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On the two–loop Yukawa corrections to the MSSM Higgs boson masses at large $\tan\beta$

A. Dedes [a,1], G. Degrassi [b,2], P. Slavich [c,d,3]

[a] *Physik Department, Technische Universität München, D–85748 Garching, Germany*

[b] *Dipartimento di Fisica, Università di Roma III and INFN, Sezione di Roma II, Via della Vasca Navale 84, I–00146 Rome, Italy*

[c] *Institut für Theoretische Physik, Universität Karlsruhe, Kaiserstrasse 12, Physikhochhaus, D–76128 Karlsruhe, Germany*

[d] *Max Planck Institut für Physik, Föhringer Ring 6, D–80805 München, Germany*

Abstract

[1] dedes@ph.tum.de
[2] degrassi@fis.uniroma3.it
[3] slavich@mppmu.mpg.de

1 Introduction

The Minimal Supersymmetric Standard Model (MSSM) [1] naturally accomodates a light Higgs boson [2]. Although the experiments at LEP and the Tevatron could not find a conclusive evidence for it, the experimental search for a light Higgs boson is the major task of Tevatron Run II and of the LHC hadron colliders. Within the MSSM, the tree–level masses of the neutral Higgs bosons can be parameterised in terms of three input parameters: the mass of the CP-odd Higgs m_A, the Z boson mass m_Z and the ratio of the two Higgs vacuum expectation values, $\tan\beta$. At tree level, at least one of the MSSM Higgs bosons is bound to be lighter than the Z boson, thus the failure of detecting it at LEP indicates that the MSSM is a correct theory only after the radiative corrections to the Higgs boson masses have been taken into account.

The radiative corrections arise from loop diagrams involving Standard Model particles and their superpartners. Although the first computations [3] of radiative corrections to the MSSM Higgs masses date back to the eighties, it was first realized in Ref. [4] that the inclusion of the one–loop top/stop corrections at $\mathcal{O}(\alpha_t)$, where $\alpha_t = h_t^2/(4\pi)$ and h_t is the superpotential top coupling, may push the light Higgs mass well above the tree–level bound. In the subsequent years, an impressive theoretical effort has been devoted to the precise determination of the MSSM Higgs masses: full one–loop computations have been provided [5, 6], leading logarithmic effects at two loop have been included via appropriate renormalization group equations [7, 8], and genuine two–loop corrections of $\mathcal{O}(\alpha_t\alpha_s)$ [9, 10, 11, 12, 13], $\mathcal{O}(\alpha_t^2)$ [9, 12, 14], and $\mathcal{O}(\alpha_b\alpha_s)$ [15] have been evaluated in the limit of zero external momentum. The tadpole corrections, needed to justify the requirement of radiative electroweak symmetry breaking, have also been calculated [16] to the same perurbative order. Furthermore, the full two-loop corrections to the MSSM effective potential have been calculated in Ref. [17], together with a first study of the effect of the two–loop corrections to the Higgs masses controlled by the electroweak gauge couplings [18].

The corrections controlled by the top Yukawa coupling dominate over the parameter space unless the parameter $\tan\beta$ is large. In this case the superpotential bottom coupling h_b may also be large, $h_b \simeq h_t$, and at the one–loop level the bottom-sbottom $\mathcal{O}(\alpha_b)$ corrections, where $\alpha_b = h_b^2/(4\pi)$, compete with those of $\mathcal{O}(\alpha_t)$. The two–loop corrections of $\mathcal{O}(\alpha_b\alpha_s)$ *have been first addressed in the masterpiece [15]... ;-)*

The purpose of this article is the calculation of the full two-loop corrections to the Higgs boson masses arising from the Yukawa sector at the order $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ in a mass independent renormalization scheme, such as $\overline{\rm DR}$, as well in an on–shell scheme which we describe in detail. As a byproduct, we also calculate the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections to the minimization conditions of the effective potential. These corrections are of high relevance in the particular region of large $\tan\beta$. Our results for the Higgs masses and tadpoles are available upon request [1] in the form of a Fortran code.

The importance of the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections is threefold: *i)* Tevatron searches for the MSSM Higgs bosons at large $\tan\beta$ exist [19] and our two-loop corrections are of significant relevance in improving the Higgs bounds or otherwise a precise mass upon discovery *ii)* they improve Higgs boson mass predictions in GUT models which predict large $\tan\beta$ values i.e., minimal SO(10) [20] *iii)* they can be incorporated in various existing codes [21, 22] in order to

[1]E-mail: slavich@mppmu.mpg.de

relate high energy $\overline{\rm DR}$ input parameters with accurate predictions for the Higgs boson masses.

The structure of this paper is the following: In section 2 we recall some general issues of the effective potential approach in the calculating the Higgs masses. Section 3 describes our two–loop computation of the $\overline{\rm DR}$ tadpoles and CP-odd, CP-even Higgs mass matrices while section 4 addresses our on-shell renormalization prescription. Numerical results are given in section 5 and in section 6 we conclude with...

2 Higgs masses in the effective potential approach

We begin our discussion by recalling some general results for computation of the MSSM Higgs masses in the effective potential approach. The effective potential, which we write from the start in terms of $\overline{\rm DR}$–renormalized fields and parameters, can be decomposed as $V_{\rm eff} = V_0 + \Delta V$, where V_0 is the tree–level scalar potential and ΔV contains the radiative corrections. Keeping only the dependence on the neutral Higgs fields H_1^0 and H_2^0, the tree–level MSSM potential reads

$$V_0 = (\mu^2 + m_{H_1}^2)\, \left|H_1^0\right|^2 + (\mu^2 + m_{H_2}^2)\, \left|H_2^0\right|^2 + m_3^2 \left(H_1^0 H_2^0 + {\rm h.c.}\right) + \frac{g^2 + g'^2}{8} \left(|H_1^0|^2 - |H_2^0|^2\right)^2 , \quad (1)$$

where: μ is the Higgs mass term in the superpotential (we assume it to be real, neglecting all possible CP–violating phases); $m_{H_1}^2$, $m_{H_2}^2$ and m_3^2 are soft SUSY–breaking masses; g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, respectively. The neutral Higgs fields can be decomposed into their vacuum expectation values (VEVs) plus their CP–even and CP–odd fluctuations as $H_i^0 = (v_i + S_i + i P_i)/\sqrt{2}$ $(i = 1, 2)$. The VEVs v_i are determined by solving the minimization conditions on the effective potential, i.e.

$$\left.\frac{\partial V_{\rm eff}}{\partial S_i}\right|_{\rm min} = 0\,, \qquad \left.\frac{\partial V_{\rm eff}}{\partial P_i}\right|_{\rm min} = 0\,, \quad (2)$$

the second equality being automatically satisfied since CP is conserved. However, it is also possible to take v_1 and v_2, or equivalently $v^2 \equiv v_1^2 + v_2^2$ and $\tan\beta \equiv v_2/v_1$, as input parameters [we recall that v^2 is related to the squared running mass of the Z boson through $m_Z^2 = (g^2 + g'^2)\, v^2/4$]. In this case, the minimization conditions on $V_{\rm eff}$ can be translated into conditions on μ^2 and m_3^2:

$$\mu^2 = -\frac{m_Z^2}{2} + \frac{m_{H_1}^2 + \Sigma_1 - (m_{H_2}^2 + \Sigma_2)\, \tan^2\beta}{\tan^2\beta - 1}\,, \quad (3)$$

$$m_3^2 = \frac{m_Z^2}{2}\, \sin 2\beta + \frac{1}{2}\, \tan 2\beta \left(m_{H_1}^2 - m_{H_2}^2 + \Sigma_1 - \Sigma_2\right)\,, \quad (4)$$

where the "tadpoles" Σ_1 and Σ_2 are defined as

$$\Sigma_i \equiv \frac{1}{v_i} \left.\frac{\partial \Delta V}{\partial S_i}\right|_{\rm min}\,. \quad (5)$$

In the effective potential approach, the mass matrices for the neutral CP–odd and CP–even Higgs bosons can be approximated by

$$\left(\mathcal{M}_P^2\right)_{ij}^{\rm eff} = \left.\frac{\partial^2 V_{\rm eff}}{\partial P_i \partial P_j}\right|_{\rm min}\,, \qquad \left(\mathcal{M}_S^2\right)_{ij}^{\rm eff} = \left.\frac{\partial^2 V_{\rm eff}}{\partial S_i \partial S_j}\right|_{\rm min}\,. \quad (6)$$

Exploiting the minimization conditions of the effective potential, Eq. (2), the CP–odd mass matrix can be written as

$$\left(\mathcal{M}_P^2\right)^{\rm eff}_{ij} = -m_3^2\,\frac{v_1 v_2}{v_i v_j} - \delta_{ij}\,\Sigma_i + \left.\frac{\partial^2 \Delta V}{\partial P_i \partial P_j}\right|_{\rm min} . \tag{7}$$

$(\mathcal{M}_P^2)^{\rm eff}$ has a single non–vanishing eigenvalue that, in the approximation of zero external momentum, can be identified with the squared physical mass of the A boson. We denote it as $\overline{m}_A^2 = m_A^2 + \Delta m_A^2$, where $m_A^2 = -2\,m_3^2/\sin 2\beta$ is the squared running mass of the A boson. The CP–even mass matrix can in turn be decomposed as

$$\left(\mathcal{M}_S^2\right)^{\rm eff} = \left(\mathcal{M}_S^2\right)^{0,\,\rm eff} + \left(\Delta\mathcal{M}_S^2\right)^{\rm eff} , \tag{8}$$

where the first term in the sum is the tree–level mass matrix expressed in terms of $\overline{m}_A$:

$$\left(\mathcal{M}_S^2\right)^{0,\,\rm eff} = \left(\begin{array}{cc} m_Z^2\,c^2_\beta + \overline{m}_A^2\,s^2_\beta & -\left(m_Z^2 + \overline{m}_A^2\right) s_\beta\, c_\beta \\ -\left(m_Z^2 + \overline{m}_A^2\right) s_\beta\, c_\beta & m_Z^2\,s^2_\beta + \overline{m}_A^2\,c^2_\beta \end{array}\right) , \tag{9}$$

($c_\beta \equiv \cos\beta\,,\ s_\beta \equiv \sin\beta$ and so on), while the second term contains the radiative corrections:

$$\left(\Delta\mathcal{M}_S^2\right)^{\rm eff}_{ij} = \left.\frac{\partial^2 \Delta V}{\partial S_i \partial S_j}\right|_{\rm min} - (-1)^{i+j}\left.\frac{\partial^2 \Delta V}{\partial P_i \partial P_j}\right|_{\rm min} . \tag{10}$$

It is clear from Eqs. (7)–(10) that, in order to make contact with the physical A mass, the effective potential should be computed as a function of both CP–even and CP–odd fields.

3 Computation of the two–loop Yukawa corrections

We shall now describe our two–loop computation of the tadpoles Σ_i, the A boson mass correction Δm_A^2 and the matrix $(\Delta\mathcal{M}_S^2)^{\rm eff}$, including terms controlled by the top and/or the bottom Yukawa couplings. The resulting corrections are proportional to various combinations of couplings and masses: e.g., terms of $\mathcal{O}(\alpha_t^2\, m_b^2)$ might as well be interpreted as $\tan\beta$–suppressed terms of $\mathcal{O}(\alpha_t\alpha_b\, m_t^2)$. To simplify our notation, we will refer to all such "mixed" terms as to $\mathcal{O}(\alpha_t\alpha_b)$ corrections. Our computation will thus provide us with the $\mathcal{O}(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ corrections, extending the $\mathcal{O}(\alpha_t^2)$ results presented in Ref. [14].

The computation is consistently performed in the gaugeless limit, i.e. by setting to zero all the gauge couplings, and by keeping $h_t = \sqrt{4\pi\alpha_t}$ and $h_b = \sqrt{4\pi\alpha_b}$ as the only non–vanishing Yukawa couplings. In this limit, the tree–level (field–dependent) spectrum of the MSSM simplifies considerably: gauginos and Higgsinos do not mix; charged and neutral Higgsinos combine into Dirac spinors with degenerate mass eigenvalues $|\mu|^2$; the only massive SM fermions are the top and bottom quarks, while all other fermions and gauge bosons have vanishing masses; the only sfermions with non–vanishing couplings are the stop and sbottom squarks; the lighter CP–even Higgs boson, h, is massless, and the same is true for the Goldstone bosons; all the remaining Higgs states, $(H, A, H^\pm)$, have degenerate mass eigenvalues m_A^2. The tree–level mixing angle in the CP–even sector is just $\alpha = \beta - \pi/2$.

The renormalization of the effective potential is performed according to the lines of Ref. [16], i. e. we express $V_{\rm eff}$, from the beginning, in terms of $\overline{\rm DR}$–renormalized fields and parameters.

In practice, this amounts to dropping all the divergent terms in ΔV and replacing the two–loop integrals $I(m_1^2, m_2^2, m_3^2)$ and $J(m_1^2, m_2^2)$ (see e. g. Ref. [16] for the definitions) with their "subtracted" counterparts $\hat{I}$ and $\hat{J}$, first introduced in Ref. [23]. Alternatively, we could follow the procedure of Refs. [13, 14]: express ΔV in terms of bare parameters and then renormalize the derivatives of ΔV (i. e. the tadpoles and the corrections to the Higgs masses), checking explicitly the cancellation of the divergent terms. The general formulae for the tadpoles and the corrections to the Higgs masses would look slightly more complicated in the latter case. However, we have checked that the two renormalization procedures lead to the same final result, as they should.

According to Eqs. (5), (7) and (10), the tadpoles and the corrections to the Higgs mass matrices can be computed by taking the derivatives of ΔV with respect to the CP–even and CP–odd fields, evaluated at the minimum of $V_{\rm eff}$. Following the strategy of Refs. [13, 14], we compute ΔV in terms of a set of field–dependent parameters (masses and angles), and use the chain rule to express the corrections in terms of derivatives of ΔV with respect to those parameters. In each sector, the field–dependent parameters can be chosen as

$$m_q\,,\;\; m^2_{\tilde{q}_1}\,,\;\; m^2_{\tilde{q}_2}\,,\;\; \bar{\theta}_{\tilde{q}}\,,\;\; \varphi_q\,,\;\; \widetilde{\varphi}_q \qquad (q=t,b) \tag{11}$$

where: m_q and $m^2_{\tilde{q}_i}$ are the quark and squark masses; $\bar{\theta}_{\tilde{q}}$ is the field–dependent squark mixing angle, defined in such a way that $0 \leq \bar{\theta}_{\tilde{q}} < \pi/2$ (to be contrasted with the usual field–independent mixing angle $\theta_{\tilde{q}}$, such that $-\pi/2 \leq \theta_{\tilde{q}} < \pi/2$); φ_q is the phase in the complex quark mass; $\widetilde{\varphi}_q$ is the phase in the off–diagonal element of the squark mass matrix. For the explicit Higgs field dependence of these parameters, see Refs. [13, 14]. In the expression of ΔV relevant to the $\mathcal{O}(\alpha_t^2)$ corrections (i.e., with h_b set to zero), the top and stop phases always combine in the difference $\varphi_t - \widetilde{\varphi}_t$, so that a convenient choice for the field–dependent parameter is $c_{\varphi_t - \widetilde{\varphi}_t} \equiv \cos(\varphi_t - \widetilde{\varphi}_t)$. On the other hand, when both h_t and h_b are nonzero [as it is the case in the computation of the $\mathcal{O}(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ corrections] the situation becomes more complicated: besides the terms involving $\varphi_t - \widetilde{\varphi}_t$ and $\varphi_b - \widetilde{\varphi}_b$, we find other terms, coming from diagrams with a charged Higgs or Goldstone boson, that involve the combinations $\varphi_t + \widetilde{\varphi}_b\,,\; \varphi_b + \widetilde{\varphi}_t\,,\; \varphi_t + \varphi_b$ and $\widetilde{\varphi}_t + \widetilde{\varphi}_b$.

Exploiting the field–dependence of the various masses and angles, we get the following general formulae for the $\mathcal{O}(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ corrections in the $\overline{\rm DR}$ renormalization scheme:

$$\begin{aligned}
\left(\Delta\mathcal{M}^2_S\right)^{\rm eff}_{11} &= 2\,h_b^2\,m_b^2\,F_1^b + 2\,h_b^2\,A_b\,m_b\,s_{2\theta_b}\,F_2^b + \frac{1}{2}\,h_b^2\,A_b^2\,s^2_{2\theta_b}\,F_3^b \\
&+ \frac{1}{2}\,h_t^2\,\mu^2\,s^2_{2\theta_t}\,F_3^t + 2\,h_t\,h_b\,m_b\,\mu\,s_{2\theta_t}\,F_4^t + h_t\,h_b\,\mu\,A_b\,s_{2\theta_t}\,s_{2\theta_b}\,F_5\,,
\end{aligned} \tag{12}$$

$$\begin{aligned}
\left(\Delta\mathcal{M}^2_S\right)^{\rm eff}_{12} &= h_t^2\,\mu\,m_t\,s_{2\theta_t}\,F_2^t + \frac{1}{2}\,h_t^2\,A_t\,\mu\,s^2_{2\theta_t}\,F_3^t + h_t\,h_b\,m_b\,A_t\,s_{2\theta_t}\,F_4^t \\
&+ h_b^2\,\mu\,m_b\,s_{2\theta_b}\,F_2^b + \frac{1}{2}\,h_b^2\,A_b\,\mu\,s^2_{2\theta_b}\,F_3^b + h_t\,h_b\,m_t\,A_b\,s_{2\theta_b}\,F_4^b \\
&+ \frac{1}{2}\,h_t\,h_b\,s_{2\theta_t}\,s_{2\theta_b}\,(A_t\,A_b + \mu^2)\,F_5 + 2\,h_t\,h_b\,m_t\,m_b\,F_6\,,
\end{aligned} \tag{13}$$

$$\begin{aligned}
\left(\Delta\mathcal{M}^2_S\right)^{\rm eff}_{22} &= 2\,h_t^2\,m_t^2\,F_1^t + 2\,h_t^2\,A_t\,m_t\,s_{2\theta_t}\,F_2^t + \frac{1}{2}\,h_t^2\,A_t^2\,s^2_{2\theta_t}\,F_3^t \\
&+ \frac{1}{2}\,h_b^2\,\mu^2\,s^2_{2\theta_b}\,F_3^b + 2\,h_t\,h_b\,m_t\,\mu\,s_{2\theta_b}\,F_4^b + h_t\,h_b\,\mu\,A_t\,s_{2\theta_t}\,s_{2\theta_b}\,F_5\,,
\end{aligned} \tag{14}$$

$$v_1^2\,\Sigma_1 \;=\; m_t\,\mu\,\cot\beta\, s_{2\theta_t}\,F^t + m_b\,A_b\,s_{2\theta_b}\,F^b + 2\,m_b^2\,G^b\,, \tag{15}$$

$$v_2^2\,\Sigma_2 \;=\; m_b\,\mu\,\tan\beta\, s_{2\theta_b}\,F^b + m_t\,A_t\,s_{2\theta_t}\,F^t + 2\,m_t^2\,G^t\,, \tag{16}$$

$$\Delta m_A^2 \;=\; -\frac{1}{c_\beta\,s_\beta}\left(\frac{h_t^2\,\mu\,A_t}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\,F^t + \frac{h_b^2\,\mu\,A_b}{m_{\tilde b_1}^2 - m_{\tilde b_2}^2}\,F^b + 2\,h_t\,h_b\,F_A\right)\,. \tag{17}$$

In the equations above, A_t and A_b are the soft SUSY–breaking trilinear couplings of the Higgs fields to the stop and sbottom squarks, and $s_{2\theta_q} \equiv \sin 2\theta_{\tilde q}$ $(q = t, b)$ refer to the usual field–independent squark mixing angles. The functions F_i^q $(i = 1,2,3,4)$, F_5, F_6, F^q, G^q and F_A are combinations of the derivatives of ΔV with respect to the field–dependent parameters, computed at the minimum of the effective potential; their definitions are given in the appendix. It can be noticed that, as it is predictable from the form of the MSSM Lagrangian, the above results are fully symmetric with respect to the simultaneous replacements $t \leftrightarrow b$ and $H_1 \leftrightarrow H_2$ [the latter resulting into $\tan\beta \leftrightarrow \cot\beta$, $v_1 \leftrightarrow v_2$, $(\Delta\mathcal{M}_S^2)_{11}^{\rm eff} \leftrightarrow (\Delta\mathcal{M}_S^2)_{22}^{\rm eff}$ and $\Sigma_1 \leftrightarrow \Sigma_2$].

An explicit expression of the two–loop top and bottom Yukawa contribution to ΔV can be found in Ref. [12], while the complete two–loop effective potential for the MSSM was given in the second paper of Ref. [17]. However, those expressions were computed for vanishing CP–odd fields, thus omitting the dependence on the phases φ_q and $\widetilde\varphi_q$. Since these phases appear in ΔV in many different combinations, it is not possible to obtain the general field–dependent expression of ΔV by means of simple substitutions in Eq. (D.6) of Ref. [12], as it was the case in the computation of the $\mathcal{O}(\alpha_t\alpha_s + \alpha_t^2)$ corrections [2]. We worked out the general expression of the two–loop top and bottom Yukawa contribution to ΔV in terms of all the field–dependent parameters of Eq. (11), then we computed its derivatives in order to obtain explicit formulae for the various functions appearing in Eqs. (12)–(17). The use of a recursive relation for the derivatives of $I(m_1^2, m_2^2, m_3^2)$, presented in Ref. [16], helped us to keep the number of terms involved under control. However, the resulting analytical formulae are too long even for an appendix, thus we make them available, upon request, in the form of a Fortran code.

4 On–shell renormalization scheme and input parameters

The results presented in the previous section are valid when the MSSM input parameters are expressed in the $\overline{\rm DR}$ renormalization scheme. This way of presenting the results is convenient for analysing models that predict, via the MSSM renormalization group equations, the low–energy $\overline{\rm DR}$ values of the parameters in terms of a set of boundary conditions assigned at some scale M_{GUT} much larger than the weak scale (see Ref. [21] for a list of public codes that are commonly used in this kind of analyses, and Ref. [24] for a comparison among them). General low–energy analyses of the MSSM, however, do not refer to boundary conditions at high scales, and are usually performed in terms of parameters with a more direct physical interpretation, such as pole masses and appropriately defined mixing angles in the squark sector. Such an approach requires modifications of our two–loop results, induced by the variation of the one–loop parameters when

[2] Also, we disagree with Ref. [12] on the sign of the penultimate line of Eq. (D.6).

moving from the $\overline{\rm DR}$ scheme to a different scheme (for a generic parameter x, we define the shift from the $\overline{\rm DR}$ value $\hat{x}$ as $\delta x \equiv \hat{x} - x$).

While an On–Shell (OS) renormalization scheme for the parameters in the top/stop sector can be rather easily devised (see e. g. Refs. [13, 14]), some additional care is required in the choice of an OS scheme for the parameters in the bottom/sbottom sector, due to the potentially large one–loop threshold corrections [25], proportional to $\tan\beta$, that contribute to the pole bottom mass. For example, a definition of A_b in terms of the pole bottom and sbottom masses, similar to the usual definition of A_t, would produce a shift δA_b proportional to $\tan^2\beta$ [26]. When $\tan\beta$ is large, this would induce very large corrections to the Higgs masses at two loops, questioning the validity of the perturbative expansion. To overcome this problem, we adopt a set of renormalization prescriptions for the parameters in the the bottom/sbottom sector, first introduced in Ref. [15] for the case of the strong corrections, that avoids the occurrence of unphysically large threshold effects. Generalizing these prescriptions to the case of the Yukawa corrections, and combining them with the usual prescriptions for the top/stop parameters [14], we obtain a convenient OS renormalization scheme for the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ part of the corrections to the Higgs masses. Since the corrections controlled by the bottom Yukawa coupling can be sizeable only for large values of $\tan\beta$, we work directly in the physically relevant limit of $\tan\beta \to \infty$, i. e. $v_1 \to 0\,,\; v_2 \to v$.

For the OS squark masses and mixing angles, top quark mass and electroweak parameter $v \equiv (\sqrt{2}\,G_\mu)^{-1/2}$ we adopt the definitions

$$\delta m^2_{\tilde{q}_i} = \Pi^{\tilde{q}}_{ii}(m^2_{\tilde{q}_i})\,, \quad \delta\theta_{\tilde{q}} = \frac{1}{2}\,\frac{\Pi^{\tilde{q}}_{12}(m^2_{\tilde{q}_1}) + \Pi^{\tilde{q}}_{12}(m^2_{\tilde{q}_2})}{m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2}}\,, \quad \delta m_t = \Sigma_t(m_t)\,, \quad \delta v = \frac{v}{2}\,\frac{\Pi^T_{WW}(0)}{m^2_W}\,, \tag{18}$$

where $\tilde{q} = (\tilde{t}\,,\tilde{b})$, while $\Pi^{\tilde{q}}_{ij}(p^2)$, $\Sigma_t(p)$ and $\Pi^T_{WW}(p^2)$ denote the real and finite parts of the self–energies of squarks, top quark and W boson, respectively. Following Ref. [14], we further treat μ as a $\overline{\rm DR}$ parameter (i. e., $\delta\mu = 0$), and h_t and A_t as derived quantities, that can be computed by means of the tree–level formulae for m_t and $s_{2\theta_t}$. In principle, we still have to define $m_b\,, h_b$ and A_b. However, in the large $\tan\beta$ limit, the bottom mass is just zero, and the sbottom mixing angle becomes

$$s_{2\theta_b} \;=\; \frac{\sqrt{2}\,h_b\,\mu\,v}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}}\,, \tag{19}$$

which is independent of m_b and A_b. We can thus treat h_b as a quantity derived from the sbottom mixing, and use eq. (19) to obtain a prescription for δh_b:

$$\delta h_b = h_b\left(\frac{\delta m^2_{\tilde{b}_1} - \delta m^2_{\tilde{b}_2}}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}} + \frac{\delta s_{2\theta_b}}{s_{2\theta_b}} - \frac{\delta v}{v}\right)\,. \tag{20}$$

Finally, following Ref. [15], we provide an OS definition for the quantity $\widetilde{A}_b \equiv h_b\,A_b$, in terms of the $(\tilde{b}_1\tilde{b}_2^* A)$ proper vertex $i\Lambda_{12A}(p_1^2,p_2^2,p_A^2)$. We thus have $\delta A_b = (\delta\widetilde{A}_b - \delta h_b\,A_b)/h_b$, where

$$\begin{aligned}
\delta\widetilde{A}_b \;&=\; -\frac{i}{\sqrt{2}}\left[\Lambda_{12A}(m^2_{\tilde{q}_1},m^2_{\tilde{q}_1},0) + \Lambda_{12A}(m^2_{\tilde{q}_2},m^2_{\tilde{q}_2},0)\right]\\
&\quad + \frac{\widetilde{A}_b}{2}\left[\frac{\Pi^{\tilde{b}}_{11}(m^2_{\tilde{b}_1}) - \Pi^{\tilde{b}}_{11}(m^2_{\tilde{b}_2})}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}} + \left(\Pi^{\tilde{b}}_{11} \to \Pi^{\tilde{b}}_{22}\right) + \left(\Pi^{\tilde{b}}_{11} \to \Pi_{AA}\right)\right]\,.
\end{aligned} \tag{21}$$

In order to obtain the formulae for the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections to the Higgs masses in our OS scheme, three steps have to be taken: first, we take the limit of $\tan\beta \to \infty$, $m_b \to 0$ in the general $\overline{\rm DR}$ results for the $\mathcal{O}(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ corrections; then we add the contributions due to the shifts of the parameters entering the one–loop corrections (this requires the computation of the $\mathcal{O}(\alpha_t + \alpha_b)$ part of the counterterms in the large $\tan\beta$ limit); finally, we subtract the pure $\mathcal{O}(\alpha_t^2)$ part, which, being relevant for all values of $\tan\beta$, must be computed separately with the formulae of Ref. [14]. Notice that we do not encounter any divergent terms when taking the limit of large $\tan\beta$ in the $\overline{\rm DR}$ results: unphysically large contributions could only be introduced by hand, as the result of a poor choice of the renormalization conditions for the parameters in the bottom/sbottom sector.

After defining our OS renormalization scheme, we discuss the parameters that we will actually use as input of our calculation. In particular, although we have used eqs. (19)–(20) to define the OS bottom Yukawa coupling h_b through the sbottom mixing, we still need to exploit the experimental information on the bottom mass in order to obtain the $\overline{\rm DR}$ running coupling $\hat{h}_b$. The OS coupling will then be computed through the relation $h_b = \hat{h}_b - \delta h_b$. Following Ref. [15], we define the running coupling $\hat{h}_b$ at the reference scale $Q_0 = 175$ GeV to be

$$\hat{h}_b \equiv h_b(Q_0)^{\overline{\rm DR}}_{\rm MSSM} = \frac{\overline{m}_b\sqrt{2}}{v_1}\,\frac{1+\delta_b}{|1+\epsilon_b|}\,, \qquad (22)$$

where: $\overline{m}_b \equiv m_b(Q_0)^{\overline{\rm DR}}_{\rm SM} = 2.74 \pm 0.05$ GeV is the Standard Model bottom mass, evolved up to the scale Q_0 to take into account the resummation of the universal large QCD logarithms; ϵ_b contains the $\tan\beta$–enhanced threshold corrections from both the gluino–sbottom and the higgsino–stop loops (denoted as ϵ_b and ϵ_b', respectively, in eqs. (25) and (26) of Ref. [15]); δ_b contains the residual threshold corrections that are not enhanced by $\tan\beta$. Notice that, as shown in Ref. [27], keeping ϵ_b in the denominator of eq. (22) allows to resum the $\tan\beta$–enhanced threshold corrections to all orders in the perturbative expansion. On the other hand, there is no preferred way of including the threshold corrections parametrized by δ_b, whose effect on the value of $\hat{h}_b$ is anyway very small.

For the top/stop sector, we take as input the top pole mass, $M_t = 174.3$ GeV, and the parameters $(m_{Q,\tilde{t}}, m_U, A_t)$ that can be derived by rotating the diagonal matrix of the OS stop masses by the angle $\theta_{\tilde{t}}$, defined in eq. (18). Concerning the sbottom sector, additional care is required, because of our non–trivial definition of h_b and of the fact that, at one loop, the parameter $m_{Q,\tilde{b}}$ entering the sbottom mass matrix differs from the corresponding stop parameter $m_{Q,\tilde{t}}$ by a finite shift [26]. We start by computing the renormalized coupling h_b as given by eq. (20) and (22). Then we compute $m_{Q,\tilde{b}}$ following the prescription of Ref. [26]. Finally, we use the parameters h_b and $m_{Q,\tilde{b}}$ to compute the actual values of the OS sbottom masses and mixing angle. The remaining numerical inputs are the physical Z–boson mass, $M_Z = 91.187$ GeV, the OS electroweak parameter $v = 246.218$ GeV, and the strong coupling constant, that we fix as $\alpha_s(Q_0) = 0.108$.

5 Numerical results

We are now ready to discuss the numerical impact of our two-loop corrections to the neutral Higgs boson masses. In doing so, we will not be refering to various experimental bounds on the Higgs boson masses from direct LEP or Tevatron searches. An analysis of the Higgs mass bounds in supergravity models using results presented here and the experimental contraints is interesting application and will appear elsewhere [28].

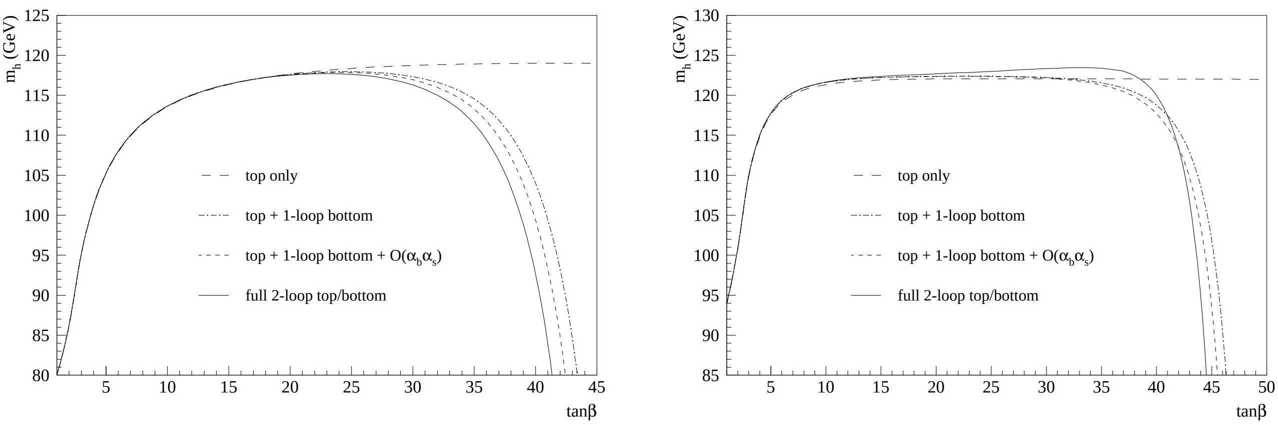
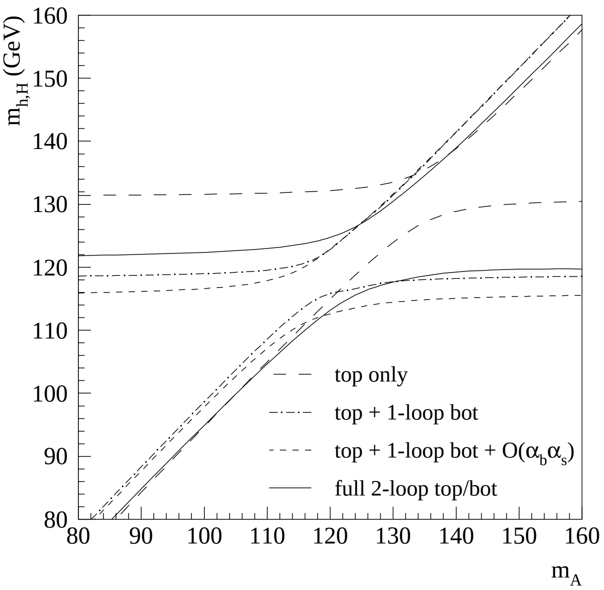


Figure 1: *The light Higgs boson mass as a function of* $\tan\beta$ *for* $m_A = 120$ *GeV (left panel),* $m_A = 200$ *GeV (right panel). As for the other input parameters we have chosen* $A_t = 1$ *TeV,* $A_b = 2$ *TeV,* $\mu = m_{Q,\tilde{t}} = m_U = m_D = m_{\tilde{g}} = 1$ *TeV. The meaning of the various curves is explained in the text.*

In Fig. (1) we show the effect on the light Higgs boson mass from the full two loop top/bottom corrections (solid lines) in comparison with the previously existed in the literature corrections, $\mathcal{O}(\alpha_t)$(long dashed line), $\mathcal{O}(\alpha_t\alpha_s + \alpha_t^2)$ (dot-dashed line) and $\mathcal{O}(\alpha_t\alpha_s + \alpha_t^2 + \alpha_b\alpha_s)$ (short dashed-line), as a function of $\tan\beta$. The following input values for the CP-odd Higgs mass m_A have been set : $m_A = 120$ GeV (left panel), $m_A = 200$ GeV (right panel) . The other input parameters have been chosen to be : $A_t = 1$ TeV, $A_b = 2$ TeV, $\mu = m_{Q,\tilde{t}} = m_U = m_D = m_{\tilde{g}} = 1$ TeV.

In Fig.(1), the full two-loop top/bottom (solid) line contains the two-loop corrections at the order $\mathcal{O}(\alpha_s\alpha_t + \alpha_s\alpha_b + \alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$. Corrections start being sensitive to $\tan\beta$ ones the bottom corrections are included. The steep dicrease of the light Higgs boson mass for large values of $\tan\beta$ is due to the one-loop $\mathcal{O}(\alpha_b)$ contributions. Our new corrections are sensitive mainly to the product $\mu\tan\beta$. In fact it is not surprising that a figure with μ vs. m_h looks identical to the Fig.(1) : our corrections depend mainly on the bottom Yukawa coupling, which in turn through its threshold corrections depend on the product $\mu\tan\beta$ and thus any variation of $\tan\beta$ with fixed μ produces the same effect as a variation of μ with fixed $\tan\beta$. Two remarks to be made for the full corrections : *i)* in passing from $m_A = 120$ GeV to $m_A = 160$ GeV the $\mathcal{O}(a_t a_b + a_b^2)$ corrections add to the Higgs mass distructively or constructively, fact which is due to the $\mathcal{O}(\alpha_t\alpha_b)$ terms in $(\Delta\mathcal{M}_S^2)_{22}^{\text{eff}}$ and *ii)* the new corrections account for several GeVs difference on the light Higgs boson mass. As $m_A \to 1$ TeV these corrections become smaller.

In Fig.2 we plot both light and heavy CP-even Higgs boson masses, m_h and m_H as functions



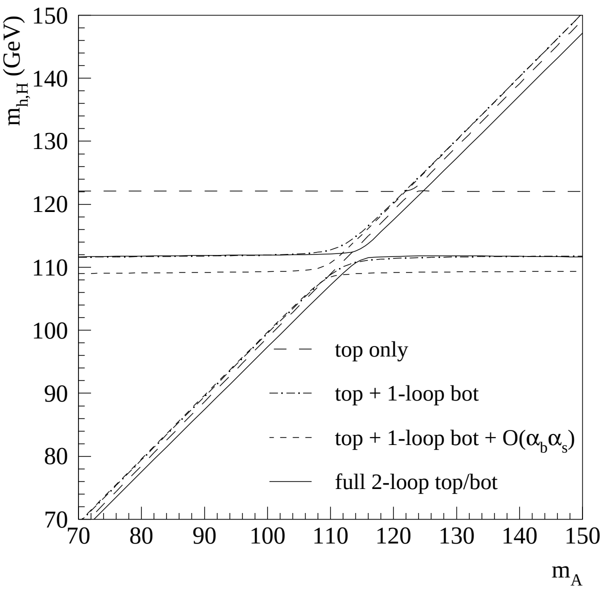


Figure 2: *The masses o the light (m_h) and the heavy (m_H) Higgs bosons versus the physical CP-odd higgs mass (m_A) for $\{\tan\beta = 40,\ A_t = 1\ TeV,\ A_b = 2\ TeV\}$ (left panel) and $\{\tan\beta = 45,\ A_t = 1\ TeV,\ A_b = 0\ TeV\}$(right panel). The other parameters have been chosen as in Fig.1 while the various curves appearing are explained in the text.*

of the CP-odd Higgs mass when the latter varies in the region 100 GeV $\leq m_A \leq$ 200 GeV and for the set of input parameters $\{\tan\beta = 40,\ A_t = 1$ TeV, $A_b = 2$ TeV$\}$ (left panel) and $\{\tan\beta = 45,\ A_t = 1$ TeV, $A_b = 0$ TeV$\}$(right panel). Regarding the light Higgs boson mass m_h, we observe that, in general, there is a tendency of cancelation between the $\mathcal{O}(\alpha_t\alpha_s + \alpha_t^2 + \alpha_b\alpha_s)$ and the new corrections $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ for $m_A > 150$ GeV and $\tan\beta = 40$ and $m_A > 110$ GeV and $\tan\beta = 45$. Significant variation ($\sim$ 5 GeV) especially on the heavy Higgs boson mass is obvious in the region where m_A is small, around 120 GeV, where the new corrections are of the same size with the $\mathcal{O}(\alpha_t\alpha_s + \alpha_t^2 + \alpha_b\alpha_s)$ ones and is mainly due to the large value of A_b chosen (see left panel of Fig.2) which in turn enhances the correction to the matrix element $(\Delta\mathcal{M}_S^2)_{11}^{\text{eff}}$. In contrast however, choosing $A_b = 0$ (right panel of Fig.2), we observe smaller effects since radiative corrections mainly affect $(\Delta\mathcal{M}_S^2)_{22}^{\text{eff}}$.

Tevatron Run I Higgs boson searches [19] are limited within the region depicted in Fig.(2) . We strongly encourage our experimental coluegues to refine the analysis of Run I results using our new corrections. Needless to say, these corrections are compulsory in comparing the theory predictions with forthcoming results for the Higgs bosons at Tevatron Run II and LHC .

Finally, and as a biproduct of our calculation we have investigated the impact of our $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ two–loop corrections on the minimization conditions of the MSSM effective potential. For definiteness, we work in the mSUGRA scenario, in which the MSSM Lagrangian at the Grand unified scale M_{GUT} contains only five independent mass parameters: a common soft SUSY-breaking scalar mass m_0, a common soft gaugino mass $m_{1/2}$, a common soft trilinear term A_0, and the weak scale parameter $\tan\beta(M_Z)$. By adopting a typical "benchmark" scenario (usually called SPS4-scenario [29])[3] SPS 4 : $m_0 = 400$ GeV, $m_{1/2} = 300$ GeV, $A_0 = 0\,, \tan\beta = 50\,, \mu < 0\,,$ and following exactly the procedure of [16] we find no significant improvement from the results presented in Fig. 5 of Ref. [16]. However, the corrections $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ should be included in the codes [21] for self consistency.

6 Conclusions

In this article, we have completed the calculation for the two-loop top/bottom Yukawa contributions to the Higgs boson masses in the MSSM, by including a significant part of the corrections of order $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$. These corrections are numerically relevant to the region where the parameter $\tan\beta$ is large ($\tan\beta \geq 10$). To this end, we have calculated the effective potential $V_{\rm eff}$ as a function of $\overline{DR}$ fields and parameters (as they come from the MSSM at high energies) and following [16] we replace the integrals with their subtracted forms [23]. We have cross checked the validity of our method by instead calculating the effective potential in terms of bare parameters and then renormalize (as in [13, 14]) and checking explicitly the cancelation of divergencies. Both procedures lead to the same result, and we take this as a hint that our calculation is correct. The resulting expressions for the Higgs boson masses are too lengthy to be written in an article and make them available in the form of a Fortran code. In addition, we have calculated the MSSM Higgs boson masses in terms of physical imput parameters. To this end we have devised an On-Shell renormalization scheme (slightly extended version of Ref. [15]) in order to accomodate potential large threshold corrections on the bottom mass in the large $\tan\beta$ regime.

The results of our calculation are depicted in Figs.(1,2). In general our corrections account for several GeVs on the Higgs masses compared with the previous results. We believe that they are of a particular interest in testing the MSSM from Higgs boson searches at Tevatron and LHC.

This article has to be regarded as a continuation of previous efforts [13, 14, 15, 16] appeared in the literature. What remains to be done? First our results can be extended including corrections controlled by the lepton Yukawa couplings and in particular corrections proportional to the τ-Yukawa. In the DRbar scheme, they can be obtained for free from the $\mathcal{O}(\alpha_t^2)$ corrections , with obvious replacements [4]. We have added these corrections to our tadpoles and Higgs masses but found negligible effects. On the other hand, a full analysis of the electroweak two-loop corerctions specificly at large $\tan\beta$ and external momentum contributions remain to be seen.

Acknowledgments

We would like to thank A. Djouadi, M. Drees, S. Heinemeyer and A. Pilaftsis for useful comments and discussions.

[3]Notice that our convention for the sign of μ differs from the one in Ref. [29]

[4]Just change in the $\mathcal{O}(\alpha_t^2)$ subroutine $M_{11} \leftrightarrow M_{22}, \tan\beta \to 1/\tan\beta, m_t \to m_\tau, m_b \to m_\nu, m_{\tilde{b}_L} \to m_{\tilde{\nu}}, N_c \to 1$.

Appendix

We present here the expressions for the functions F_i^t $(i = 1,2,3,4)$, F_5 , F_6 , F^t , G^t and F_A, appearing in Eqs. (12)–(17), in terms of derivatives of the $\overline{\rm DR}$–renormalized ΔV, computed at the minimum of $V_{\rm eff}$:

$$
\begin{aligned}
F_1^t &= \frac{\partial^2 \Delta V}{(\partial m_t^2)^2} + \frac{\partial^2 \Delta V}{(\partial m_{\tilde t_1}^2)^2} + \frac{\partial^2 \Delta V}{(\partial m_{\tilde t_2}^2)^2} + 2\,\frac{\partial^2 \Delta V}{\partial m_t^2 \partial m_{\tilde t_1}^2} + 2\,\frac{\partial^2 \Delta V}{\partial m_t^2 \partial m_{\tilde t_2}^2} + 2\,\frac{\partial^2 \Delta V}{\partial m_{\tilde t_1}^2 \partial m_{\tilde t_2}^2} \\
&+ \frac{1}{4\,m_t^4}\left(4\,\frac{\partial \Delta V}{\partial c_{\varphi_t+\varphi_b}} + z_t\,\frac{\partial \Delta V}{\partial c_{\varphi_t-\tilde\varphi_t}} + z_b\,\frac{\partial \Delta V}{\partial c_{\varphi_t+\tilde\varphi_b}}\right)\,, \qquad \text{(A1)}
\end{aligned}
$$

$$
\begin{aligned}
F_2^t &= \frac{\partial^2 \Delta V}{(\partial m_{\tilde t_1}^2)^2} - \frac{\partial^2 \Delta V}{(\partial m_{\tilde t_2}^2)^2} + \frac{\partial^2 \Delta V}{\partial m_t^2 \partial m_{\tilde t_1}^2} - \frac{\partial^2 \Delta V}{\partial m_t^2 \partial m_{\tilde t_2}^2} \\
&- \frac{4\,c^2_{2\theta_t}}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\left(\frac{\partial^2 \Delta V}{\partial c^2_{2\bar\theta_t}\partial m_t^2} + \frac{\partial^2 \Delta V}{\partial c^2_{2\bar\theta_t}\partial m_{\tilde t_1}^2} + \frac{\partial^2 \Delta V}{\partial c^2_{2\bar\theta_t}\partial m_{\tilde t_2}^2}\right) - \frac{z_t}{s^2_{2\theta_t}\,m_t^2\,(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)}\,\frac{\partial \Delta V}{\partial c_{\varphi_t-\tilde\varphi_t}}\,, \qquad \text{(A2)}
\end{aligned}
$$

$$
\begin{aligned}
F_3^t &= \frac{\partial^2 \Delta V}{(\partial m_{\tilde t_1}^2)^2} + \frac{\partial^2 \Delta V}{(\partial m_{\tilde t_2}^2)^2} - 2\,\frac{\partial^2 \Delta V}{\partial m_{\tilde t_1}^2 \partial m_{\tilde t_2}^2} - \frac{2}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\left(\frac{\partial \Delta V}{\partial m_{\tilde t_1}^2} - \frac{\partial \Delta V}{\partial m_{\tilde t_2}^2}\right) \\
&+ \frac{16\,c^2_{2\theta_t}}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)^2}\left(c^2_{2\theta_t}\,\frac{\partial^2 \Delta V}{(\partial c^2_{2\bar\theta_t})^2} + 2\,\frac{\partial \Delta V}{\partial c^2_{2\bar\theta_t}}\right) - \frac{8\,c^2_{2\theta_t}}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\left(\frac{\partial^2 \Delta V}{\partial c^2_{2\bar\theta_t}\partial m_{\tilde t_1}^2} - \frac{\partial^2 \Delta V}{\partial c^2_{2\bar\theta_t}\partial m_{\tilde t_2}^2}\right) \\
&+ \frac{4\,z_t}{s^4_{2\theta_t}\,(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)^2}\left(\frac{\partial \Delta V}{\partial c_{\varphi_t-\tilde\varphi_t}} + \frac{\partial \Delta V}{\partial c_{\varphi_b+\tilde\varphi_t}} + z_b\,\frac{\partial \Delta V}{\partial c_{\tilde\varphi_t+\tilde\varphi_b}}\right)\,, \qquad \text{(A3)}
\end{aligned}
$$

$$
\begin{aligned}
F_4^t &= \frac{\partial^2 \Delta V}{\partial m_{\tilde t_1}^2 \partial m_b^2} + \frac{\partial^2 \Delta V}{\partial m_{\tilde t_1}^2 \partial m_{\tilde b_1}^2} + \frac{\partial^2 \Delta V}{\partial m_{\tilde t_1}^2 \partial m_{\tilde b_2}^2} - \frac{\partial^2 \Delta V}{\partial m_{\tilde t_2}^2 \partial m_b^2} - \frac{\partial^2 \Delta V}{\partial m_{\tilde t_2}^2 \partial m_{\tilde b_1}^2} - \frac{\partial^2 \Delta V}{\partial m_{\tilde t_2}^2 \partial m_{\tilde b_2}^2} \\
&- \frac{4\,c^2_{2\theta_t}}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\left(\frac{\partial^2 \Delta V}{\partial m_{\tilde b_1}^2\,\partial c^2_{2\bar\theta_t}} + \frac{\partial^2 \Delta V}{\partial m_{\tilde b_2}^2\,\partial c^2_{2\bar\theta_t}} + \frac{\partial^2 \Delta V}{\partial m_b^2 \partial c^2_{2\bar\theta_t}}\right) - \frac{z_t}{s^2_{2\theta_t}\,m_b^2\,(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)}\,\frac{\partial \Delta V}{\partial c_{\varphi_b+\tilde\varphi_t}}\,, \qquad \text{(A4)}
\end{aligned}
$$

$$
\begin{aligned}
F_5 &= \frac{\partial^2 \Delta V}{\partial m_{\tilde t_1}^2 \partial m_{\tilde b_1}^2} - \frac{\partial^2 \Delta V}{\partial m_{\tilde t_1}^2 \partial m_{\tilde b_2}^2} - \frac{\partial^2 \Delta V}{\partial m_{\tilde t_2}^2 \partial m_{\tilde b_1}^2} + \frac{\partial^2 \Delta V}{\partial m_{\tilde t_2}^2 \partial m_{\tilde b_2}^2} \\
&- \frac{4\,c^2_{2\theta_t}}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\left(\frac{\partial^2 \Delta V}{\partial m_{\tilde b_1}^2\,\partial c^2_{2\bar\theta_t}} - \frac{\partial^2 \Delta V}{\partial m_{\tilde b_2}^2\,\partial c^2_{2\bar\theta_t}}\right) - \frac{4\,c^2_{2\theta_b}}{m_{\tilde b_1}^2 - m_{\tilde b_2}^2}\left(\frac{\partial^2 \Delta V}{\partial m_{\tilde t_1}^2\,\partial c^2_{2\bar\theta_b}} - \frac{\partial^2 \Delta V}{\partial m_{\tilde t_2}^2\,\partial c^2_{2\bar\theta_b}}\right) \\
&+ \frac{16\,c^2_{2\theta_t} c^2_{2\theta_b}}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)(m_{\tilde b_1}^2 - m_{\tilde b_2}^2)}\,\frac{\partial^2 \Delta V}{\partial c^2_{2\bar\theta_t}\partial c^2_{2\bar\theta_b}} - \frac{4\,z_t\,z_b}{s^2_{2\theta_t}\,s^2_{2\theta_b}\,(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)\,(m_{\tilde b_1}^2 - m_{\tilde b_2}^2)}\,\frac{\partial \Delta V}{\partial c_{\tilde\varphi_t+\tilde\varphi_b}}\,, \qquad \text{(A5)}
\end{aligned}
$$

$$
\begin{aligned}
F_6 &= \frac{\partial^2 \Delta V}{\partial m_t^2 \partial m_b^2} + \frac{\partial^2 \Delta V}{\partial m_{\tilde t_1}^2 \partial m_b^2} + \frac{\partial^2 \Delta V}{\partial m_{\tilde t_2}^2 \partial m_b^2} + \frac{\partial^2 \Delta V}{\partial m_{\tilde b_1}^2 \partial m_t^2} + \frac{\partial^2 \Delta V}{\partial m_{\tilde b_2}^2 \partial m_t^2} \\
&+ \frac{\partial^2 \Delta V}{\partial m_{\tilde t_1}^2 \partial m_{\tilde b_1}^2} + \frac{\partial^2 \Delta V}{\partial m_{\tilde t_1}^2 \partial m_{\tilde b_2}^2} + \frac{\partial^2 \Delta V}{\partial m_{\tilde t_2}^2 \partial m_{\tilde b_1}^2} + \frac{\partial^2 \Delta V}{\partial m_{\tilde t_2}^2 \partial m_{\tilde b_2}^2} - \frac{1}{m_t^2\, m_b^2}\, \frac{\partial \Delta V}{\partial c_{\varphi_t+\varphi_b}} ,
\end{aligned} \tag{A6}
$$

$$
F^t = \frac{\partial \Delta V}{\partial m_{\tilde t_1}^2} - \frac{\partial \Delta V}{\partial m_{\tilde t_2}^2} - \frac{4\, c_{2\theta_t}^2}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\, \frac{\partial \Delta V}{\partial c_{2\theta_t}^2} , \tag{A7}
$$

$$
G^t = \frac{\partial \Delta V}{\partial m_t^2} + \frac{\partial \Delta V}{\partial m_{\tilde t_1}^2} + \frac{\partial \Delta V}{\partial m_{\tilde t_2}^2} , \tag{A8}
$$

$$
\begin{aligned}
F_A &= \frac{1}{m_t\, m_b}\, \frac{\partial \Delta V}{\partial c_{\varphi_t+\varphi_b}} + \frac{4\,(A_t\, A_b - \mu^2)^2\, m_t\, m_b\, z_t\, z_b}{s_{2\theta_t}^2\, s_{2\theta_b}^2\, (m_{\tilde t_1}^2 - m_{\tilde t_2}^2)^2\, (m_{\tilde b_1}^2 - m_{\tilde b_2}^2)^2}\, \frac{\partial \Delta V}{\partial c_{\tilde\varphi_t+\tilde\varphi_b}} \\
&+ \frac{m_t\, z_t}{s_{2\theta_t}^2\, m_b\, (m_{\tilde t_1}^2 - m_{\tilde t_2}^2)^2} \left(A_t^2\, \frac{\partial \Delta V}{\partial c_{\varphi_b+\tilde\varphi_t}} + \mu^2\, \cot^2\beta\, \frac{\partial \Delta V}{\partial c_{\varphi_t-\tilde\varphi_t}} \right) \\
&+ \frac{m_b\, z_b}{s_{2\theta_b}^2\, m_t\, (m_{\tilde b_1}^2 - m_{\tilde b_2}^2)^2} \left(A_b^2\, \frac{\partial \Delta V}{\partial c_{\varphi_t+\tilde\varphi_b}} + \mu^2\, \tan^2\beta\, \frac{\partial \Delta V}{\partial c_{\varphi_b-\tilde\varphi_b}} \right) .
\end{aligned} \tag{A9}
$$

In the above formulae, $z_q \equiv \mathrm{sign}(X_q)$. [5] The functions F_i^b, F^b and G^b can be obtained from their top counterparts through the replacement $t \leftrightarrow b$.

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