)}{2}$$

complex phases.

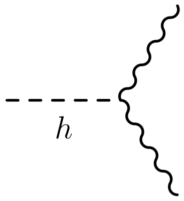
(*1 credit*)

(f) Physical complex phases in the CKM matrix lead to $\mathcal{CP}$ violating processes. What is the minimal amount of families to observe $\mathcal{CP}$ violation in the quark sector?

(*1 credit*)

23. Higgs production in an electroweak process *(14 credits)*

(a) We want to consider the process $e^- e^+ \to Zh$. The Feynman rule for interactions between the Z bosons and the Higgs h is



$$= -i\frac{g}{\cos\theta_{\mathrm{W}}} M_Z \eta^{\mu\nu}.$$

An external Z boson is labeled by a polarization vector $\epsilon_\mu(p)$ if it is incoming and $\epsilon^*_\mu(p)$ if it is outgoing. An external Higgs h contributes with 1. Draw the two possible Feynman graphs at leading order. Which is the dominant one?

(2 credits)

(b) Write down the matrix element $\mathcal{M}$ for the dominant graph from part (a).

(2 credits)

(c) Calculate the squared matrix element for averaged initial spin but fixed Z boson polarization. Assume that the scattering angle is θ and as usual the beam direction is along the z-axis. Neglect the electron mass m_e in your calculation and use the polarization vectors

$$\epsilon^{\pm} = \frac{1}{\sqrt{2}}(0, 1, \pm i\cos\theta, \mp i\sin\theta), \qquad \epsilon^0 = \frac{1}{M_Z}(p, 0, E_Z\sin\theta, E_Z\cos\theta)$$

where p labels the absolute value of the Z boson momentum and E_Z its energy. You will find a different result for each polarization of the Z boson.

(6 credits)

(d) Instead of working with the polarization vectors given in part (c) repeat the analysis using

$$\sum_{\text{polarizations}} \epsilon^*_\mu(p)\epsilon_\nu(p) = -\eta_{\mu\nu} + \frac{p_\mu p_\nu}{M_Z^2}.$$

Show that this agrees with the sum over the three different polarizations obtained in part (c).

(3 credits)

(e) Simplify your result from part (c) by imposing the high energy limit $s \gg (M_Z + m_h)^2$.

(1 credit)