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Resummed Effective Lagrangian for Higgs-Mediated FCNC Interactions in the CP-Violating MSSM

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ABSTRACT

We derive the properly resummed effective Lagrangian for Higgs-mediated flavour-changing neutral-current (FCNC) interactions in the Minimal Supersymmetric Standard Model (MSSM), without resorting to particular assumptions based on the squark-mass or the quark-Yukawa structure of the theory. In our derivation we also include the possibility of explicit CP violation through the Cabibbo–Kobayashi–Maskawa mixing matrix and soft supersymmetry-breaking mass terms. The advantages of our resummed FCNC effective Lagrangian are explicitly demonstrated within the context of phenomenologically motivated scenarios. We obtain new testable predictions in the large $\tan\beta$ regime of the theory for CP-conserving and CP-violating observables related to the K- and B-meson systems, such as $\Delta M_{K,B}$, ϵ_K, ϵ'/ϵ, $\mathcal{B}(B_{d,s} \to \ell^+\ell^-)$ and their associated leptonic CP asymmetries.

1 Introduction

The appearance of too large FCNC interactions of Higgs bosons to fermionic matter is a generic feature of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ theories with two and more Higgs doublets. Unless there is a symmetry to forbid these Higgs-mediated FCNC interactions to occur in the bare Lagrangian of the model [1], their unsuppressed existence will inevitably lead to predictions for rare processes in the kaon and B-meson systems that often violate experimental limits by several orders of magnitude [1, 2]. In the minimal realization of softly-broken supersymmetry (SUSY), the Minimal Supersymmetric Standard Model (MSSM), the holomorphicity of the superpotential prevents the occurrence of Higgs-boson FCNCs by coupling the one Higgs-doublet superfield, $\widehat{H}_1$, to the down-quark sector, and the other one, $\widehat{H}_2$, to the up-quark sector. However, the above holomorphic property of the superpotential is violated by finite radiative (threshold) corrections due to soft SUSY-breaking interactions [3, 4, 5]. As a consequence, Higgs-mediated FCNCs reappear at the one-loop level, but are naturally suppressed for low and intermediate values of $\tan\beta = \langle \widehat{H}_2 \rangle / \langle \widehat{H}_1 \rangle$, i.e. for $\tan\beta \lesssim 20$. For larger values of $\tan\beta$, e.g. $\tan\beta \gtrsim 30$, the FCNCs partially overcome the loop suppression factor $1/(16\pi^2)$ and become phenomenologically very relevant [6, 8], especially for the K- and B-meson systems.

Recently, the topic of Higgs-boson FCNCs in the large-$\tan\beta$ limit of the MSSM has received much attention [6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. Several approaches have been devised to implement the non-holomorphic finite radiative corrections into the phenomenological analysis of FCNC processes, such as $K^0\bar{K}^0$ and $B^0\bar{B}^0$ mixings, $B_s \to X_s\gamma$ and $B_s \to \ell^+\ell^-$. In most cases, however, the suggested approaches to threshold radiative effects involve certain explicit or implicit assumptions pertinent to the squark-mass and the quark-Yukawa structures of the theory, such as the dominance of the top quark in the FCNC transition amplitudes. We term the latter assumption the t-quark dominance hypothesis. Moreover, some of the approaches neglect higher-order terms in the resummation of threshold corrections, which become very important in the large-$\tan\beta$ regime of the theory [16].

In this paper we derive the effective Lagrangian that properly takes into account the resummation of higher-order threshold effects on Higgs-boson FCNC interactions to fermions. To accomplish this in Section 2, we also avoid the imposition of particular assumptions on the structure of the soft squark masses and the quark-Yukawa couplings of the theory. Moreover, we do not rely on specific kinematic approximations to the transition amplitudes, such as the aforementioned t-quark dominance hypothesis in the FCNC matrix elements. In our derivation of the effective Lagrangian, we also consider the possibility of CP violation through two sources: (i) the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [17] and (ii) the soft SUSY-violating mass terms. As we explicitly demonstrate in Section 3, our resummed FCNC effective Lagrangian gives rise to new testable predictions for CP-conserving as well as CP-violating observables related to the K- and B-meson systems. In the same section, we also qualitatively discuss the implications of $\tan\beta$-enhanced Higgs-mediated interactions for ϵ'/ϵ and the decays $B_d \to \phi K_S$ and $B_d \to J/\Psi K_S$. Section 4 is devoted to our numerical analysis of a number of K- and B-meson observables, such as ΔM_K, ϵ_K, $\Delta M_{B_{d,s}}$, $\mathcal{B}(B_{d,s} \to \ell^+\ell^-)$ and their associated leptonic CP asymmetries. Our conclusions are summarized in Section 5.

2 Resummed FCNC effective Lagrangian

To elucidate our points, let us first consider the following simple form for the effective Yukawa Lagrangian governing the Higgs-mediated FCNC interactions in the d-quark sector:

$$-\mathcal{L}_Y \;=\; \bar{d}^0_R\, \mathbf{h_d} \Big[\, \Phi_1^{0*} \;+\; \Phi_2^{0*} \Big(\hat{\mathbf{E}}_\mathbf{g} \;+\; \hat{\mathbf{E}}_\mathbf{u}\, \mathbf{h_u}{}^\dagger \mathbf{h_u}\Big)\,\Big]\, d^0_L \;+\; \Phi^0_2\, \bar{u}^0_R\, \mathbf{h_u}\, u^0_L \;+\; \text{H.c.}\,, \tag{2.1}$$

where $\Phi^0_{1,2}$ are the neutral components of the two Higgs doublets[1] and the superscript '0' on the d- and u-type quarks denotes fields in the interaction basis. In (2.1), $\mathbf{h_d}$ and $\mathbf{h_u}$ are 3×3-dimensional down- and up-quark Yukawa matrices, and

$$\hat{\mathbf{E}}_\mathbf{g} \;=\; \mathbf{1}\, \frac{2\alpha_s}{3\pi}\, m^*_{\tilde{g}} \mu^*\, I(m^2_{\tilde{Q}_L}, m^2_{\tilde{d}_R}, |m_{\tilde{g}}|^2)\,, \tag{2.2}$$

$$\hat{\mathbf{E}}_\mathbf{u} \;=\; \mathbf{1}\, \frac{1}{16\pi^2}\, \mu^* A^*_U\, I(m^2_{\tilde{Q}_L}, m^2_{\tilde{u}_R}, |\mu|^2) \tag{2.3}$$

are finite non-holomorphic radiative effects, where the loop integral $I(x,y,z)$ is given by

$$I(x,y,z) \;=\; \frac{xy\ln(x/y) \;+\; yz\ln(y/z) \;+\; xz\ln(z/x)}{(x-y)\,(y-z)\,(x-z)}\;, \tag{2.4}$$

with $I(x,x,x) = 1/(2x)$. To keep things simple in the beginning, we have assumed that the bilinear and trilinear soft SUSY-breaking masses in (2.2) and (2.3), $m_{\tilde{Q}_L}$, $m_{\tilde{u}_R}$, $m_{\tilde{d}_R}$ and A_U, are flavour-diagonal and universal already at the SUSY scale. The consequences of relaxing this assumption will be discussed later on.

From (2.1), we can easily write down the effective Lagrangian relevant to the effective d- and u-type quark masses:

$$-\mathcal{L}_\text{mass} \;=\; \frac{v_1}{\sqrt{2}}\, \bar{d}^0_R\, \mathbf{h_d} \Big[\, \mathbf{1} \;+\; \tan\beta\, \Big(\hat{\mathbf{E}}_\mathbf{g} \;+\; \hat{\mathbf{E}}_\mathbf{u}\, \mathbf{h_u}{}^\dagger \mathbf{h_u}\Big)\,\Big]\, d^0_L \;+\; \frac{v_2}{\sqrt{2}}\, \bar{u}^0_R\, \mathbf{h_u}\, u^0_L \;+\; \text{H.c.} \tag{2.5}$$

[1]Throughout the paper, we follow the notation and the CP-phase conventions of [18].

Our next step is to redefine the quark fields as follows:

$$u_L^0 \;=\; \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}\, u_L\,,\quad d_L^0 \;=\; \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}\,\mathbf{V}\, d_L\,,\quad u_L^0 \;=\; \mathcal{U}_{\mathbf{R}}^{\mathbf{u}}\, u_R\,,\quad d_R^0 \;=\; \mathcal{U}_{\mathbf{R}}^{\mathbf{d}}\, d_R\,, \tag{2.6}$$

where $\mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}$, $\mathcal{U}_{\mathbf{R}}^{\mathbf{u}}$, $\mathcal{U}_{\mathbf{R}}^{\mathbf{d}}$ and $\mathbf{V}$ are 3-by-3 unitary matrices that relate the weak to mass eigenstates of quarks. Evidently, $\mathbf{V}$ is by construction the physical CKM matrix. Substituting (2.6) into (2.5) yields

$$\begin{aligned} -\mathcal{L}_{\rm mass} &= \frac{v_1}{\sqrt{2}}\,\bar{d}_R\,\mathcal{U}_{\mathbf{R}}^{\mathbf{d}\,\dagger}\mathbf{h_d}\mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}\left[\mathbf{1}\;+\;\tan\beta\left(\hat{\mathbf{E}}_{\mathbf{g}}\;+\;\hat{\mathbf{E}}_{\mathbf{u}}|\hat{\mathbf{h}}_{\mathbf{u}}|^2\right)\right]\mathbf{V}\,d_L\;+\;\frac{v_2}{\sqrt{2}}\,\bar{u}_R\,\hat{\mathbf{h}}_{\mathbf{u}}\,u_L\;+\;\text{H.c.}\\ &=\;\bar{d}_R\,\hat{\mathbf{M}}_{\mathbf{d}}\,d_L\;+\;\bar{u}_R\,\hat{\mathbf{M}}_{\mathbf{u}}\,u_L\;+\;\text{H.c.}\,, \end{aligned} \tag{2.7}$$

where $\hat{\mathbf{M}}_{\mathbf{d}}$ and $\hat{\mathbf{M}}_{\mathbf{u}}$ are the physical d- and u-quark mass matrices, respectively. Consistency of (2.7) implies

$$\hat{\mathbf{M}}_{\mathbf{u}} \;=\; \frac{v_2}{\sqrt{2}}\,\hat{\mathbf{h}}_{\mathbf{u}}\,, \tag{2.8}$$

$$\mathcal{U}_{\mathbf{R}}^{\mathbf{d}\,\dagger}\mathbf{h_d}\mathcal{U}_{\mathbf{L}}^{\mathbf{Q}} \;=\; \frac{\sqrt{2}}{v_1}\,\hat{\mathbf{M}}_{\mathbf{d}}\mathbf{V}^\dagger\,\hat{\mathbf{R}}^{-1}\,, \tag{2.9}$$

with

$$\hat{\mathbf{R}} \;=\; \mathbf{1}\;+\;\hat{\mathbf{E}}_{\mathbf{g}}\tan\beta\;+\;\hat{\mathbf{E}}_{\mathbf{u}}\tan\beta\,|\hat{\mathbf{h}}_{\mathbf{u}}|^2\,. \tag{2.10}$$

Observe that the matrix $\hat{\mathbf{R}}$ cannot be zero, as this would result in massless d quarks.

With the help of (2.6) and (2.9), we can now express our original Yukawa Lagrangian (2.1) in terms of the mass eigenstates $d_{L,R}$ and $u_{L,R}$ in a resummed form:

$$\begin{aligned} -\mathcal{L}_Y \;&=\; \frac{\sqrt{2}}{v_1}\,\bar{d}_R\,\hat{\mathbf{M}}_{\mathbf{d}}\mathbf{V}^\dagger\hat{\mathbf{R}}^{-1}\Big[\Phi_1^{0*}\;+\;\Phi_2^{0*}\Big(\hat{\mathbf{E}}_{\mathbf{g}}\;+\;\hat{\mathbf{E}}_{\mathbf{u}}|\hat{\mathbf{h}}_{\mathbf{u}}|^2\Big)\Big]\mathbf{V}\,d_L\;+\;\Phi_2^0\,\bar{u}_R\hat{\mathbf{h}}_{\mathbf{u}}u_L\;+\;\text{H.c.}\\ &=\;\frac{\sqrt{2}}{v_1}\left(\Phi_1^{0*}\;-\;\frac{\Phi_2^{0*}}{\tan\beta}\right)\bar{d}_R\,\hat{\mathbf{M}}_{\mathbf{d}}\mathbf{V}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{V}\,d_L\;+\;\frac{\sqrt{2}}{v_1}\,\frac{\Phi_2^{0*}}{\tan\beta}\,\bar{d}_R\,\hat{\mathbf{M}}_{\mathbf{d}}\,d_L\\ &\quad+\;\Phi_2^0\,\bar{u}_R\hat{\mathbf{h}}_{\mathbf{u}}u_L\;+\;\text{H.c.} \end{aligned} \tag{2.11}$$

In deriving the last equality in (2.11), we have employed the relation: $\hat{\mathbf{R}}^{-1}(\hat{\mathbf{E}}_{\mathbf{g}}+\hat{\mathbf{E}}_{\mathbf{u}}|\hat{\mathbf{h}}_{\mathbf{u}}|^2) = (\mathbf{1}-\hat{\mathbf{R}}^{-1})/\tan\beta$.

It will be very illuminating to see how the FCNC part of (2.11) compares with the literature, e.g. with that obtained in Ref. [8]. To this end, let us first assume that $\mathbf{1} + \hat{\mathbf{E}}_{\mathbf{g}}\tan\beta \neq 0$ and decompose $\hat{\mathbf{R}}^{-1}$ as follows:

$$\hat{\mathbf{R}}^{-1} \;=\; \frac{\mathbf{1}}{\mathbf{1}+\hat{\mathbf{E}}_{\mathbf{g}}\tan\beta}\;+\;\chi_{\rm FC}\,, \tag{2.12}$$

where $\chi_{\rm FC}$ is the diagonal matrix

$$\chi_{\rm FC} \ = \ - \, \frac{\hat{\mathbf{E}}_{\mathbf{u}}\, |\hat{\mathbf{h}}_{\mathbf{u}}|^2 \, \tan\beta}{(\mathbf{1} \, + \, \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta)\, \hat{\mathbf{R}}} \ . \tag{2.13}$$

Making use of the above linear decomposition of $\hat{\mathbf{R}}^{-1}$ and the unitarity of the CKM matrix in (2.11), the FCNC part of our resummed effective Lagrangian reads

$$-\mathcal{L}_{\rm FCNC} \ = \ \frac{\sqrt{2}}{v_1} \left(\Phi_1^{0*} \ - \ \frac{\Phi_2^{0*}}{\tan\beta} \right) \bar{d}_{iR}\, m_{d_i} \left(V^*_{ti} \chi^{(t)}_{\rm FC} V_{tj} \ + \ V^*_{ci} \chi^{(c)}_{\rm FC} V_{cj} \ + \ V^*_{ui} \chi^{(u)}_{\rm FC} V_{uj} \right) d_{jL} \, , \tag{2.14}$$

where $\chi^{(u,c,t)}_{\rm FC}$ are the diagonal entries of $\chi_{\rm FC}$ and summation over $i,j = d,s,b$ is understood. The term proportional to $\chi^{(t)}_{\rm FC}$ gives the top quark contribution, which is the result of [8] and subsequent articles [9, 12, 13]. However, we should remark here that the frequently-used approximation called above the top-quark dominance hypothesis, which results from assuming that only the top quark dominates the FCNC Higgs couplings to d quarks, cannot be entirely justified from considerations based only on minimal flavour-violation models [8, 9, 12, 13], for which the CKM matrix is the only source of flavour and CP violations. In fact, the other terms in (2.14) and especially the one proportional to $\chi^{(c)}_{\rm FC}$ due to the charm-quark contribution becomes very relevant in the limit $\mathbf{1} + \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta \to 0$. Also, in the same limit in (2.14), the singularity in $\chi^{(t)}_{\rm FC}$ is canceled against the singularities of $\chi^{(c)}_{\rm FC}$ and $\chi^{(u)}_{\rm FC}$ as a result of the unitarity of $\mathbf{V}$. In this context, we should note that the limit $\mathbf{1} + \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta \to 0$ is not attainable before the theory itself reaches a non-perturbative regime. Requiring that all d-quark Yukawa couplings are perturbative, we can estimate the lower bound, $|\mathbf{1} + \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta| \gtrsim 2.5 \times 10^{-2}$, for $\tan\beta = 50$.[2] Although $|\mathbf{1} \, + \, \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta|$ must not vanish in perturbation theory, it can be sufficiently close to zero, so that the c-quark contribution becomes competitive with the t-quark one.

So far, we have assumed that the radiatively-induced matrices $\hat{\mathbf{E}}_{\mathbf{g}}$ and $\hat{\mathbf{E}}_{\mathbf{u}}$ in the effective Yukawa Lagrangian (2.1) are proportional to the unity matrix. However, this assumption of flavour universality is rather specific, as it gets generally invalidated by the mixing of the squark generations, the soft trilinear Yukawa couplings and renormalization-group (RG) running of the soft SUSY-breaking parameters from the unification to the low-energy scale. In this respect, the minimal-flavour-violation hypothesis, although better motivated, should also be viewed as a very particular way from minimally departing from universality.

[2] To obtain this lower limit, we simply take the trace of the square of (2.9) and demand that ${\rm Tr}|\hat{\mathbf{h}}_{\mathbf{d}}|^2 \ < \ 3$, or equivalently ${\rm Tr}[(\hat{\mathbf{R}}^{-1})^\dagger V |\hat{\mathbf{M}}_{\mathbf{d}}|^2 V^\dagger \hat{\mathbf{R}}^{-1}] \ < \ 3v_1^2/2$. The latter implies that $|\mathbf{1} \, + \, \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta| \ \gtrsim \ (m_s \tan\beta)/(\sqrt{3}\, m_t) \ = \ 5 \times 10^{-4} \tan\beta$. Finally, it is amusing to notice that if $\Im m \hat{\mathbf{E}}_{\mathbf{g}} = 0$ and $\Re e \hat{\mathbf{E}}_{\mathbf{g}} < 0$, an additional upper bound on $\tan\beta$, $\tan\beta \lesssim 1/|\hat{\mathbf{E}}_{\mathbf{g}}|$, may be derived.

Nevertheless, given that threshold radiative effects on the up sector are negligible, especially for large values of $\tan\beta$, it is straightforward to derive the general resummed form for the Higgs-mediated FCNC effective Lagrangian. Starting from general non-diagonal matrices $\tilde{\mathbf{E}}_{\mathbf{g}}$ and $\tilde{\mathbf{E}}_{\mathbf{u}}$ in (2.1) and following steps very analogous to those from (2.1) to (2.11), we arrive at the same form as in (2.11) for the resummed FCNC effective Lagrangian, but with $\hat{\mathbf{E}}_{\mathbf{g}}$, $\hat{\mathbf{E}}_{\mathbf{u}}$ and $\hat{\mathbf{R}}$ replaced by

$$
\begin{aligned}
\mathbf{E_g} &= \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}\dagger} \tilde{\mathbf{E}}_{\mathbf{g}} \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}\,, \qquad\qquad \mathbf{E_u} = \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}\dagger} \tilde{\mathbf{E}}_{\mathbf{u}} \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}\,, & (2.15)\\
\mathbf{R} &= \mathbf{1} + \mathbf{E_g} \tan\beta + \mathbf{E_u} \tan\beta\, |\hat{\mathbf{h}}_{\mathbf{u}}|^2 + \dots\,, & (2.16)
\end{aligned}
$$

where the ellipses in (2.16) denote additional (generically sub-dominant) threshold effects [19]. Notice that besides the possibility of squark non-universality, the unitary matrix $\mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}$ in (2.15), which is only constrained by the relation $|\hat{\mathbf{h}}_{\mathbf{u}}|^2 = \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}\dagger} \mathbf{h_u}^\dagger \mathbf{h_u} \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}$, introduces additional non-trivial model dependence in the matrices $\mathbf{E_g}$, $\mathbf{E_u}$ and $\mathbf{R}$. Moreover, even in minimal flavour-violating scenarios, $\mathbf{R}$ may contain additional threshold contributions proportional to $\mathcal{U}_{\mathbf{L}}^{\mathbf{Q}\dagger} \mathbf{h_d}^\dagger \mathbf{h_d} \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}$ induced by RG running of the squark masses from the unification to the soft SUSY-breaking scale. Those contributions may also be resummed individually by taking appropriately the hermitian square of the modified (2.9) and solving for $\mathcal{U}_{\mathbf{L}}^{\mathbf{Q}\dagger} \mathbf{h_d}^\dagger \mathbf{h_d} \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}$. This last step may involve the use of iterative or other numerical methods. Thus, even in this general case with a non-universal squark sector, the resummed FCNC couplings of the Higgs bosons to down-type quarks can be successfully parameterized in terms of a well-defined set of parameters, i.e. the soft SUSY-breaking masses, the u- and d-quark masses, and the unitary matrices $\mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}$ and $\mathbf{V}$.

In our last step in deriving the resummed FCNC effective Lagrangian, we express the Higgs fields $\Phi^0_{1,2}$ in terms of their mass eigenstates $H_{1,2,3}$ and the neutral would-be Goldstone boson G^0 in the presence of CP violation [20]. Following the conventions of [18], we relate the weak to mass eigenstates through the linear transformations:

$$
\begin{aligned}
\Phi_1^0 &= \frac{1}{\sqrt{2}}\Big[O_{1i} H_i + i\Big(\cos\beta\, G^0 - \sin\beta\, O_{3i} H_i\Big)\Big]\,,\\
\Phi_2^0 &= \frac{1}{\sqrt{2}}\Big[O_{2i} H_i + i\Big(\sin\beta\, G^0 + \cos\beta\, O_{3i} H_i\Big)\Big]\,, & (2.17)
\end{aligned}
$$

where O is a 3-by-3 orthogonal matrix that accounts for CP-violating Higgs-mixing effects [21]. If we substitute the weak Higgs fields $\Phi^0_{1,2}$ by virtue of (2.17) into (2.11), we obtain the properly resummed effective Lagrangian for the diagonal as well as off-diagonal Higgs interactions to d quarks,

$$
\mathcal{L}_{H_i \bar{d} d'} = -\frac{g_w}{2M_W} \sum_{i=1}^{3} H_i\, \bar{d}\, \Big(\hat{\mathbf{M}}_{\mathbf{d}}\, \mathbf{g}^L_{H_i \bar{d} d'}\, P_L + \mathbf{g}^R_{H_i \bar{d} d'}\, \hat{\mathbf{M}}_{\mathbf{d'}} P_R \Big)\, d'\,, \qquad (2.18)
$$

where $P_{L\,(R)} = [1 - (+)\,\gamma_5]/2$ and

$$\mathbf{g}^L_{H_i\bar{d}d'} \;=\; \mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V}\,\frac{O_{1i}}{\cos\beta} \;+\; \Big(\mathbf{1} \;-\; \mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V}\Big)\,\frac{O_{2i}}{\sin\beta} \;-\; i\,\Big(\mathbf{1} \;-\; \frac{1}{\cos^2\beta}\,\mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V}\Big)\,\frac{O_{3i}}{\tan\beta}\;,$$

$$\mathbf{g}^R_{H_i\bar{d}d'} \;=\; (\,\mathbf{g}^L_{H_i\bar{d}d'}\,)^\dagger\,. \tag{2.19}$$

The 3×3-dimensional matrix $\mathbf{R}$ in (2.19), which resums all radiative threshold effects, is given by (2.16). Equation (2.18), along with (2.19), constitutes the major result of the present paper, which will be extensively used in our phenomenological discussions in Section 3.

Finally, let us summarize the most important properties of the resummed effective Lagrangian (2.18):

(i) The FCNC interactions in (2.18) are described by $\tan^2\beta$-enhanced terms that are proportional to O_{1i} and O_{3i} and to $\mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V}$ in (2.19). These $\tan^2\beta$-enhanced FCNC terms properly take into account resummation,[3] squark-mass non-universality and CP-violating effects.

(ii) The resummation matrix $\mathbf{R}$ controls the strength of the Higgs-mediated FCNC effects. For instance, if $\mathbf{R}$ is proportional to unity, then a kind of a Glashow–Iliopoulos–Maiani (GIM) cancellation mechanism [22] becomes operative and the Higgs-boson contributions to all FCNC observables vanish identically in this case. Furthermore, as well as the top quark, the other two lighter up-type quarks can also give significant contributions to FCNC transition amplitudes, which are naturally included in (2.18) through the resummation matrix $\mathbf{R}$.

(iii) In the CP-invariant limit of the theory, the effective couplings $\mathbf{g}^{L,R}_{H_i\bar{d}d'}$ are either pure real or pure imaginary numbers. Moreover, in the limit $\mathbf{V}\to\mathbf{1}$, the effective Lagrangian (2.18) of the diagonal Higgs-couplings to down quarks is in excellent agreement with the one presented in [18].

(iv) If $M_{H^+}\sim M_{H_{2,3}}\gg M_{H_1}$, one can show that $O_{11}\approx\cos\beta$, $O_{21}\approx\sin\beta$ and $O_{31}\approx 0$. In this case, the H_1-coupling to d quarks becomes SM like and so H_1-mediated FCNC

[3] In addition to the non-holomorphic contributions we have been considering here, there are in general holomorphic radiative effects on the Φ^0_1 coupling to d quarks which have an analogous matrix structure, i.e. $\varepsilon_{\mathbf{g}} + \varepsilon_{\mathbf{u}}|\hat{\mathbf{h}}_{\mathbf{u}}|^2$. These additional holomorphic terms are generally small, typically of order 10^{-2}, and only slightly modify the form of the matrix $\mathbf{R}$ to: $\mathbf{R} = \mathbf{1} + (\mathbf{1} + \varepsilon_{\mathbf{g}} + \varepsilon_{\mathbf{u}}|\hat{\mathbf{h}}_{\mathbf{u}}|^2)^{-1}(\mathbf{E}_{\mathbf{g}} + \mathbf{E}_{\mathbf{u}}|\hat{\mathbf{h}}_{\mathbf{u}}|^2)\tan\beta$. Obviously, such a modification does not alter the general form of the effective couplings in (2.19) and is beyond the one-loop order of our resummation. Therefore, these additional small holomorphic terms can be safely neglected.

effects are getting very suppressed. However, in the above decoupling limit of the two heavier Higgs bosons, FCNC effects in the H_2-H_3 sector do not decouple, are $\tan\beta$-enhanced and can therefore be sizeable.

In the next section, we will closely study the above properties of the $\tan\beta$-enhanced FCNC effects mediated by Higgs bosons on rare processes and CP asymmetries related to the K- and B-meson systems.

3 Applications to K- and B-meson systems

We shall now analyze the impact of our resummed effective Lagrangian (2.18) for Higgs-mediated FCNC interactions on representative K- and B-meson observables. For comprehensive reviews on K- and B-meson physics, we refer the reader to [23, 24, 25].

3.1 ΔM_K, ϵ_K and ϵ'/ϵ

Our starting point is the effective Hamiltonian for the $\Delta S = 2$ interactions,

$$H_{\rm eff}^{\Delta S=2} \;=\; \frac{G_F^2}{16\pi^2}\, M_W^2 \sum_i C_i(\mu)\, Q_i(\mu)\ , \tag{3.1}$$

where $G_F = 1.16639 \times 10^{-5}$ GeV^{-2} is the Fermi constant, $C_i(\mu)$ are the scale-dependent Wilson coefficients associated to the $\Delta S = 2$ quark-dependent operators Q_i. Note that the CKM matrix elements in (3.1) have been absorbed into the Wilson coefficients. The $\Delta S = 2$ operators Q_i may be summarized as follows:

$$\begin{aligned}
Q_1^{\rm VLL} &= (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_L d)\,, & Q_1^{\rm VRR} &= (\bar{s}\gamma_\mu P_R d)(\bar{s}\gamma^\mu P_R d)\,,\\
Q_1^{\rm LR} &= (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_R d)\,, & Q_2^{\rm LR} &= (\bar{s}P_L d)(\bar{s}P_R d)\,,\\
Q_1^{\rm SLL} &= (\bar{s}P_L d)(\bar{s}P_L d)\,, & Q_1^{\rm SRR} &= (\bar{s}P_R d)(\bar{s}P_R d)\,,\\
Q_2^{\rm SLL} &= (\bar{s}\sigma_{\mu\nu} P_L d)(\bar{s}\sigma^{\mu\nu} P_L d)\,, & Q_2^{\rm SRR} &= (\bar{s}\sigma_{\mu\nu} P_R d)(\bar{s}\sigma^{\mu\nu} P_R d)\,,
\end{aligned} \tag{3.2}$$

with $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$. Here, much of our discussion and notation follows Ref. [26].

It now proves convenient to decompose both the K^0-$\bar{K}^0$ mass difference ΔM_K and the known CP-violating mixing parameter ϵ_K into a SM and a SUSY contribution:

$$\begin{aligned}
\Delta M_K &= M_{K_L} - M_{K_S} \;=\; \Delta M_K^{\rm SM} + \Delta M_K^{\rm SUSY}\ ,\\
\epsilon_K &= \epsilon_K^{\rm SM} + \epsilon_K^{\rm SUSY}\ .
\end{aligned} \tag{3.3}$$

To a good approximation, one has

$$\Delta M_K^{\rm SM,\,SUSY} \;=\; 2\,\Re e\,\langle \bar{K}^0|\,H_{\rm eff}^{\Delta S=2}\,|K^0\rangle_{\rm SM,\,SUSY}\ , \tag{3.4}$$

$$\epsilon_K^{\rm SM,\,SUSY} \;=\; \frac{\exp(i\pi/4)}{\sqrt{2}\,\Delta M_K}\,\Im m\,\langle \bar{K}^0|\,H_{\rm eff}^{\Delta S=2}\,|K^0\rangle_{\rm SM,\,SUSY}\ . \tag{3.5}$$

The SUSY contribution to the matrix element $\langle \bar{K}^0|H_{\rm eff}^{\Delta S=2}|K^0\rangle_{\rm SUSY}$ may be written down as [26, 27]

$$\begin{aligned}\langle \bar{K}^0|\,H_{\rm eff}^{\Delta S=2}\,|K^0\rangle_{\rm SUSY} \;=\;& \frac{G_F^2}{12\pi^2}\,M_W^2 m_K F_K^2\eta_2\hat{B}_K\,\Big[\,\bar{P}_1^{\rm VLL}\,\Big(C_1^{\rm VLL}\,+\,C_1^{\rm VRR}\Big)\\ &+\bar{P}_1^{\rm LR}\,C_1^{\rm LR}\;+\;\bar{P}_2^{\rm LR}\,C_2^{\rm LR}\\ &+\bar{P}_1^{\rm SLL}\,\Big(C_1^{\rm SLL}\,+\,C_1^{\rm SRR}\Big)\;+\;\bar{P}_2^{\rm SLL}\,\Big(C_2^{\rm SLL}\,+\,C_2^{\rm SRR}\Big)\Big]\,,\end{aligned} \tag{3.6}$$

where $m_K = 498$ MeV, $F_K = 160$ MeV and $\bar{P}$'s are the next-to-leading order (NLO) QCD factors that include the relevant hadronic matrix elements [26, 28, 29, 31, 27]. At the scale $\mu = 2$ GeV, they are given by [26]

$$\bar{P}_1^{\rm VLL} = 0.25\,,\quad \bar{P}_1^{\rm LR} = -18.6\,,\quad \bar{P}_2^{\rm LR} = 30.6\,,\quad \bar{P}_1^{\rm SLL} = -9.3\,,\quad \bar{P}_2^{\rm SLL} = -16.6\ . \tag{3.7}$$

On obtaining (3.7), we have used the numerical values: $\eta_2 = 0.57$ and $\hat{B}_K = 0.85 \pm 0.15$.

From studies in the CP-conserving MSSM with minimal flavour violation [27, 9, 13], it is known that for large values of $\tan\beta \gtrsim 40$, the dominant contribution to $\langle \bar{K}^0|H_{\rm eff}^{\Delta S=2}|K^0\rangle_{\rm SUSY}$ comes from Higgs-mediated two-loop double penguin (DP) diagrams proportional to $C_2^{\rm LR}$. Within the framework of our large-$\tan\beta$-resummed FCNC effective Lagrangian (2.18) that includes CP violation, the Wilson coefficients due to DP graphs are found to be

$$\begin{aligned} C_1^{\rm SLL\,(DP)} \;&=\; -\,\frac{16\pi^2 m_s^2}{\sqrt{2}\,G_F M_W^2}\,\sum_{i=1}^{3}\frac{g^L_{H_i\bar{s}d}\,g^L_{H_i\bar{s}d}}{M^2_{H_i}}\ ,\\ C_1^{\rm SRR\,(DP)} \;&=\; -\,\frac{16\pi^2 m_d^2}{\sqrt{2}\,G_F M_W^2}\,\sum_{i=1}^{3}\frac{g^R_{H_i\bar{s}d}\,g^R_{H_i\bar{s}d}}{M^2_{H_i}}\ ,\\ C_2^{\rm LR\,(DP)} \;&=\; -\,\frac{32\pi^2 m_s m_d}{\sqrt{2}\,G_F M_W^2}\,\sum_{i=1}^{3}\frac{g^L_{H_i\bar{s}d}\,g^R_{H_i\bar{s}d}}{M^2_{H_i}}\ ,\end{aligned} \tag{3.8}$$

where the couplings $g^{L,R}_{H_i\bar{s}d}$ are given in (2.19). Note that the DP Wilson coefficients in (3.8) exhibit a $\tan^4\beta$-dependence and, although being two-loop suppressed, they become very significant for large values of $\tan\beta \gtrsim 40$. Analogous DP-graphs have been recently [30] found to dominate the lepton flavour violating B-decays such as $B_{s,d} \to l^+ l^{-'}$. In this case, the second vertex in (3.8) has to be replaced by $g^{L,R}_{H_i \bar{l} l'}$ which can straightforwardly been derived by following our analysis in Section 2 with appropriate replacements for the lepton sector.

In addition to the aforementioned DP contributions due to Higgs-boson exchange graphs, there exist relevant one-loop contributions to $\langle \bar{K}^0 | H^{\Delta S=2}_{\rm eff} | K^0 \rangle_{\rm SUSY}$ at large $\tan\beta$: (i) the t-$H^\pm$ box contribution to $C_2^{\rm LR}$ of the two-Higgs-doublet-model (2HDM) type, and (ii) the one-loop chargino-stop box diagram contributing to $C_1^{\rm SLL}$. The first contribution (i) becomes significant, up to 10%, only in the kinematic region $M_{H^\pm} \approx m_t$. In this case, to a good approximation, $C_1^{\rm SLL}$ may be given by [27, 9]

$$C_2^{\rm LR\,(2HDM)} \approx -\frac{2 m_s m_d}{M_W^2} \, (V_{ts}^* V_{td})^2 \, \tan^2\beta \, . \tag{3.9}$$

Note that the light-quark masses contained in (3.8) and (3.9) are running and are evaluated at the top-quark mass scale, i.e. $m_s(m_t) \simeq 61$ MeV, $m_d(m_t) \simeq 4$ MeV. The second contribution (ii) becomes non-negligible only for small values of the μ-parameter [9, 27], i.e. for $|\mu| \lesssim 200$ GeV.

In view of the above discussion, the kinematic parameter range of interest to us is:

$$M_{\rm SUSY}, \, \mu \; \gg \; m_t \, , \qquad \tan\beta \; \gtrsim \; 40 \, , \tag{3.10}$$

including the case $M_{H^\pm} \approx m_t$, for which the Higgs-mediated DP effects can dominate the K^0-$\bar{K}^0$ transition amplitude. Thus, taking also into account the sub-dominant 2HDM contribution (3.9), formula (3.6) simplifies to

$$\begin{aligned} \langle \bar{K}^0 | \, H^{\Delta S=2}_{\rm eff} \, | K^0 \rangle_{\rm SUSY} \; &= \; 4.6 \times 10^{-11} \ {\rm GeV} \left(\frac{F_K}{160 \ {\rm MeV}} \right)^2 \left(\frac{\eta_2}{0.57} \right) \left(\frac{\hat{B}_K}{0.85} \right) \\ &\times \Big[30.6 \left(C_2^{\rm LR\,(DP)} \; + \; C_2^{\rm LR\,(2HDM)} \right) \; - \; 9.3 \left(C_1^{\rm SLL\,(DP)} \; + \; C_1^{\rm SRR\,(DP)} \right) \Big] \, . \end{aligned} \tag{3.11}$$

Observe that the Wilson coefficients C_1 and C_2 contribute with opposite signs to $\langle \bar{K}^0 | \, H^{\Delta S=2}_{\rm eff} \, | K^0 \rangle_{\rm SUSY}$. Based on (3.11), we will give numerical estimates of ΔM_K and ϵ_K in Section 4.

We now turn our attention to the computation of the direct CP-violation parameter ϵ'/ϵ in the kaon system, due to CP-violating Higgs-mediated FCNC interactions. In the SM, unlike the Z-penguin graphs, the Higgs-penguin contribution to $K \to \pi\pi$, which is proportional to the operator

$$Q_H \; = \; (\bar{s} P_L d) \sum_{q=u,d,s} (\bar{q} q) \; , \tag{3.12}$$

has a suppressed Wilson coefficient proportional to m_q^2/M_H^2, where the SM Higgs-boson mass M_H is subject into the experimental bound: $M_H \gtrsim 114$ GeV. One might even think of the possibility that the operator Q_H in (3.12), which has enhanced Wilson coefficients for $q = c, b$, mixes with the gluonic and electroweak penguins, as well as with the other basis operators in (3.2). However, as was already pointed out in [32], this is not the case, and so the SM-Higgs penguin effects remain negligible.

The situation changes drastically in the MSSM with explicit CP violation, since the Higgs-boson FCNC couplings to down-type quarks are substantially enhanced by $\tan^2\beta$-dependent terms, for large values of $\tan\beta$. Furthermore, besides the CKM phase, the presence of complex soft SUSY-breaking masses with large CP-violating phases may further increase the Higgs-boson FCNC effects on ϵ'/ϵ. In fact, we note that soft CP-odd phases could even be the only source [3] to account for direct CP violation.

To reliably estimate the new SUSY effect on ϵ'/ϵ due to Higgs-boson FCNC interactions, we normalize each individual contribution with respect to the dominant SM contribution arising from the operator $Q_6 = \sum_{q=u,d,s} (\bar{s} P_R q)(\bar{q} P_L d)$ [33, 34], with Wilson coefficient y_6, viz.

$$\frac{\epsilon'}{\epsilon} \;=\; \left(\frac{\epsilon'}{\epsilon}\right)_6 \Big(\Omega_{\rm SM} \;+\; \Omega_{\rm SUSY} \;+\; \Omega^{\rm Higgs}_{\rm SUSY} \Big)\;. \tag{3.13}$$

In the MSSM with minimal flavour violation, the non-Higgs SUSY contribution $\Omega_{\rm SUSY}$ is small [35]. Sizable contributions may be obtained if one relaxes the assumptions of universality and CP conservation in the squark sector [36, 37].

Here, we compute a novel contribution to ϵ'/ϵ, namely the quantity $\Omega^{\rm Higgs}_{\rm SUSY}$ in (3.13), which entirely originates from Higgs-boson exchange diagrams in the CP-violating MSSM. Based on our resummed FCNC effective Lagrangian (2.18), we obtain in the zero strong-phase approximation

$$\begin{aligned}
\Omega^{\rm Higgs}_{\rm SUSY} \;=\; & 2 \sum_{i=1}^{3} \sum_{q=u,d,s} \frac{m_s\, m_q}{M^2_{H_i}} \\
& \times \Bigg[\frac{\Im m(g^L_{H_i\bar{s}d})\, g^S_{H_i\bar{q}q}}{A^2\lambda^5\eta} \left(\frac{\langle \pi^+\pi^-|(\bar{s}P_L d)(\bar{q}q)|K^0\rangle_0}{y_6\, \langle \pi^+\pi^-|Q_6|K^0\rangle_0} \;-\; \frac{1}{|\omega|}\, \frac{\langle \pi^+\pi^-|(\bar{s}P_L d)(\bar{q}q)|K^0\rangle_2}{y_6\, \langle \pi^+\pi^-|Q_6|K^0\rangle_0} \right) \\
& + \frac{\Re e(g^L_{H_i\bar{s}d})\, g^P_{H_i\bar{q}q}}{A^2\lambda^5\eta} \left(\frac{\langle \pi^+\pi^-|(\bar{s}P_L d)(\bar{q}\gamma_5 q)|K^0\rangle_0}{y_6\, \langle \pi^+\pi^-|Q_6|K^0\rangle_0} \;-\; \frac{1}{|\omega|}\, \frac{\langle \pi^+\pi^-|(\bar{s}P_L d)(\bar{q}\gamma_5 q)|K^0\rangle_2}{y_6\, \langle \pi^+\pi^-|Q_6|K^0\rangle_0} \right) \Bigg] \\
& + 2 \sum_{i=1}^{3} \sum_{q=u,d,s} \frac{m_d\, m_q}{M^2_{H_i}}
\end{aligned} \tag{3.14}$$

$$\times \left[\frac{\Im m(g^R_{H_i\bar{s}d})\, g^S_{H_i\bar{q}q}}{A^2\lambda^5\eta} \left(\frac{\langle\pi^+\pi^-|(\bar{s}P_R d)(\bar{q}q)|K^0\rangle_0}{y_6\,\langle\pi^+\pi^-|Q_6|K^0\rangle_0} \;-\; \frac{1}{|\omega|}\, \frac{\langle\pi^+\pi^-|(\bar{s}P_R d)(\bar{q}q)|K^0\rangle_2}{y_6\,\langle\pi^+\pi^-|Q_6|K^0\rangle_0} \right) \right.$$

$$\left. +\; \frac{\Re e(g^R_{H_i\bar{s}d})\, g^P_{H_i\bar{q}q}}{A^2\lambda^5\eta} \left(\frac{\langle\pi^+\pi^-|(\bar{s}P_R d)(\bar{q}\gamma_5 q)|K^0\rangle_0}{y_6\,\langle\pi^+\pi^-|Q_6|K^0\rangle_0} \;-\; \frac{1}{|\omega|}\, \frac{\langle\pi^+\pi^-|(\bar{s}P_R d)(\bar{q}\gamma_5 q)|K^0\rangle_2}{y_6\,\langle\pi^+\pi^-|Q_6|K^0\rangle_0} \right) \right],$$

where the subscripts 0, 2 adhered to the hadronic matrix elements denote the total isospin I of the final states, and $A^2\lambda^5\eta$ is a CKM combination in the Wolfenstein parameterization, which has the value $A^2\lambda^5\eta \approx 1.3\times 10^{-4}$ in the SM. Furthermore, for $\Lambda_{\rm QCD} = 325$ MeV and $m_s = 150$ MeV, the SM Wilson coefficient y_6 and the matrix element of Q_6 take on the values [34]: $y_6 = -0.089$ and $\langle\pi^+\pi^-|Q_6|K^0\rangle_0 = -0.35\ {\rm GeV}^3$, respectively. Also, experimental analyses suggest the value $|\omega| = 0.045$, approximately yielding the SM contribution $\Omega_{\rm SM} \sim 1$ to ϵ'/ϵ in (3.13). Finally, the parameters $g^{S,P}_{H_i\bar{d}d}$ and $g^{S,P}_{H_i\bar{u}u}$ that occur in (3.14) are the diagonal scalar and pseudoscalar couplings of the H_i bosons to u- and d-type quarks [18], whose strengths are normalized to the SM Higgs-boson coupling. These reduced H_i-couplings are given by

$$g^S_{H_i\bar{d}d} \;=\; \tfrac{1}{2}\,(\,g^L_{H_i\bar{d}d} \,+\, g^R_{H_i\bar{d}d}\,)\,, \qquad g^P_{H_i\bar{d}d} \;=\; \tfrac{i}{2}\,(\,g^L_{H_i\bar{d}d} \,-\, g^R_{H_i\bar{d}d}\,)\,, \tag{3.15}$$

$$g^S_{H_i\bar{u}u} \;=\; O_{2i}/\sin\beta\,, \qquad g^P_{H_i\bar{u}u} \;=\; -O_{3i}\cot\beta\,, \tag{3.16}$$

where we have neglected the small radiative threshold effects in the up sector.

On the experimental side, the latest world average result for $\Re e(\epsilon'/\epsilon)$ is [38]

$$\Re e\Big(\frac{\epsilon'}{\epsilon}\Big) \;=\; (1.66\pm 0.16)\times 10^{-3}\,, \tag{3.17}$$

at the 1-σ confidence level. In the light of the experimental result (3.17) and the discussion given above, we may conservatively require that

$$|\Omega^{\rm Higgs}_{\rm SUSY}| \;\lesssim\; 1\,. \tag{3.18}$$

The biggest contribution in the sum over quarks in (3.14) comes from the d-quark and exhibits the qualitative scaling behaviour

$$\Omega^{\rm Higgs}_{\rm SUSY} \;\simeq\; \frac{2m_s m_d}{M^2_H}\,\frac{\tan^3\beta}{|\omega|}\;\times\;\mathcal{O}(1)\,. \tag{3.19}$$

For instance, for $\tan\beta = 50$ and $M_H = 200$ GeV, (3.19) gives $\Omega^{\rm Higgs}_{\rm SUSY} \simeq 0.1\times\mathcal{O}(1)$. Obviously, such a contribution is, in principle, non-negligible, but very sensitively depends on the actual values of the new hadronic matrix elements:

$$(Q^{L,R}_{S,P})_I \;=\; \langle\pi^+\pi^-|\,(\bar{s}P_{L,R}d)\,(\bar{q}(1,\gamma_5)q)\,|K^0\rangle_I\,, \tag{3.20}$$

with $I = 0, 2$. A detailed calculation of the hadronic matrix elements $(Q^{L,R}_{S,P})_I$ in (3.20) will be given elsewhere. In Section 4, however, we will present numerical estimates of ΔM_K and ϵ_K within the context of generic soft SUSY-breaking models.

3.2 ΔM_{B_q}, $B_q \to \ell^+\ell^-$ and associated CP asymmetries

We start our discussion of a set of observables in the B-meson system by first analyzing the B^0_q-$\bar{B}^0_q$ mass difference, ΔM_{B_q} with $q = s, d$, in the CP-violating MSSM at large $\tan\beta$. In the applicable limit of equal B-meson lifetimes, ΔM_{B_q} may be written as the modulus of a sum of a SM and a SUSY term:

$$\Delta M_{B_q} \;=\; 2\,|\langle \bar{B}^0_q|\, H_{\rm eff}^{\Delta B=2}\,|B^0_q\rangle_{\rm SM} \;+\; \langle \bar{B}^0_q|\, H_{\rm eff}^{\Delta B=2}\,|B^0_q\rangle_{\rm SUSY}|\;, \tag{3.21}$$

where the effective $\Delta B = 2$ Hamiltonian $H_{\rm eff}^{\Delta B=2}$ may be obtained from the $\Delta S = 2$ one stated in (3.1), after making the obvious replacements: $s \to b$ and $d \to q$, with $q = d, s$. Proceeding as in Section 3.1, we arrive at analogous closed expressions for the SUSY contributions to the $\Delta B = 2$ transition amplitudes:

$$\begin{aligned}
\langle \bar{B}^0_d|\, H_{\rm eff}^{\Delta B=2}\,|B^0_d\rangle_{\rm SUSY} \;&=\; 1711\ {\rm ps}^{-1}\left(\frac{\hat{B}^{1/2}_{B_d}\,F_{B_d}}{230\ {\rm MeV}}\right)^2\left(\frac{\eta_B}{0.55}\right)\\
&\times\Big[0.88\left(C_2^{\rm LR\,(DP)} \;+\; C_2^{\rm LR\,(2HDM)}\right) \;-\; 0.52\left(C_1^{\rm SLL\,(DP)} \;+\; C_1^{\rm SRR\,(DP)}\right)\Big]\,,\\
\langle \bar{B}^0_s|\, H_{\rm eff}^{\Delta B=2}\,|B^0_s\rangle_{\rm SUSY} \;&=\; 2310\ {\rm ps}^{-1}\left(\frac{\hat{B}^{1/2}_{B_s}\,F_{B_s}}{265\ {\rm MeV}}\right)^2\left(\frac{\eta_B}{0.55}\right)\\
&\times\Big[0.88\left(C_2^{\rm LR\,(DP)} \;+\; C_2^{\rm LR\,(2HDM)}\right) \;-\; 0.52\left(C_1^{\rm SLL\,(DP)} \;+\; C_1^{\rm SRR\,(DP)}\right)\Big]\,.
\end{aligned} \tag{3.22}$$

In deriving (3.22), we have also substituted the values determined in [26, 28, 29, 31, 27] for the NLO-QCD factors, along with their hadronic matrix elements at the scale $\mu = 4.2$ GeV:

$$\bar{P}_1^{\rm LR} \;=\; -0.58\;,\qquad \bar{P}_2^{\rm LR} \;=\; 0.88\;,\qquad \bar{P}_1^{\rm SLL} \;=\; -0.52\;,\qquad \bar{P}_2^{\rm SLL} \;=\; -1.1\;. \tag{3.23}$$

Moreover, the corresponding Wilson coefficients appearing in (3.22) may be recovered from those in (3.8) and (3.9), after performing the quark replacements mentioned above.

Another observable, which is enhanced at large $\tan\beta$, is the pure leptonic decay of B mesons [6, 7, 8, 9, 10, 11, 12, 13, 14], $\bar{B}^0_q \to \ell^+\ell^-$, with $\ell = \mu$, τ. Neglecting contributions proportional to the lighter quark masses $m_{d,s}$, the relevant effective Hamiltonian for $\Delta B = 1$ FCNC transitions, such as $b \to q\ell^+\ell^-$ with $q = d, s$, is given by

$$H_{\rm eff}^{\Delta B=1} \;=\; -\,2\,\sqrt{2}\,G_F\,V_{tb}V^*_{tq}\left(C_S\,\mathcal{O}_S \;+\; C_P\,\mathcal{O}_P \;+\; C_{10}\,\mathcal{O}_{10}\right)\,, \tag{3.24}$$

where

$$\mathcal{O}_S \;=\; \frac{e^2}{16\pi^2}\,m_b\,(\bar{q}P_R b)\,(\bar{\ell}\ell)\;,$$

$$\mathcal{O}_P = \frac{e^2}{16\pi^2}\, m_b\, (\bar{q} P_R b)\, (\bar{\ell}\gamma_5 \ell)\ ,$$
$$\mathcal{O}_{10} = \frac{e^2}{16\pi^2}\, (\bar{q}\gamma^\mu P_L b)\, (\bar{\ell}\gamma_\mu\gamma_5 \ell)\ . \tag{3.25}$$

Employing our resummed FCNC effective Lagrangian (2.18), it is not difficult to compute the Wilson coefficients C_S and C_P at large values of $\tan\beta$:

$$C_S = \frac{2\pi m_\ell}{\alpha_{\rm em}}\, \frac{1}{V_{tb}V^*_{tq}} \sum_{i=1}^{3} \frac{g^R_{H_i\bar{q}b}\, g^S_{H_i\bar{\ell}\ell}}{M^2_{H_i}}\ ,$$
$$C_P = i\, \frac{2\pi m_\ell}{\alpha_{\rm em}}\, \frac{1}{V_{tb}V^*_{tq}} \sum_{i=1}^{3} \frac{g^R_{H_i\bar{q}b}\, g^P_{H_i\bar{\ell}\ell}}{M^2_{H_i}}\ , \tag{3.26}$$

while $C_{10} = -4.221$ is the leading SM contribution. In analogy to (3.15), the reduced scalar and pseudoscalar Higgs couplings to charged leptons $g^{S,P}_{H_i\bar{\ell}\ell}$ in (3.26) are given by

$$g^S_{H_i\bar{\ell}\ell} = \frac{O_{1i}}{\cos\beta}\ , \qquad g^P_{H_i\bar{\ell}\ell} = -\ \tan\beta\, O_{3i}\ , \tag{3.27}$$

where non-holomorphic vertex effects on the leptonic sector have been omitted as being negligibly small.

With the approximations mentioned above, the branching ratio for the $\bar{B}^0_q$ meson decay to $\ell^+\ell^-$ acquires the simple form [7]

$$\mathcal{B}(\bar{B}^0_q \to \ell^+\ell^-) = \tag{3.28}$$
$$\frac{G_F^2\alpha^2_{\rm em}}{16\pi^3}\, M_{B_q}\tau_{B_q}\, |V_{tb}V^*_{tq}|^2 \sqrt{1-\frac{4m^2_\ell}{M^2_{B_q}}}\ \left[\left(1-\frac{4m^2_\ell}{M^2_{B_q}}\right) |F^q_S|^2\ +\ |F^q_P\ +\ 2m_\ell F^q_A|^2 \right]\ ,$$

where τ_{B_q} is the total lifetime of the B_q meson and

$$F^q_{S,P} = -\,\frac{i}{2}\, M^2_{B_q} F_{B_q}\, \frac{m_b}{m_b+m_q}\, C_{S,P}\ , \qquad F^q_A = -\,\frac{i}{2}\, F_{B_q}\, C_{10}\ . \tag{3.29}$$

In our numerical estimates, we ignore the contribution from C_{10}, as being subdominant in the region of large $\tan\beta$, i.e. for $\tan\beta \gtrsim 40$, where all Higgs-particle masses are well below the TeV scale, The SM predictions as well as the current experimental bounds pertinent to $\mathcal{B}(\bar{B}^0_d \to \ell^+\ell^-)$ can be read off from Table 1 in [39].

In the CP-violating MSSM, an equally important class of observables related to $\mathcal{B}(\bar{B}^0_{d,s} \to \ell^+\ell^-)$ [15] is the one probing possible CP asymmetries that can take place in the same leptonic B-meson decays. The leptonic CP asymmetries may shed even light on the CP nature of possible new-physics effects, as the SM prediction for these observables turns out to be dismally small of order 10^{-3} [40]. This SM result is a consequence of the fact that the CP-violating phase in B^0-$\bar{B}^0$-mixing parameter q/p is opposite to the one entering the ratio of the amplitudes $\bar{\mathcal{A}}_{L(R)}(\bar{B}^0_{d,s} \to l^+_{L(R)} l^-_{L(R)})/\mathcal{A}_{L(R)}(B^0_{d,s} \to l^+_{L(R)} l^-_{L(R)})$, such that the net CP-violating effect on the observable parameter $\lambda_{L(R)} = (q/p)(\bar{\mathcal{A}}_{L(R)}/\mathcal{A}_{L(R)})$ almost cancels out.

There are two possible time-dependent CP asymmetries associated with leptonic B-meson decays that are physically allowed:

$$\mathcal{A}_{\rm CP}^{(B^0_q \to l^+_L l^-_L)} \;=\; \frac{\int_0^\infty dt\, \Gamma(B^0_q(t) \to l^+_L l^-_L) \;-\; \int_0^\infty dt\, \Gamma(\bar{B}^0_q(t) \to l^+_R l^-_R)}{\int_0^\infty dt\, \Gamma(B^0_q(t) \to l^+_L l^-_L) \;+\; \int_0^\infty dt\, \Gamma(\bar{B}^0_q(t) \to l^+_R l^-_R)} \tag{3.30}$$

and $\mathcal{A}_{\rm CP}^{(B^0_q \to l^+_R l^-_R)}$, with $L \leftrightarrow R$. Under the assumption that q/p is a pure phase, one finds [40]

$$\mathcal{A}_{\rm CP}^{(B^0_q \to l^+_L l^-_L)} \;=\; -\,\frac{2x_q\, \Im m \lambda_q}{(2\,+\,x_q^2)\,+\,x_q^2\,|\lambda_q|^2}\ , \qquad \mathcal{A}_{\rm CP}^{(B^0_q \to l^+_R l^-_R)} \;=\; -\,\frac{2x_q\, \Im m \lambda_q}{(2\,+\,x_q^2)\,|\lambda_q|^2\,+\,x_q^2}\ , \tag{3.31}$$

where $x_q = \Delta M_{B_q}/\Gamma_{B_q}$ and

$$\lambda_q \;=\; \frac{M_{12}^{q*}}{|M_{12}^q|}\left(\frac{V_{tb}V_{tq}^*}{V_{tb}^*V_{tq}}\right)\frac{\beta_l\, C_S \,+\, C_P \,+\, 2m_l C_{10}/(m_b M_{B_q})}{\beta_l\, C_S^* \,-\, C_P^* \,-\, 2m_l C_{10}/(m_b M_{B_q})}\ . \tag{3.32}$$

In (3.32), $\beta_l = (1 - 4m_l^2/M_{B_q}^2)^{1/2}$, M_{12}^q is the dispersive part of the B^0_q-$\bar{B}^0_q$ matrix element, and $C_{S,P}$ are Wilson coefficients given in (3.26). The miximal value that the leptonic CP asymmetries in (3.31) can reach is $\mathcal{A}_{\rm CP}^{\rm max} = 1/\sqrt{2 + x_q^2}$ and is obtained for $\Im m \lambda_q = |\lambda_q|^2$. From current experimental data [41], one may extract the values $x_d = 0.76$ and $x_s \gtrsim 19$ at the 95% CL, which leads to $\mathcal{A}_{\rm CP}^{\rm max}(B_s) \approx 5\%$ and $\mathcal{A}_{\rm CP}^{\rm max}(B_d) \approx 62\%$.

Within the framework of our resummed FCNC effective Lagrangian, we also improve earlier calculations [40, 15] of the CP asymmetries by including B^0_q-$\bar{B}^0_q$ mixing effects through $M_{12}^{q*}/|M_{12}^q|$ in (3.32). According to our standard approach of splitting the amplitude into a SM and a MSSM part, we obtain for the SM part

$$M_{12}^{q*\,({\rm SM})} \;=\; -\,\frac{G_F^2 M_W^2}{12\pi^2}\; M_{B_q}\eta_B \hat{B}_{B_q} F_{B_q}^2\,(V_{tq}V_{tb}^*)^2\, S_{tt}\ , \tag{3.33}$$

where $S_{tt} \approx 2.38$ is the value of the dominant m_t-dependent loop function for a top-pole mass $m_t = 175$ GeV. The SUSY contribution to $M_{12}^{q*\,({\rm SUSY})} = \langle \bar{B}^0_d|\, H_{\rm eff}^{\Delta B=2}\, |B^0_d\rangle_{\rm SUSY}$ may be obtained from (3.22).

In our last (but not least) application of our resummed effective Lagrangian (2.18) to the B-meson system, we will qualitatively discuss the CP asymmetries related to the time evolution and decay of the B^0_q and $\bar{B}^0_q$ mesons into non-leptonic final states $f_{\rm CP}$ of definite CP. The time-dependence of this category of exclusive CP asymmetries is determined by [42]

$$\mathcal{A}_{f_{\rm CP}}(t) \;=\; -\,C_{f_{\rm CP}}\cos(\Delta M_{B_q} t) \;+\; S_{f_{\rm CP}}\sin(\Delta M_{B_q} t)\ , \tag{3.34}$$

where the CP-violating factors $C_{f_{\rm CP}}$ and $S_{f_{\rm CP}}$ are given by

$$C_{f_{\rm CP}} \ = \ \frac{1 \ - \ |\lambda_{f_{\rm CP}}|^2}{1 \ + \ |\lambda_{f_{\rm CP}}|^2}\ , \qquad S_{f_{\rm CP}} \ = \ \frac{2\Im m\lambda_{f_{\rm CP}}}{1 \ + \ |\lambda_{f_{\rm CP}}|^2}\ . \tag{3.35}$$

The CP-violating parameter $\lambda_{f_{\rm CP}}$ in (3.35) is defined as

$$\lambda_{f_{\rm CP}} \ = \ \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}\ \frac{\langle f_{\rm CP}|\,\mathcal{H}\,|\bar{B}^0\rangle}{\langle f_{\rm CP}|\,\mathcal{H}\,|B^0\rangle}\ , \tag{3.36}$$

where M_{12} and Γ_{12} are the dispersive and absorptive parts of the $B^0_q \to \bar{B}^0_q$ transition amplitude, with $q = d, s$. Experimentally, it is $M_{12} \gg \Gamma_{12}$, which enables one to approximate $\lambda_{f_{\rm CP}}$ in (3.36) as

$$\lambda_{f_{\rm CP}} \ \approx \ e^{-2i\phi_q}\,\frac{\bar{A}_q}{A_q}, \tag{3.37}$$

where

$$-\,2\phi_q \ = \ \arg(M_{12}^*) \ = \ \arg\langle\bar{B}^0_q|\,H^{\Delta B=2}_{\rm eff}|B^0_q\rangle\ , \tag{3.38}$$

and $A_q(\bar{A}_q) = \langle f_{\rm CP}|\,\mathcal{H}\,|B^0_q(\bar{B}^0_q)\rangle$.

In the SM, one finds that $\phi_d^{\rm SM} = \beta = \arg\,[-V_{cd}V^*_{cb}/(V_{td}V^*_{tb})]$ and $\phi_s^{\rm SM} \lesssim \lambda^2 \approx 0.05$. On the other hand, experimental data for the decays $B_d \to J/\Psi K_S$ (or $\bar{b} \to \bar{s}c\bar{c}$ at the quark level) imply that $\sin 2\phi_d = 0.734 \pm 0.054$ [38]. According to our standard procedure, we write ϕ_d as a sum of two contributions: $\phi_d = \phi_d^{\rm SM} + \phi_d^{\rm MSSM}$, where the MSSM contribution can be determined from (3.22). In the same vein [43, 44], we split the decay amplitude A_d and its CP-conjugate counterpart $\bar{A}_d$ into a (dominant) SM part, $A_d^{\rm SM}$, and an MSSM part, $A_d^{\rm MSSM}$. We also introduce the ratio $r_d = |A_d^{\rm MSSM}|/|A_d^{\rm SM}|$ and denote the corresponding weak and strong phases as $\theta_d^{\rm SM,\,MSSM}$ and $\delta_d^{\rm SM,\,MSSM}$, respectively. Then, the MSSM contribution to the CP-violating factors $C_{f_{\rm CP}}$ and $S_{f_{\rm CP}}$ in (3.35) may be computed by

$$\begin{aligned} C_{f_{\rm CP}} \ &= \ 2r_d \sin(\theta_d^{\rm SM} - \theta_d^{\rm MSSM})\,\sin(\delta_d^{\rm SM} - \delta_d^{\rm MSSM})\ , \\ S_{f_{\rm CP}} \ &= \ \sin[2(\phi_d + \theta_d^{\rm SM})] \ + \ 2r_d \cos[2(\phi_d + \theta_d^{\rm SM})]\,\sin(\theta_d^{\rm SM} - \theta_d^{\rm MSSM})\,\cos(\delta_d^{\rm SM} - \delta_d^{\rm MSSM})\ . \end{aligned} \tag{3.39}$$

Even though we have concentrated on the B^0_d meson, a very analogous discussion can be followed for the B^0_s-meson decays.

The CP-violating observable (3.34) can ideally be tested at B-factories by analyzing the decays $B_d \to J/\Psi K_S$ and $B_d \to \phi K_S$, $\eta' K_S$, which correspond at the quark level to $\bar{b} \to \bar{s}c\bar{c}$ and $\bar{b} \to \bar{s}s\bar{s}$. All these decays measure almost the same value of $\phi_d^{\rm SM} = \beta$, since electroweak corrections to the ratio of the decay amplitudes in (3.37) are small [44] and $\theta_d^{\rm SM}(J/\Psi K) < \theta_d^{\rm SM}(\phi K) \lesssim \mathcal{O}(\lambda^2)$. As a consequence, one finds a small difference between the CP-violating factors $S_{J/\Psi K}$ and $S_{\phi K}$ in the SM, i.e. $D_{CP} = |S_{J/\Psi K} - S_{\phi K}| \lesssim 0.05$. However, the recently reported value,

$$D_{CP} \;=\; |S_{J/\Psi K} - S_{\phi K}| \;=\; 1.12 \pm 0.41 \,, \tag{3.40}$$

from the BaBar and Belle collaborations shows a 2.7σ deviation from the SM prediction (for more details see [45, 38]). One might then raise the question whether such a large deviation in D_{CP} could be naturally explained within the MSSM. Here, we will not perform an extensive quantitative analysis of this issue. Instead, we wish to point out that $\tan\beta$-enhanced Higgs-mediated FCNC interactions may potentially account for the observed large value of D_{CP} in the CP-violating MSSM. In particular, within the framework of our resummed effective Lagrangian (2.18), we will argue below that non-zero values for $\theta_d^{\rm MSSM}$ and r_d at a level required for explaining D_{CP} are still possible in (3.39).

The description of the aforementioned non-leptonic $\Delta B = 1$ B-meson decays is based on the effective Hamiltonian

$$H^{\Delta B=1}_{\rm eff} = \frac{G_F}{\sqrt{2}}\, V_{tb}V^*_{ts} \sum_{i=1}^{10} \Big(C_i\mathcal{O}_i \,+\, C_{7\gamma}\mathcal{O}_{7\gamma} \,+\, C_{8g}\mathcal{O}_{8g}\Big) \,+\, \frac{G_F}{\sqrt{2}} \sum_{q=s,c} \Big(C^q_S\mathcal{O}^q_S \,+\, C^q_P\mathcal{O}^q_P\Big)\,, \tag{3.41}$$

which is enlarged to include Higgs-penguin interactions in addition to electroweak penguins. The standard form for the operators $\mathcal{O}_i$, $\mathcal{O}_{7\gamma}$, $\mathcal{O}_{8g}$ may be found in [46]. The new Higgs-mediated operators $\mathcal{O}^q_{S,P}$ in (3.41) are given by

$$\mathcal{O}^q_S \;=\; (\bar{s}P_R b)\,(\bar{q}q)\,, \qquad\qquad \mathcal{O}^q_P \;=\; (\bar{s}P_R b)\,(\bar{q}\gamma_5 q)\,, \tag{3.42}$$

with $q = s, c$. Notice that we have neglected operators of the form $(\bar{s}P_L b)\,(\bar{q}q)$ and $(\bar{s}P_L b)\,(\bar{q}\gamma_5 q)$, since their respective Wilson coefficients are suppressed by extra factors m_s/m_b with respect to those considered in (3.41). In the region of large $\tan\beta$ values, the Wilson coefficients $C^q_{S,P}$ associated with $\mathcal{O}^q_{S,P}$ are dominated by Higgs-boson exchange graphs. From our resummed effective Lagrangian (2.18), we obtain (with $q = s, c$)

$$C^q_S \;=\; -\,2\,m_q m_b \sum_{i=1}^{3} \frac{g^R_{H_i\bar{s}b}\, g^S_{H_i\bar{q}q}}{M^2_{H_i}}\,, \tag{3.43}$$

$$C^q_P \;=\; -\,2i\,m_q m_b \sum_{i=1}^{3} \frac{g^R_{H_i\bar{s}b}\, g^P_{H_i\bar{q}q}}{M^2_{H_i}}\,. \tag{3.44}$$

In the large-$\tan\beta$ regime, these coefficients scale like $C^s_{S,P} \propto m_s \tan^3\beta$ and $C^c_{S,P} \propto m_c \tan\beta$. Since it is $C^c_{S,P} \ll C^s_{S,P}$ for $\tan\beta \gtrsim 40$, this allows us to carry out the crude approximations:

$$\theta_d^{\rm MSSM}(J/\Psi K_S) \;\ll\; \theta_d^{\rm MSSM}(\phi K_S) \;\sim\; \arg\left(\sum_{i=1}^{3} \frac{g^R_{H_i\bar{s}b}\, g^{S,P}_{H_i\bar{s}s}}{M^2_{H_i}} \right) \;=\; \mathcal{O}(1)\,. \tag{3.45}$$

The above large hierarchy in (3.45) for the MSSM weak phases of the two decay amplitudes, $A(B_d \to J/\Psi K_S)$ and $A(B_d \to \phi K_S)$, is very important, as it can be sufficient by itself to explain, through (3.39), the large value of D_{CP} in (3.40). Nevertheless, there can easily exist additional factors that may conspire to obtain a large D_{CP} value. To be specific, the amplitude $A(B_d \to J/\Psi K_S)$ arises at the tree level in the SM, so one expects to be the dominant one, i.e. $r_d^{J/\Psi K} \sim 0$. In such a case, CP-violating effects on $S_{J/\Psi K}$ can only come from the process-independent phase $\phi_d^{\rm MSSM}$ due to the B^0-$\bar{B}^0$ mixing and, given that $\theta_d^{\rm SM}$ is small for both the above decay modes, the phase $\phi_d^{\rm MSSM}$ almost cancels in D_{CP} against a similar contribution present in $S_{\phi K}$. However, the amplitude for the decay $B_d \to \phi K_S$ is induced at the one-loop level in the SM, and its value [46] (within the QCD factorization scheme [47]) becomes comparable, at large values of $\tan\beta$, to the amplitude generated by the operators $\mathcal{O}^q_{S,P}$. In this case, one might expect that $r_d^{\phi K} \sim 1$, which can also lead to large deviations in D_{CP} of the desired order. An extensive quantitative treatment of these asymmetries, in conjunction with the decay $B_d \to \eta' K_S$, is beyond the scope of the present article and may be given elsewhere. Instead, in the next section, we will present numerical predictions for $\Delta M_{B_{d,s}}$, $\mathcal{B}(B^0_q \to \ell^+\ell^-)$ and their associated leptonic CP asymmetries.

4 Numerical estimates

In this section, we shall numerically analyze the impact of the $\tan^2\beta$-enhanced FCNC interactions on a number of K- and B-meson observables which were discussed in detail in Sections 3.1 and 3.2, such as ΔM_K, ϵ_K, ΔM_{B_d}, ΔM_{B_s}, $B_d \to \tau^+\tau^-$, $B_s \to \mu^+\mu^-$ and their associated leptonic CP asymmetries. For our illustrations, we consider two generic low-energy soft SUSY-breaking scenarios, (A) and (B).

In scenario (A), the squark masses are taken to be universal and $\hat{\mathbf{E}}_{\mathbf{g}}$ and $\hat{\mathbf{E}}_{\mathbf{u}}$ are proportional to the unity matrix at the soft SUSY-breaking scale $M_{\rm SUSY}$. The CP-conserving version of this scenario has frequently been discussed in the literature within the context of minimal flavour-violation models, see e.g. [8].

In scenario (B) we assume the existence of a mass hierarchy between the first two generation of squarks and the third generation, namely the first two generations are degenerate and can be much heavier than the third one. In addition, although not mandatory, we assume for simplicity that the model-dependent unitary matrix $\mathcal{U}^{\mathbf{Q}}_{\mathbf{L}}$ in (2.15) equals the unity matrix $\mathbf{1}$. As a result of this choice, $\mathbf{E}_{\mathbf{g}}$ and $\mathbf{E}_{\mathbf{u}}$ become diagonal matrices in this scenario. Clearly, in the limit, in which all squarks are degenerate, scenario (B) coincides with (A).

In Fig. 1 we give a schematic representation of the generic mass spectrum that will be assumed in our numerical analysis. More explicitly, we fix the charged Higgs-boson M_{H^+} to the value 200 GeV. Since the effect of the $\mathrm{SU}(2)_L$ gauginos $\widetilde{W}$ on the resummation matrix $\mathbf{R}$ [14, 15] can be significantly reduced for $m_{\tilde{W}} \ll \mu$, we ignore this contribution by considering the relatively low value $m_{\tilde{W}} \approx 2m_t \ll M_{\mathrm{SUSY}}$ in our numerical estimates, with $m_t = 175$ GeV. As can be seen from Fig. 1, the third-generation soft squark mass $m_{\tilde{t}}$, the μ-parameter, the gluino mass $m_{\tilde{g}}$ and the trilinear soft Yukawa coupling A_U are set, for simplicity reasons, to the common soft SUSY-breaking scale M_{SUSY}, which is typically taken to be 1 TeV.

The soft squark masses of the other two generations are assumed to be equal to $m_{\tilde{q}}$ in our generic framework. To account for a possible hierarchical difference between the mass scales $m_{\tilde{t}}$ and $m_{\tilde{q}}$, we introduce the so-called hierarchy factor ρ, such that $m_{\tilde{q}} = \rho m_{\tilde{t}} = \rho M_{\mathrm{SUSY}}$. As has been mentioned above, models of minimal flavour violation correspond to scenario (A) with $\rho = 1$. As we will see in detail in Sections 4.1 and 4.2, the predictions for the K- and B-meson FCNC observables crucially depend on the values of the hierarchy factor ρ. Equally important modifications in the predictions are obtained for different values of the soft CP-violating phases $\phi_g = \arg(m_{\tilde{g}})$ and $\phi_{A_U} = \arg(A_U)$. In addition, the K- and B-meson FCNC observables exhibit a non-trivial dependence on the CKM phase δ_{CKM}, which is varied independently in our figures.

Although we primarily use $\tan\beta = 50$ and $M_{H^+} = 0.2$ TeV as inputs in our numerical analysis, approximate predictions for other values of the input parameters may be obtained by rescaling the numerical estimates by a factor

$$x_{\mathcal{O}} \;=\; \Big(\frac{\tan\beta}{50}\Big)^n \times \Big(\frac{0.2\ \mathrm{TeV}}{M_{H^+}}\Big)^k , \tag{4.1}$$

where the integers n and k depend on the FCNC observable $\mathcal{O}$ under study. Such a rescaling proves to be fairly accurate for $\tan\beta \gtrsim 40$ and $M_{H^+} \gtrsim 150$ GeV, which is the kinematic region of our interest.

4.1 ΔM_K and $|\epsilon_K|$

The SM effects on ΔM_K and $|\epsilon_K|$ were extensively discussed in the literature [48], so we will not dwell upon this issue here as well. Instead, we assume that the SM explains well the experimental results for the above two observables [41]:

$$\Delta M_K^{\mathrm{exp}} \;=\; (3.490 \pm 0.006) \times 10^{-12}\ \mathrm{MeV} \tag{4.2}$$

$$|\epsilon_K^{\mathrm{exp}}| \;=\; (2.282 \pm 0.017) \times 10^{-3}\ . \tag{4.3}$$

Given the significant uncertainties in the calculation of hadronic matrix element, however, our approach will be to constrain the soft SUSY-breaking parameters by conservatively requiring that $\Delta M_K^{\rm SUSY}$ and $|\epsilon_K^{\rm SUSY}|$ do not exceed in size the SM predictions.[4]

To start with, we display in Fig. 2 numerical values for the Higgs-boson DP effects on ΔM_K and $|\epsilon_K|$ as functions of the gluino phase $\arg m_{\tilde g}$, where the hierarchy factor ρ and the phase $\phi_{A_U} = \arg(A_{t,b})$ of the soft SUSY-breaking trilinear Yukawa couplings assume the discrete values: $(\rho,\ \phi_{A_U}) = (1,\ 0^\circ),\ (10,\ 0^\circ),\ (1,\ 90^\circ),\ (10,\ 90^\circ), (1,\ 180^\circ),\ (10,\ 180^\circ)$. According to our CP-phase conventions [18], μ is always taken be positive, while the CKM phase $\delta_{\rm CKM}$ is chosen to its maximal value 90°. The subdominant one-loop 2HDM contribution coming from $W^\pm$-$H^\mp$ box graphs [cf. (3.9)] has also been indicated by an arrow in Fig. 2. Predictions for M_{H^+} and $\tan\beta$ values other than those shown in Fig. 1 may be approximately obtained by multiplying the numerical estimates by a factor $x_{\mathcal{O}} = (\tan\beta/50)^4 \times (0.2\ {\rm TeV}/M_{H^+})^2$. We observe in Fig. 2 that the resulting values for $\Delta M_K^{\rm SUSY}$ can exceed the experimental error in (4.2) by an order of magnitude, for $\rho = 10$ and $|\phi_{A_U}|,\ |\phi_{\tilde g}| \gtrsim 90^\circ$. For the same inputs, $|\epsilon_K^{\rm SUSY}|$ takes on values comparable to the experimentally measured one (4.3). Here, we should stress the fact that universal squark-mass scenarios corresponding to $\rho = 1$ can still predict sizeable effects on ϵ_K. This non-zero result should be contrasted with the one of the gluino-squark box contributions to ΔM_K and $|\epsilon_K|$ [49] which do vanish in the limit of strictly degenerate squarks due to a SUSY-GIM-cancellation mechanism.

In our case of Higgs-mediated FCNC observables, however, the situation is slightly different. As we have discussed in Section 2, the size of the FCNC effects is encoded in the flavour structure of the 3-by-3 resummation matrix $\mathbf{R}$. To be more specific, the Higgs-mediated FCNC interactions in (2.18) are described by the $\tan\beta$-enhanced terms proportional to

$$(\mathbf{V}^\dagger\,\mathbf{R}^{-1}\,\mathbf{V})_{dd'} \;=\; V^*_{ud}\,R_u^{-1}\,V_{ud'} \;+\; V^*_{cd}\,R_c^{-1}\,V_{cd'} \;+\; V^*_{td}\,R_t^{-1}\,V_{td'}\ , \tag{4.4}$$

where d and d' collectively denote all down-type quarks. For the parameters adopted in Fig. 1, the quantities $R^{-1}_{u,c,t}$ may be simplified further to[5]

$$R_u^{-1} \;\approx\; \Big[1\,+\,\Big(\frac{\alpha_s}{3\pi\rho^2}\,e^{-i\phi_{\tilde g}}\;+\;\frac{|h_u|^2}{32\pi^2\rho^2}\,e^{-i\phi_{A_U}}\Big)\tan\beta\Big]^{-1}\ ,$$

$$R_c^{-1} \;\approx\; \Big[1\,+\,\Big(\frac{\alpha_s}{3\pi\rho^2}\,e^{-i\phi_{\tilde g}}\;+\;\frac{|h_c|^2}{32\pi^2\rho^2}\,e^{-i\phi_{A_U}}\Big)\tan\beta\Big]^{-1}\ ,$$

[4]Both ΔM_K and $|\epsilon_K|$ place important constraints on the ρ-η plane of the unitarity triangle. The so-derived limits can be used to constrain new physics. In this case, a global fit of all the relevant FCNC observables to the unitarity triangle might be more appropriate. We intend to address this issue in a future work.

[5]For $\rho \gtrsim 10$, the first two equations in (4.5) may be better approximated by replacing $\rho \to \rho/\sqrt{2}$.

$$R_t^{-1} \;=\; \Big[1 \,+\, \Big(\frac{\alpha_s}{3\pi}\, e^{-i\phi_{\tilde g}} \,+\, \frac{|h_t|^2}{32\pi^2}\, e^{-i\phi_{A_U}} \Big) \tan\beta \Big]^{-1} . \tag{4.5}$$

Then, from (4.5), it is easy to see that the off-diagonal elements of $\mathbf{V}^\dagger \mathbf{R}^{-1} \mathbf{V}$ increase if ϕ_{A_U}, $\phi_{\tilde g} = \pm\pi$ and so the effective couplings $g^{L,R}_{H_i \bar d d'}$, thereby giving rise to enhanced predictions. This is a very generic feature which is reflected in Fig. 2 and, as we will see in Section 4.2, also holds true for our numerical estimates of B-meson FCNC observables.

Neglecting the small Yukawa couplings of the first two generations and making use of the unitarity of $\mathbf{V}$, we find for $d \neq d'$

$$(\mathbf{V}^\dagger \mathbf{R}^{-1} \mathbf{V})_{dd'} \;\propto\; V^*_{td} V_{td'} \left[\frac{\alpha_s}{3\pi}\, e^{-i\phi_{\tilde g}} \left(\frac{1}{\rho^2} \,-\, 1 \right) \,-\, \frac{|h_t|^2}{32\pi^2}\, e^{-i\phi_{A_U}} \right] \tan\beta\, . \tag{4.6}$$

If $\rho = 1$, the dominant FCNC effect originates from the second term in the square brackets of (4.6), provided $|1 + \frac{\alpha_s}{3\pi}\, e^{-i\phi_{\tilde g}} \tan\beta| \gg 5 \times 10^{-4}\, \tan\beta$ (see also footnote 2). If $\rho \gg 1$, then gluino corrections become dominant; they are larger by a factor $\frac{\alpha_s}{3\pi} / \frac{1}{32\pi^2} \simeq 3.6$. However, between the low and high ρ-regime, there is an intermediate value of ρ, where $(\mathbf{V}^\dagger \mathbf{R}^{-1} \mathbf{V})_{dd'}$ does exactly vanish for $d \neq d'$, and so the effective couplings $g^{L,R}_{H_i \bar d d'}$. In this case, one has $R_u^{-1} = R_c^{-1} = R_t^{-1}$ in (4.5), implying that $\mathbf{R}$ is proportional to the unity matrix. Then, it is $(\mathbf{V}^\dagger \mathbf{R}^{-1} \mathbf{V})_{dd'} = 0$, as a result of a GIM-cancellation mechanism due to the unitarity of the CKM matrix. We call such a point in the parameter space *GIM operative point*. The ρ value, for which the GIM-cancellation mechanism becomes fully operative, may easily be determined from (4.6), i.e.

$$\rho^2_{\rm GIM} \;=\; \left(1 \,+\, \frac{3|h_t|^2}{32\pi\alpha_s}\, e^{i(\phi_g - \phi_{A_U})} \right)^{-1} . \tag{4.7}$$

For the MSSM parameter space under study, there is always a GIM-operative value for the hierarchy factor ρ, iff $\phi_{A_U} - \phi_{\tilde g} = 0$ or $\pm\pi$. If $\phi_{A_U} - \phi_{\tilde g} = 0$, we have $\rho_{\rm GIM} < 1$, whereas it is $\rho_{\rm GIM} > 1$, for $\phi_{A_U} - \phi_{\tilde g} = \pi$. In fact, the second case is realized in Fig. 3 for $\rho_{\rm GIM} \approx 1.22$, where $\Delta M_K^{\rm SUSY}$ and $|\epsilon_K^{\rm SUSY}|$ vanish independently of the value of the CKM phase. Here, we should emphasize the fact that the value of $\rho_{\rm GIM}$ does not depend on the FCNC observable under consideration and is in excellent agreement with the one determined by (4.7). Because of the above flavour-universal property of $\rho_{\rm GIM}$, one may even face the very unusual possibility of discovering SUSY at high-energy colliders, without accompanying such a discovery with any new-physics signal in low-energy K- and B-meson FCNC observables.

It is now instructive to gauge the relative size of the different DP-induced Wilson coefficients in (3.8). For simplicity, let us take $\rho = 1$. Then, each individual DP-induced

Wilson coefficient in (3.8) may be approximately given by

$$\begin{aligned}
\sum_{i=1}^{3} (g^L_{H_i\bar{s}d})^2 &\approx (\chi^{(t)}_{\rm FC})^2\, (V^*_{ts}V_{td})^2 \sum_{i=1}^{3} \frac{O^2_{1i} - O^2_{3i} + 2\, i\, O_{1i}O_{3i}}{M^2_{H_i}}\ , \\
\sum_{i=1}^{3} (g^R_{H_i\bar{s}d})^2 &\approx (\chi^{(t)*}_{\rm FC})^2\, (V^*_{ts}V_{td})^2 \sum_{i=1}^{3} \frac{O^2_{1i} - O^2_{3i} - 2\, i\, O_{1i}O_{3i}}{M^2_{H_i}}\ , \\
\sum_{i=1}^{3} (g^L_{H_i\bar{s}d}\, g^R_{H_i\bar{s}d}) &\approx |\chi^{(t)}_{\rm FC}|^2\, (V^*_{ts}V_{td})^2 \sum_{i=1}^{3} \frac{O^2_{1i} + O^2_{3i}}{M^2_{H_i}}\ ,
\end{aligned} \tag{4.8}$$

where $\chi^{(i)}_{\rm FC}$ are the diagonal entries of the matrix $\chi_{\rm FC}$ defined in (2.13). For $M_{H^+} \gtrsim 180$ GeV, CP-violation and Higgs-mixing effects start to decouple from the lightest H_1-sector [18]. Moreover, in the region $\tan\beta \gtrsim 40$, the Φ_2-component in the H_2- and H_3-boson mass-eigenstates is very suppressed. As a consequence of the latter, we obtain

$$\sum_{i=1}^{3} \frac{O^2_{1i}\ -\ O^2_{3i}}{M^2_{H_i}} = \Big(O^2_{11}\ -\ O^2_{31}\Big) \Big(\frac{1}{M^2_{H_1}} - \frac{1}{M^2_{H_{23}}}\Big) + \mathcal{O}\Big(\frac{M^2_{H_2} - M^2_{H_3}}{M^2_{H_{23}}}\Big), \tag{4.9}$$

$$\sum_{i=1}^{3} \frac{O_{1i}\, O_{3i}}{M^2_{H_i}} = O_{11}\, O_{31} \Big(\frac{1}{M^2_{H_1}} - \frac{1}{M^2_{H_{23}}}\Big) + \mathcal{O}\Big(\frac{M^2_{H_2} - M^2_{H_3}}{M^2_{H_{23}}}\Big), \tag{4.10}$$

where $M^2_{H_{23}} = \frac{1}{2}\,(M^2_{H_2} + M^2_{H_3})$ and the orthogonality of the O matrix has been used. Since it is $O_{11},\ O_{31} \ll 1$ in the kinematic region of our interest, then on account of (4.9) and (4.10) and for maximal CKM phase $\delta_{\rm CKM} = 90^\circ$, the dominant contribution to $\Delta M^{\rm SUSY}_K$ and $\epsilon^{\rm SUSY}_K$ comes from the last DP expression in (4.8), namely from the Wilson coefficient $C^{\rm LR\,(DP)}_2$ in (3.8), despite the additional suppression factor $m_d/m_s \approx 1/10$ with respect to $C^{\rm SLL\,(DP)}_1$. If $\delta_{\rm CKM} = 0$, $\Delta M^{\rm SUSY}_K$ still receives its largest contribution from $C^{\rm LR\,(DP)}_2$, while $|\epsilon^{\rm SUSY}_K|$ is dominated by the first DP expression in (4.8), i.e. from $C^{\rm SLL\,(DP)}_1$; the second DP expression in (4.8) is very suppressed with respect to the first one by two powers of the ratio m_d/m_s. From Fig. (2), we also see that in addition to the CKM phase $\delta_{\rm CKM}$, the soft SUSY-breaking CP-phases, such as $\arg m_{\tilde{g}}$ and $\arg\,(A_U)$, may also give rise by themselves to enhancements of $|\Delta M^{\rm SUSY}_K|$ and $|\epsilon^{\rm SUSY}_K|$ even up to one order of magnitude. Analogous remarks and observations also hold true for the B-meson FCNC observables which are to be discussed in the next section.

Finally, we should comment on the fact that the 2HDM contribution by itself due to $C^{\rm LR\,(2HDM)}_2$ in (3.9) can only give rise to the undetectably small numerical values, $|\Delta M^{\rm 2HDM}_K| = 5\times 10^{-17}$ MeV and $|\epsilon^{\rm 2HDM}_K| = 5\times 10^{-6}$ (indicated by an arrow in the Fig. 2), for $\delta_{\rm CKM} = 90^\circ$.

4.2 ΔM_{B_q}, $B_q \to \ell^+\ell^-$ and associated leptonic CP asymmetries

In this section, we will present numerical estimates for a number of B-meson FCNC observables, such as the mass difference ΔM_{B_q}, the branching ratio for $B_q \to \ell^+\ell^-$ and the CP asymmetries associated with the B-meson leptonic decays. The current experimental status of these observables is as follows [41]:

$$\Delta M_{B_d} \;=\; 0.489 \,\pm\, 0.008 \text{ ps}^{-1}\,, \tag{4.11}$$

$$\Delta M_{B_s} \;>\; 13.1 \text{ ps}^{-1}\,, \tag{4.12}$$

$$\mathcal{B}(B_s \to \mu^+\mu^-) \;<\; 2.0 \times 10^{-6} \tag{4.13}$$

and [50]

$$\mathcal{B}(B_d \to \tau^+\tau^-) \;<\; 0.015\,. \tag{4.14}$$

Future experiments at an upgraded phase of the Tevatron collider may reach higher sensitivity to $\mathcal{B}(B_s \to \mu^+\mu^-)$ up to the 10^{-8}-level [51, 11, 10].

Let us start our discussion by numerically analyzing the B-meson mass differences ΔM_{B_d} and ΔM_{B_s}. As in the case of the K-meson observables, we use the same input values as those shown in Fig. 1, i.e. $M_{\rm SUSY} = 1$ TeV, $M_{H^+} = 0.2$ TeV and $\tan\beta = 50$. Then, Fig. 4 displays the combined, SM and Higgs-DP, contributions to ΔM_{B_d} and ΔM_{B_s} as functions of the gluino phase $\arg(m_{\tilde g})$, for $\delta_{\rm CKM} = 90^\circ$ and different choices of hierarchy factor ρ and ϕ_{A_U}. Note that the SM contributions alone for $\delta_{\rm CKM} = 90^\circ$ are displayed by horizontal dashed lines. Even though the SM predictions for $\Delta M_{B_{d,s}}$ may adequately describe by themselves the experimental values in (4.11) and (4.12), they cannot yet decisively exclude possible new-physics contributions due to the inherent uncertainties in the calculation of hadronic matrix elements such as those induced by SUSY Higgs-mediated FCNC interactions. In particular, we observe in Fig. 4 that SM and Higgs-DP effects may add constructively or destructively to the mass differences $\Delta M_{B_{d,s}}$. Similar features are found in Fig. 5, where the SM and SUSY Higgs-DP contributions to ΔM_{B_d} and ΔM_{B_s} are plotted versus the hierarchy factor ρ, for discrete values of $\delta_{\rm CKM}$ and ϕ_{A_U}. In our SM CKM-phase convention [41], unlike the CKM matrix element V_{ts}, the matrix element V_{td} is very sensitive to $\delta_{\rm CKM}$ values. As a result, the SM predictions for ΔM_{B_d} strongly depend on the selected value of $\delta_{\rm CKM}$, as can be seen from Fig. 5.

In Fig. 6, we exhibit numerical values for the branching ratios $\mathcal{B}(\bar B^0_s \to \mu^+\mu^-)$ and $\mathcal{B}(\bar B^0_s \to \tau^+\tau^-)$ as functions for the gluino phase $\arg(m_{\tilde g})$, for $M_{\rm SUSY} = 1$ TeV, $M_{H^+} = 0.2$ TeV, $\tan\beta = 50$, and $\delta_{\rm CKM} = 90^\circ$, where ρ and ϕ_{A_U} are varied discretely. Since the branching ratios are driven by Higgs-penguin effects in the region of large $\tan\beta$, predictions for other inputs of $\tan\beta$ and M_{H^+} may be easily estimated by rescaling the numerical values by a factor $x_{\mathcal{O}} = (\tan\beta/50)^6 \times (0.2\ TeV/M_{H^+})^4$, for $\tan\beta \gtrsim 40$. Thus, confronting the predictions for $\mathcal{B}(\bar{B}_s^0 \to \mu^+\mu^-)$ with experiment data in (4.13), combined bounds on the $\tan\beta$-M_{H^+} plane may be obtained for a given set of soft SUSY-breaking parameters. As we see in Fig. 6, these combined bounds become even more restrictive for large gluino phases, $\arg m_{\tilde{g}} \gtrsim 90°$, in agreement with our discussions in Section 4.1.

However, there is an additional factor that may crucially affect our predictions for the branching ratios of the decays $\bar{B}_s^0 \to \mu^+\mu^-$ and $\bar{B}_s^0 \to \tau^+\tau^-$, namely the hierarchy parameter ρ. As we show in Fig. 7, even for the extreme choice of a gluino phase $\arg m_{\tilde{g}} = 180°$, $\mathcal{B}(\bar{B}_d^0 \to \mu^+\mu^-)$ and $\mathcal{B}(\bar{B}_s^0 \to \mu^+\mu^-)$ can get very suppressed for a specific value of ρ in certain soft SUSY-breaking scenarios that can realize a GIM-operative point in their parameter space. As we detailed in Section 4.1, this phenomenon occurs for the universal value of $\rho = \rho_{\rm GIM} = 1.22$, when $\phi_{A_U} - \phi_{\tilde{g}} = 0$ or $\pm 180°$. As can be seen in Fig. 7, the predicted values for $\bar{B}_s^0 \to \mu^+\mu^-$ and $\bar{B}_s^0 \to \tau^+\tau^-$, where $\phi_{A_U} = 0°$, confirm the above observation.

Let us now investigate the size of the CP asymmetries in the leptonic B_d-meson decays in the CP-violating MSSM; the corresponding CP asymmetries for the B_s meson are experimentally constrained to be rather small, less than 5% (see also discussion after (3.32)). In Fig. 8, we display numerical values for the CP asymmetries $\mathcal{A}_{\rm CP}^{(B_d^0 \to \mu_L^+\mu_L^-)}$ and $\mathcal{A}_{\rm CP}^{(B_d^0 \to \mu_R^+\mu_R^-)}$ as functions of the gluino phase $\arg(m_{\tilde{g}})$, for $M_{\rm SUSY} = 1$ TeV, $M_{H^+} = 0.2$ TeV, $\tan\beta = 50$, and $\delta_{\rm CKM} = 90°$. As usual, we independently vary the parameters ρ and ϕ_{A_U} to take on the discrete values $\rho = 1,\ 10$ and $\phi_{A_U} = 0°, 90°$ and $180°$. We find that if the B_d^0-$\bar{B}_d^0$ mixing is consistently taken into account, the typical size of $\mathcal{A}_{\rm CP}^{(B_d^0 \to \mu_L^+\mu_L^-)}$ and $\mathcal{A}_{\rm CP}^{(B_d^0 \to \mu_R^+\mu_R^-)}$ does not exceed 0.7% and 3%, respectively. If B_d^0-$\bar{B}_d^0$ mixing is not included, the CP asymmetries can reach slightly higher values up to 1.2% and 6%, respectively. The apparent reason for the smallness of the CP asymmetries is due to the occurrence of an accidental cancellation in the sum $C_S + C_P$ in (3.32) at large $\tan\beta$, as the muon velocity is $\beta_\mu \approx 1$.

Having gained some insight from the above exercise, one may seek alternative ways to enhance the di-muon asymmetries $\mathcal{A}_{\rm CP}^{(B_d^0 \to \mu^+_{L,(R)}\mu^-_{L,(R)})}$. To this end, the first attempt would be to suppress the effect of the $B_d^0 - \bar{B}_d^0$ mixing by considering smaller $\tan\beta$ values, e.g. $\tan\beta \lesssim 10$. In this intermediate region of $\tan\beta$, the above cancellation in the sum $C_S + C_P$ does not occur due to non-trivial CP-violating Higgs-mixing effects and so the CP asymmetries $\mathcal{A}_{\rm CP}^{(B_d^0 \to \mu^+_{L,(R)}\mu^-_{L,(R)})}$ can be significantly increased. To get an idea of the magnitude of the CP asymmetries in this case, we consider the so-called CPX scenario introduced in [52] to maximize CP-violating effects in the lightest Higgs sector of an effective MSSM. In the CPX scenario, the μ-parameter and the soft trilinear Yukawa coupling A_U are set by the relations: $\mu = 4M_{\rm SUSY}$ and $A_U = 2M_{\rm SUSY}$. Thus, for $M_{H^+} = 0.15$ TeV,

$M_{\rm SUSY} = 1$ TeV, $\tan\beta = 7$, $\rho = 10$, $\phi_{A_U} = 45^\circ$ and $\phi_{\tilde{g}} = 0^\circ$, we find that CP-violating Higgs-penguin effects can give rise to the CP asymmetries:

$$\mathcal{A}_{\rm CP}^{(B_d^0\to\mu_L^+\mu_L^-)} \;\approx\; -9\ \%\,, \qquad \mathcal{A}_{\rm CP}^{(B_d^0\to\mu_R^+\mu_R^-)} \;\approx\; -37\ \% \tag{4.15}$$

where $\mathcal{B}(B_d \to \mu^+\mu^-) = 3.6 \times 10^{-10}$, which is an order of magnitude larger than the SM prediction [39]. Another variant of the CPX scenario of equally phenomenological importance utilizes the parameters: $\tan\beta = 10$, $\phi_{A_U} = 45^\circ$ and $\phi_{\tilde{g}} = 90^\circ$, with $M_{\rm SUSY} =$ 1 TeV. In this case, we obtain

$$\mathcal{A}_{\rm CP}^{(B_d^0\to\mu_L^+\mu_L^-)} \;\approx\; 11\ \%\,, \qquad \mathcal{A}_{\rm CP}^{(B_d^0\to\mu_R^+\mu_R^-)} \;\approx\; 43\ \% \tag{4.16}$$

and $\mathcal{B}(B_d \to \mu^+\mu^-) = 4.1\times10^{-9}$, which is two orders of magnitude above the SM prediction. The above scenario seem to pass all the experimental constraints, including those deduced from LEP2 analyses of direct Higgs searches [52, 53]. Moreover, a possible observation of a non-zero CP asymmetry in the leptonic di-muon channel will constitute the harbinger for new physics at B factories. At this stage, it is important to comment on the fact that the numerical values stated in (4.15) and (4.16) should be viewed as crude estimates, since they are obtained entirely on the basis of our resummed FCNC effective Lagrangian (2.18) at the intermediate $\tan\beta$ regime. However, in this region of $\tan\beta$, we expect additional one-loop effects to start getting relevant, such as supersymmetric Z-penguin and box diagrams. Even though our initial estimates given above appear to yield rather encouraging results, a complete study of the leptonic B-meson branching ratios and the respective CP asymmetries for all values of $\tan\beta$ needs be performed.

In the case of τ-lepton CP asymmetries, the velocities of the decayed τ-leptons β_τ is roughly 0.5, so one naturally gets an appreciably higher value for the expression $\beta_\tau\, C_S + C_P \approx 0.5 C_P$ in (3.32). As a result, larger values for the τ-lepton CP asymmetries are expected. Indeed, in Fig. 9, we display numerical predictions for $\mathcal{A}_{\rm CP}^{(B_d^0\to\tau_L^+\tau_L^-)}$ and $\mathcal{A}_{\rm CP}^{(B_d^0\to\tau_R^+\tau_R^-)}$ versus the gluino phase $\arg\,(m_{\tilde{g}})$, for the same values of the input parameters as in Fig. 8. Then, the CP asymmetries $\mathcal{A}_{\rm CP}^{(B_d^0\to\tau_L^+\tau_L^-)}$ and $\mathcal{A}_{\rm CP}^{(B_d^0\to\tau_R^+\tau_R^-)}$ can be as high as 9% and 36%, respectively.

We conclude this section with some general remarks. In addition to the K- and B-meson observables we have been studying here, there is a large number of other FCNC observables which have to be studied in a full-fledged combined analysis. For example, a central rôle in such a global analysis plays the decay $B_s \to X_s\gamma$ [5, 54], which will enable us to delineate more accurately the CP-conserving/CP-violating soft SYSY-breaking parameter space favoured by low-energy FCNC observables. Finally, in our numerical analysis, we have concentrated on scenarios that minimally depart from the minimal flavour-violation assumption through the presence of diagonal, but non-universal squark masses. In the most general case, however, the squark mass matrices and consequently the resummation matrix $\mathbf{R}$ of the radiative threshold effects may not be diagonal. Such low-energy realizations with off-diagonal soft squark-mass matrices can still be treated exactly within the context of our resummed FCNC effective Lagrangian (2.18), by appropriately considering non-trivial quark-squark CKM-like matrices, such as the 3-by-3 unitary matrices $\mathcal{U}^{\mathbf{Q}}_{\mathbf{L}}$ and $\mathcal{U}^{\mathbf{d}}_{\mathbf{R}}$ in (2.9).

5 Conclusions

We have derived the proper general form for the effective Lagrangian of Higgs-mediated FCNC interactions to d-type quarks, where large-$\tan\beta$ radiative threshold effects have been resummed consistently (cf. (2.18) and (2.19)). Our properly resummed FCNC effective Lagrangian is free from pathological singularities, which mainly emanate from the top-quark dominance hypothesis frequently adopted in the literature, and has been appropriately generalized to include effects of squark-mass non-universality, as well as CP-violation effects originating from the CKM-mixing matrix and the complex soft SUSY-breaking masses. In particular, our resummed effective Lagrangian can be applied to study Higgs-mediated FCNC effects in more general soft SUSY-breaking scenarios, beyond those that have already been discussed within the restricted framework of models with minimal flavour violation. Also, an approach to resumming radiative threshold effects, very analogous to the one developed in Section 2, can straightforwardly be applied to see-saw SUSY models, so as to properly describe Higgs-mediated lepton-flavour-violating interactions.

Within the context of generic soft SUSY-breaking scenarios, we have analyzed a number of K- and B-meson observables, such as $\Delta M_{K,B}$, ϵ_K, ϵ'/ϵ, $\mathcal{B}(B_{s,d} \to \ell^+\ell^-)$, and their associated leptonic asymmetries, which are enhanced by Higgs-boson FCNC interactions for large values of $\tan\beta$. In the same context, we have also discussed the consequences of $\tan\beta$-enhanced Higgs-mediated FCNC effects on the decays $B_d \to \phi K_S$ and $B_d \to J/\Psi K_S$. Moreover, we find that the predictions crucially depend on the choice of soft CP-violating phases in a given set of soft SUSY-breaking parameters. For example, for certain values of the gluino and stop phases, the predictions can reach and even exceed the current experimental limits, whereas for other values of the CP-odd phases the FCNC effects can be reduced dramatically, as a result of a kind of a GIM cancellation which can be operative in the latter case.

Based on our resummed effective Lagrangian, the above analysis can now be extended to a vast number of other K- and B-meson observables. Therefore, further dedicated studies need be performed in this direction. The obtained predictions are then expected to sensitively affect other low- and high-energy observables, such as measurements of electron and neutron electric dipole moments and Higgs-boson searches, as well as studies on cosmological electroweak baryogenesis and dark matter. It would be very interesting to determine to which degree the emerging CP-violating MSSM framework with CP-mixed Higgs bosons mediating $\tan\beta$-enhanced interactions to matter could be potentially responsible for all the present and future CP-conserving/CP-violating FCNC effects observed in Nature.

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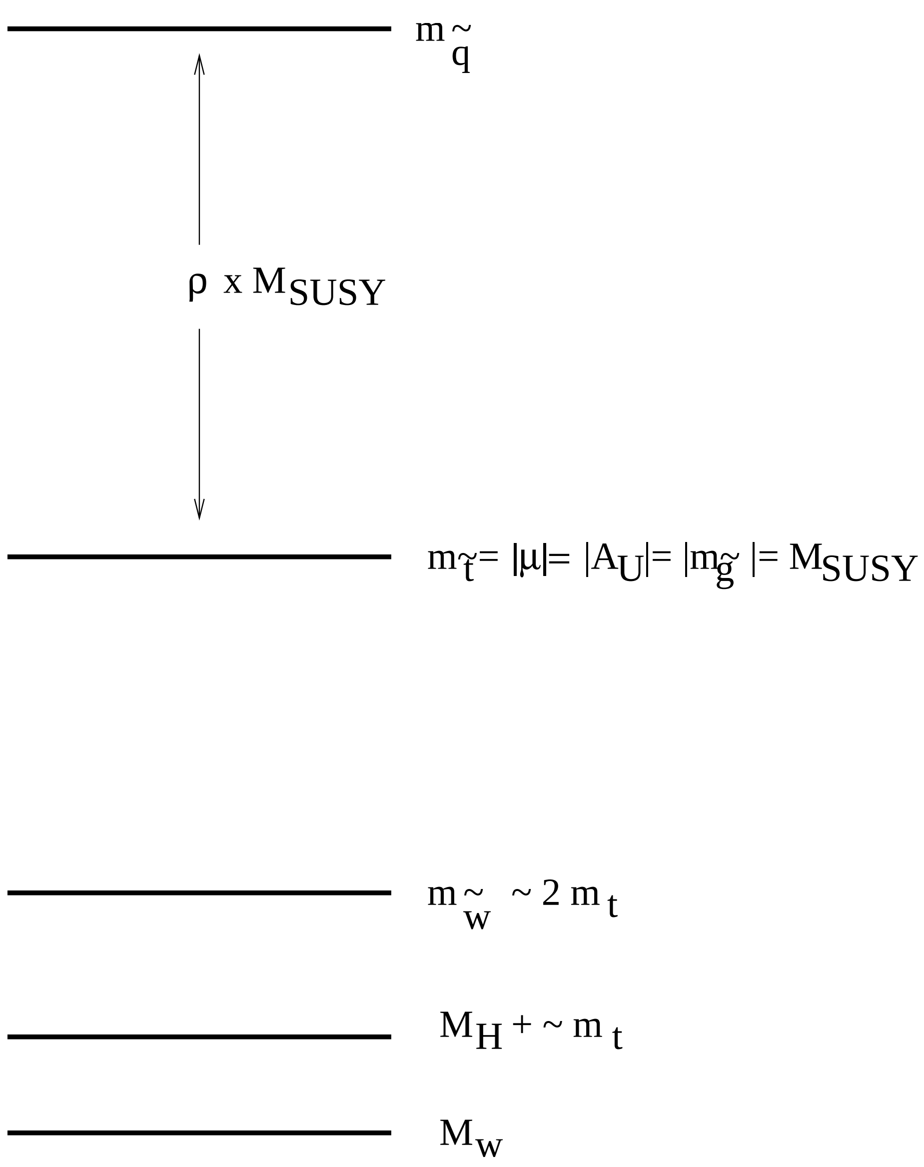

Figure 1: *Schematic representation of the SUSY mass spectrum considered in our numerical analysis, where* $m_{\tilde{q}}$ *and* $m_{\tilde{t}}$ *denote the masses of the first two and third generations of squarks, respectively. The hierarchy factor* ρ*, the phase* $\phi_{\tilde{g}}$ *of the gluino mass, and the phases* $\phi_{A_{t,b}}$ *of the soft SUSY breaking trilinear couplings, with* $\phi_{A_t} = \phi_{A_b} = \phi_{A_U}$*, are varied independently (see also discussion in the text).*

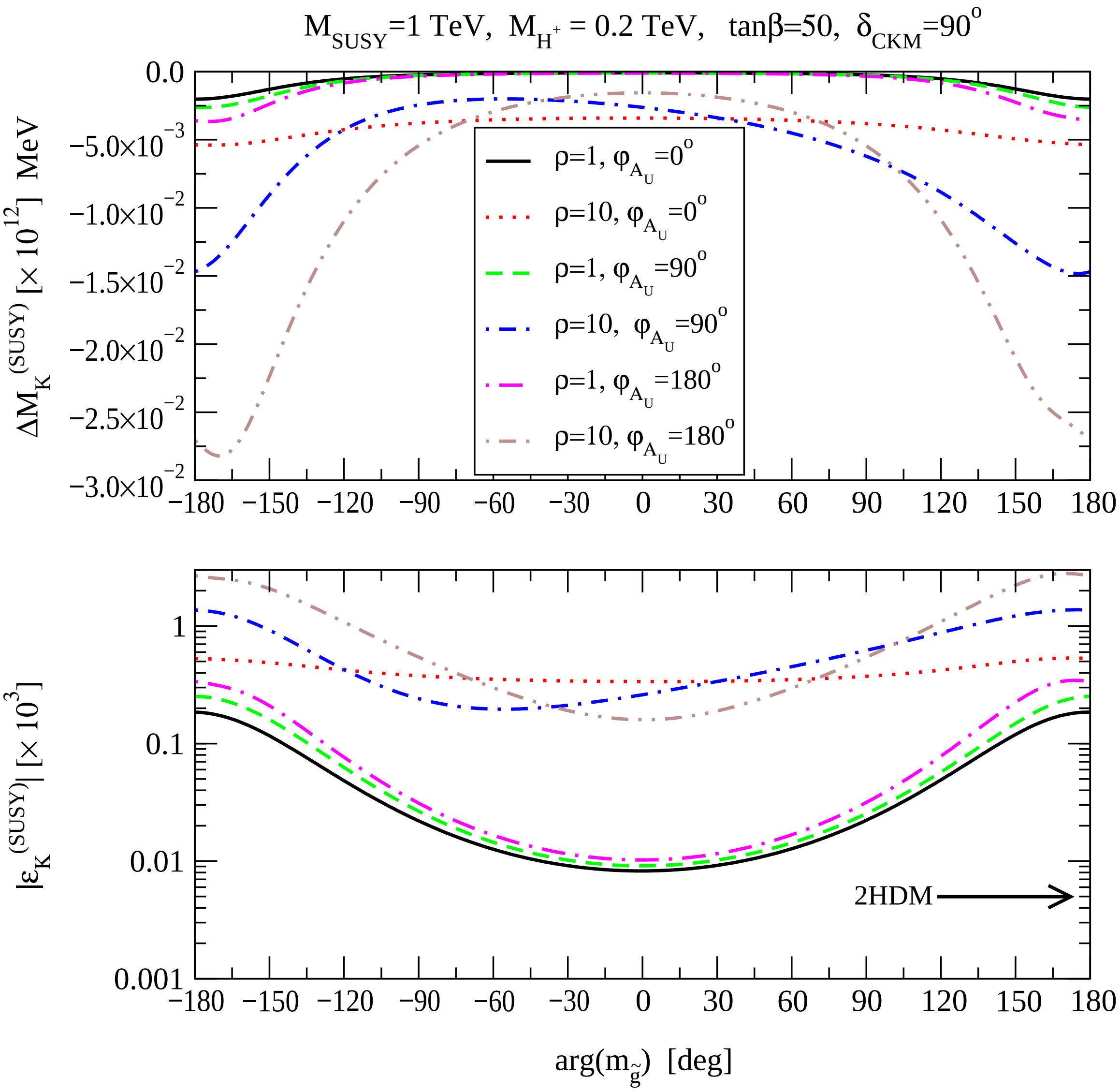

Figure 2: *SUSY Higgs-DP contributions to* ϵ_K *and* ΔM_K *given in units of* 10^{-12} *MeV and* 10^{-3}*, respectively, as functions of the gluino phase* $\arg(m_{\tilde{g}})$*, for* $M_{\rm SUSY} = 1$ *TeV,* $M_{H^+} = 0.2$ *TeV,* $\tan\beta = 50$ *and* $\delta_{\rm CKM} = 90^\circ$*. As is shown above, the different curves are obtained for selected values of* ρ *and* ϕ_{A_U}*. The size of the 2HDM effect alone on* ϵ_K *is indicated by an arrow.*

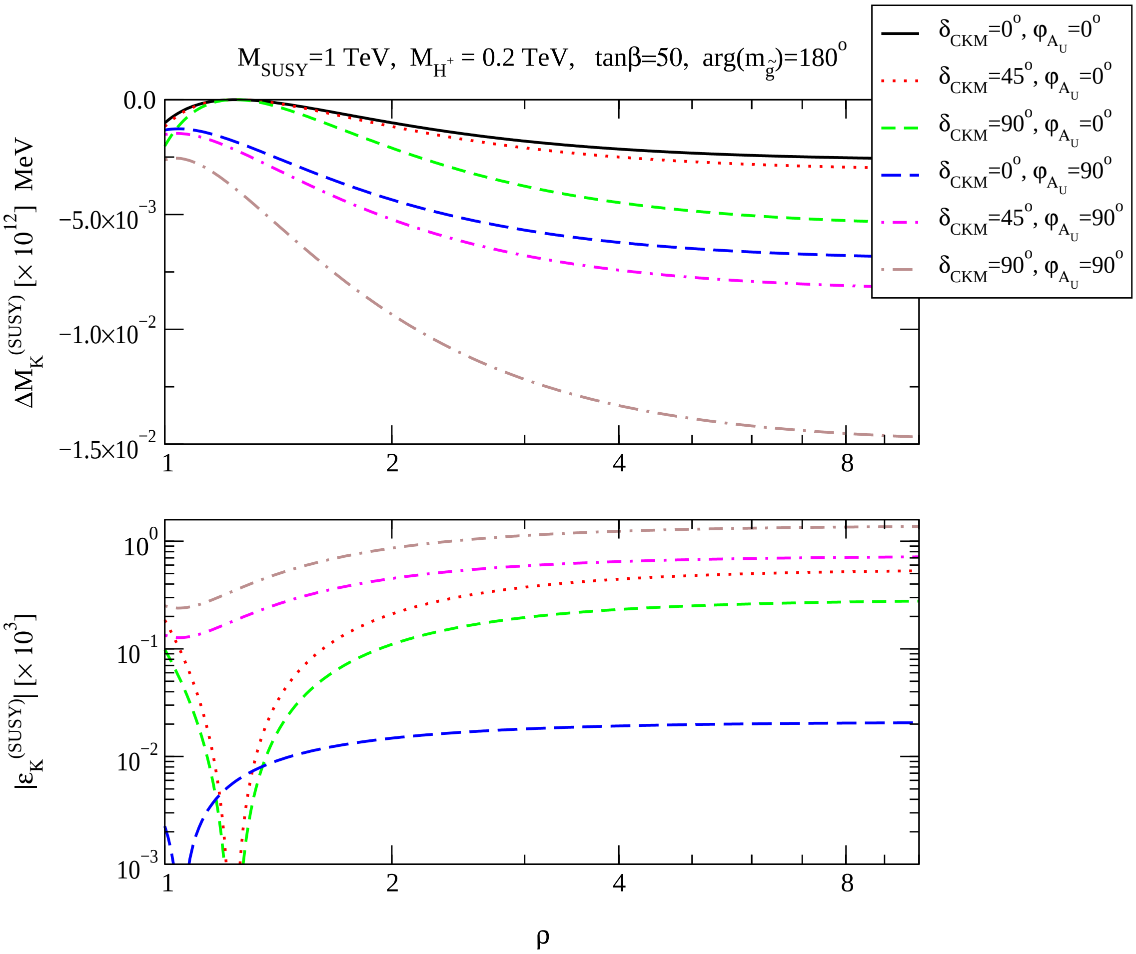

Figure 3: *SUSY Higgs-DP contributions to* ϵ_K *and* ΔM_K *given in units of* 10^{-12} *MeV and* 10^{-3}*, respectively, as functions of the hierarchy factor* ρ*, for* $M_{\rm SUSY} = 1$ *TeV,* $M_{H^+} = 0.2$ *TeV,* $\tan\beta = 50$*, and* $\arg m_{\tilde{g}} = 180^\circ$*, where the values of* $\delta_{\rm CKM}$ *and* ϕ_{A_U} *are varied discretely.*

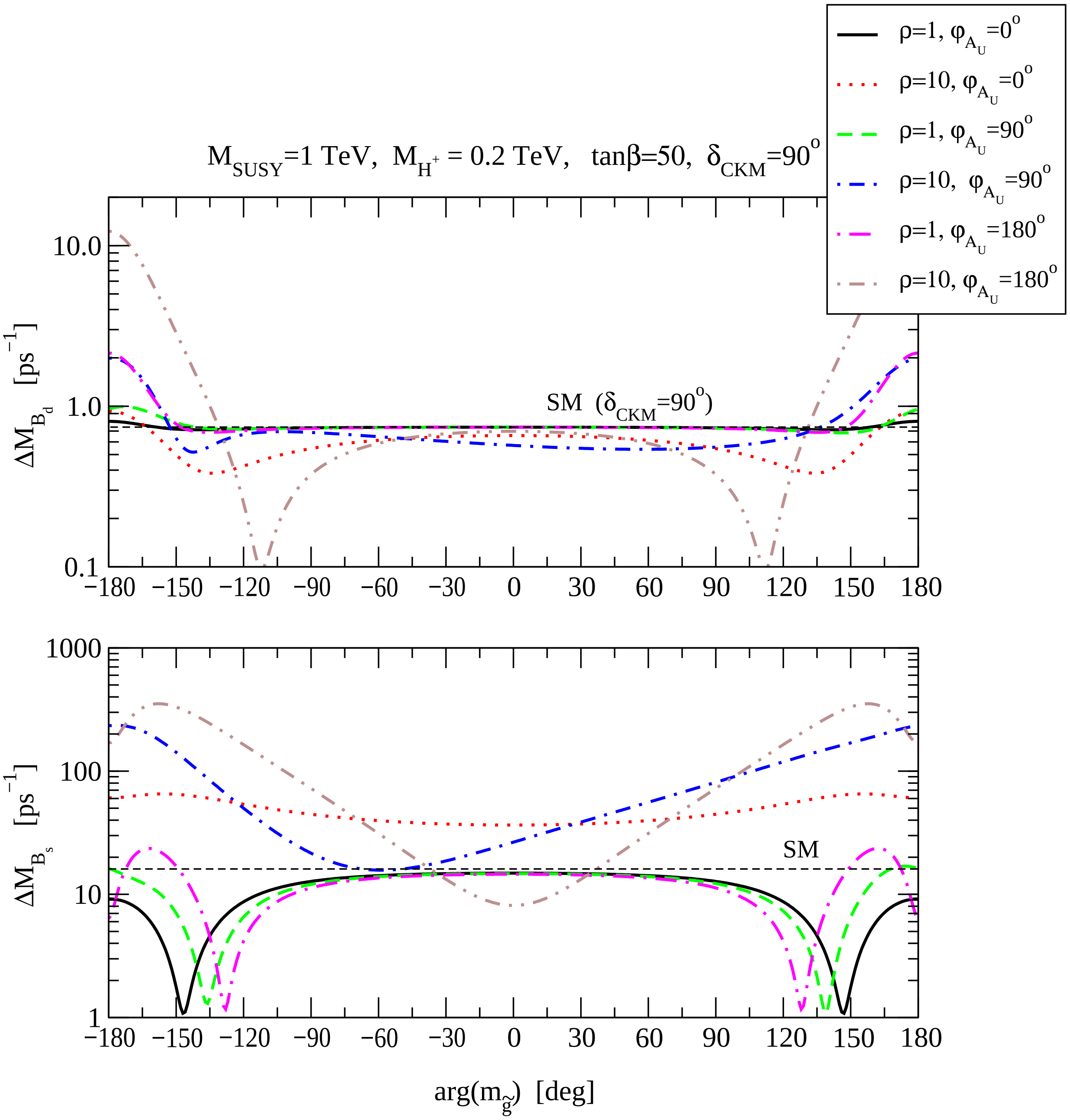

Figure 4: *SM and SUSY Higgs-DP contributions to* ΔM_{B_d} *and* ΔM_{B_s} *as functions of the gluino phase* $\arg(m_{\tilde{g}})$*, for* $M_{\rm SUSY} = 1$ *TeV,* $M_{H+} = 0.2$ *TeV,* $\tan\beta = 50$ *and* $\delta_{\rm CKM} = 90^\circ$*, where the hierarchy factor* ρ *and* ϕ_{A_U} *are varied independently as shown above. The SM contributions alone for* $\delta_{\rm CKM} = 90^\circ$ *are displayed by horizontal dashed lines.*

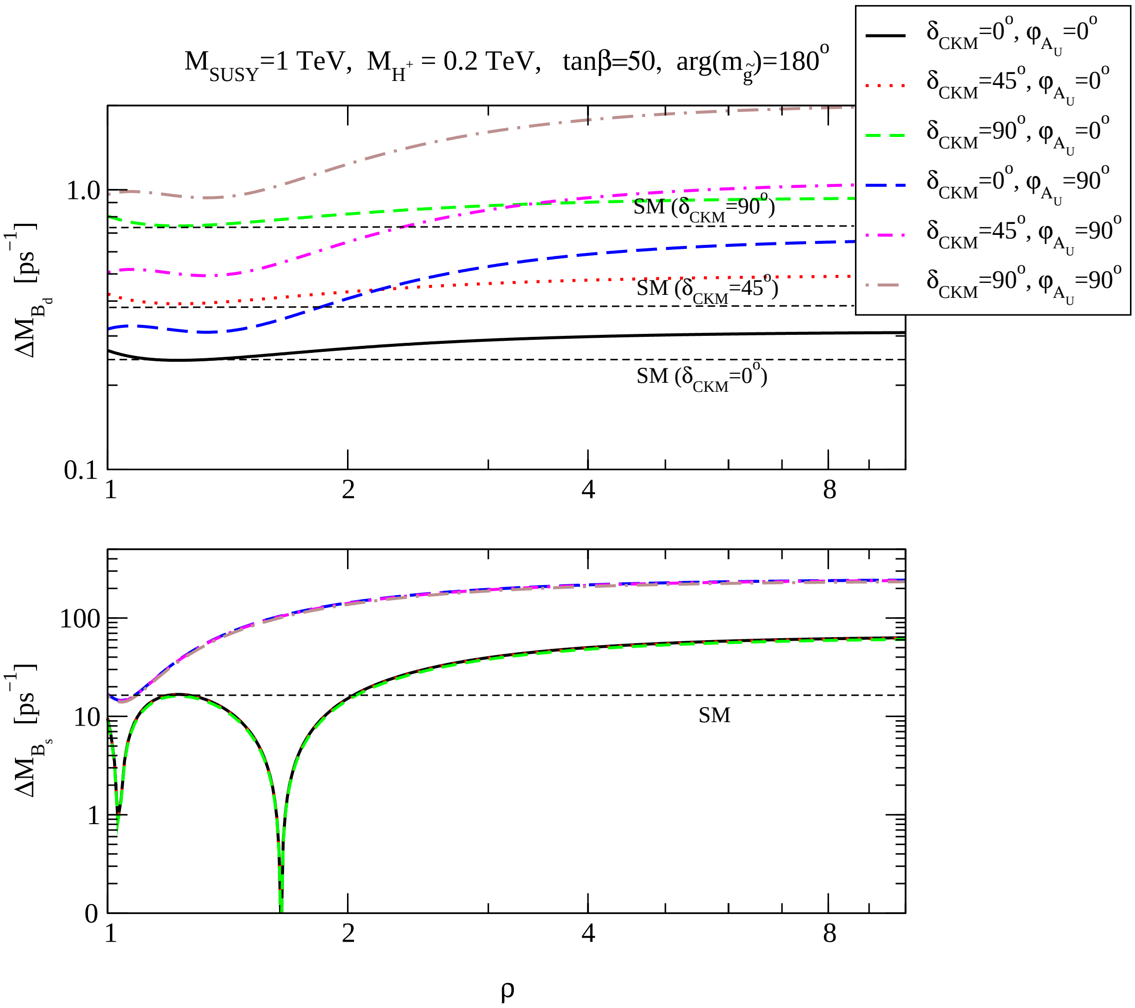

Figure 5: *SM and SUSY Higgs-DP contributions to* ΔM_{B_d} *and* ΔM_{B_s} *versus the hierarchy factor* ρ*, for* $M_{\rm SUSY} = 1$ *TeV,* $M_{H^+} = 0.2$ *TeV,* $\tan\beta = 50$*, and* $\arg m_{\tilde g} = 180^\circ$*, where* $\delta_{\rm CKM}$ *and* ϕ_{A_U} *obtain discrete values as shown above. Also shown are the SM effects alone for different choices of the CKM phase* $\delta_{\rm CKM}$ *(horizontal dashed lines).*

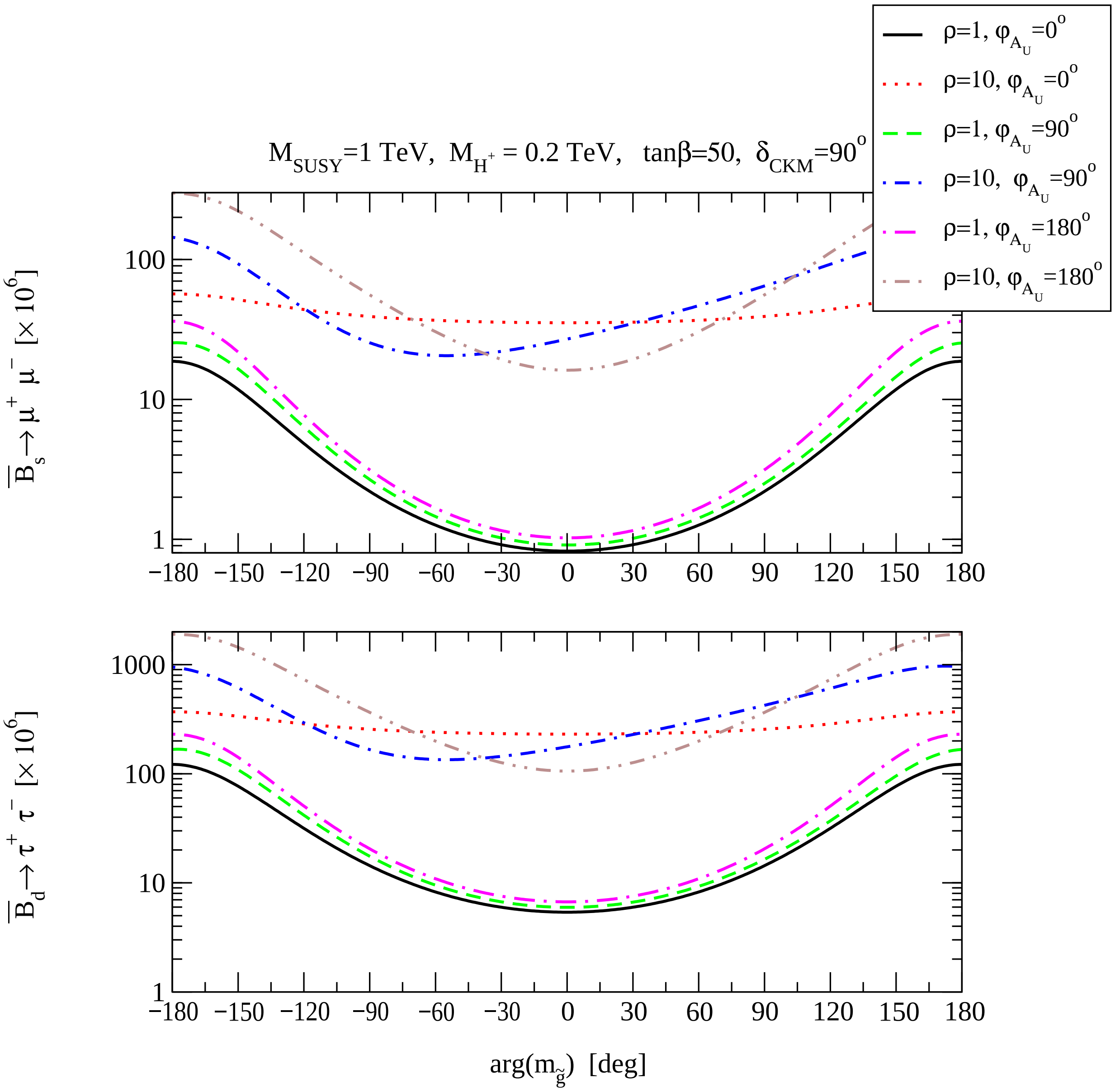

Figure 6: *SUSY Higgs-penguin contributions to* $\mathcal{B}(\bar{B}_s^0 \to \mu^+\mu^-)$ *and* $\mathcal{B}(\bar{B}_d^0 \to \tau^+\tau^-)$ *versus the gluino phase* $\arg(m_{\tilde{g}})$, *for* $M_{\rm SUSY} = 1$ *TeV*, $M_{H^+} = 0.2$ *TeV*, $\tan\beta = 50$, *and* $\delta_{\rm CKM} = 90^\circ$, *where* ρ *and* ϕ_{A_U} *are varied discretely.*

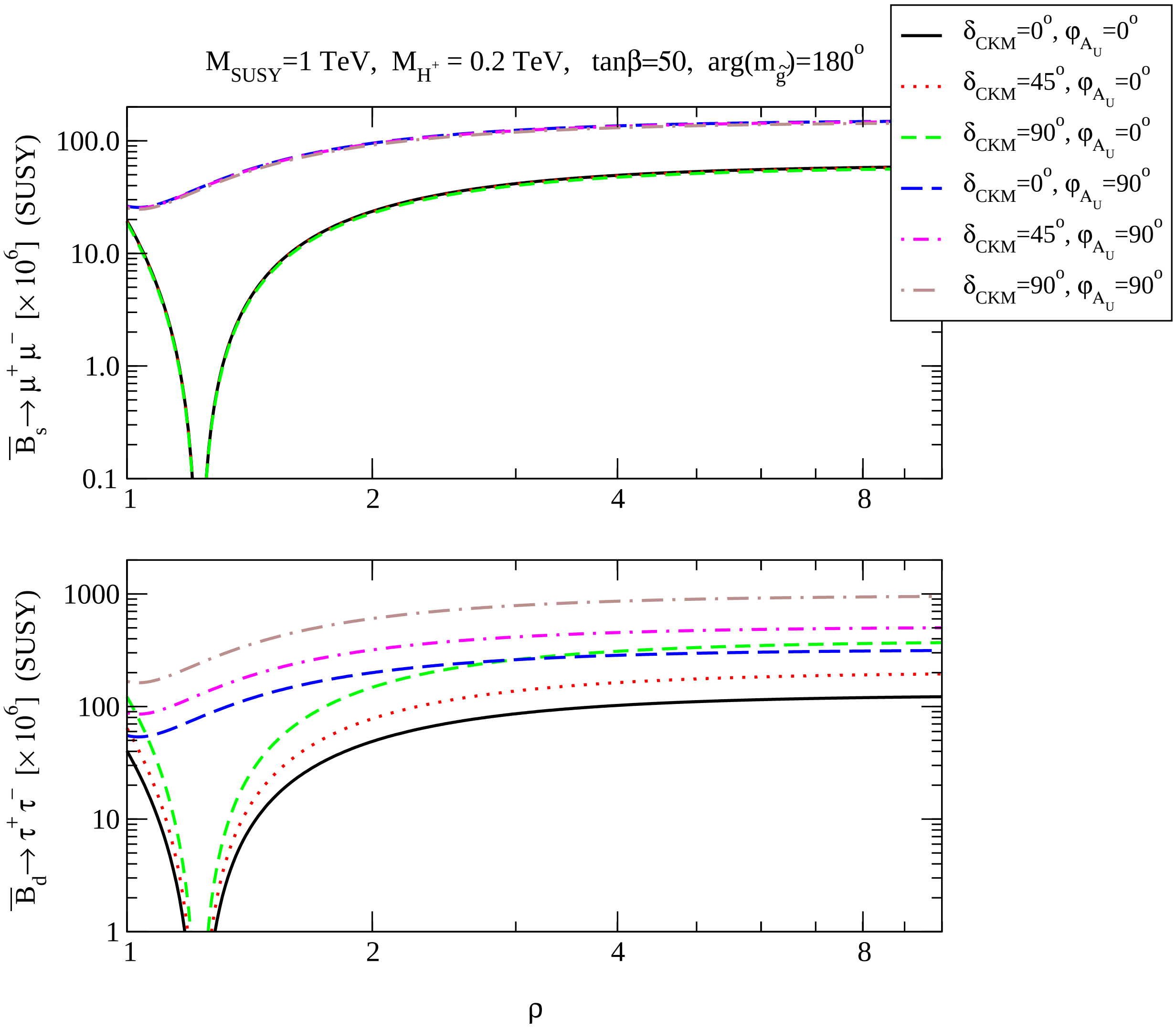

Figure 7: *SUSY Higgs-penguin contributions to* $\mathcal{B}(\bar{B}_s^0 \to \mu^+\mu^-)$ *and* $\mathcal{B}(\bar{B}_d^0 \to \tau^+\tau^-)$ *as functions of the hierarchy factor* ρ*, for* $M_{\rm SUSY} = 1$ *TeV,* $M_{H^+} = 0.2$ *TeV,* $\tan\beta = 50$*, and* $\arg m_{\tilde{g}} = 180°$*, where* $\delta_{\rm CKM}$ *and* ϕ_{A_U} *take discrete values.*

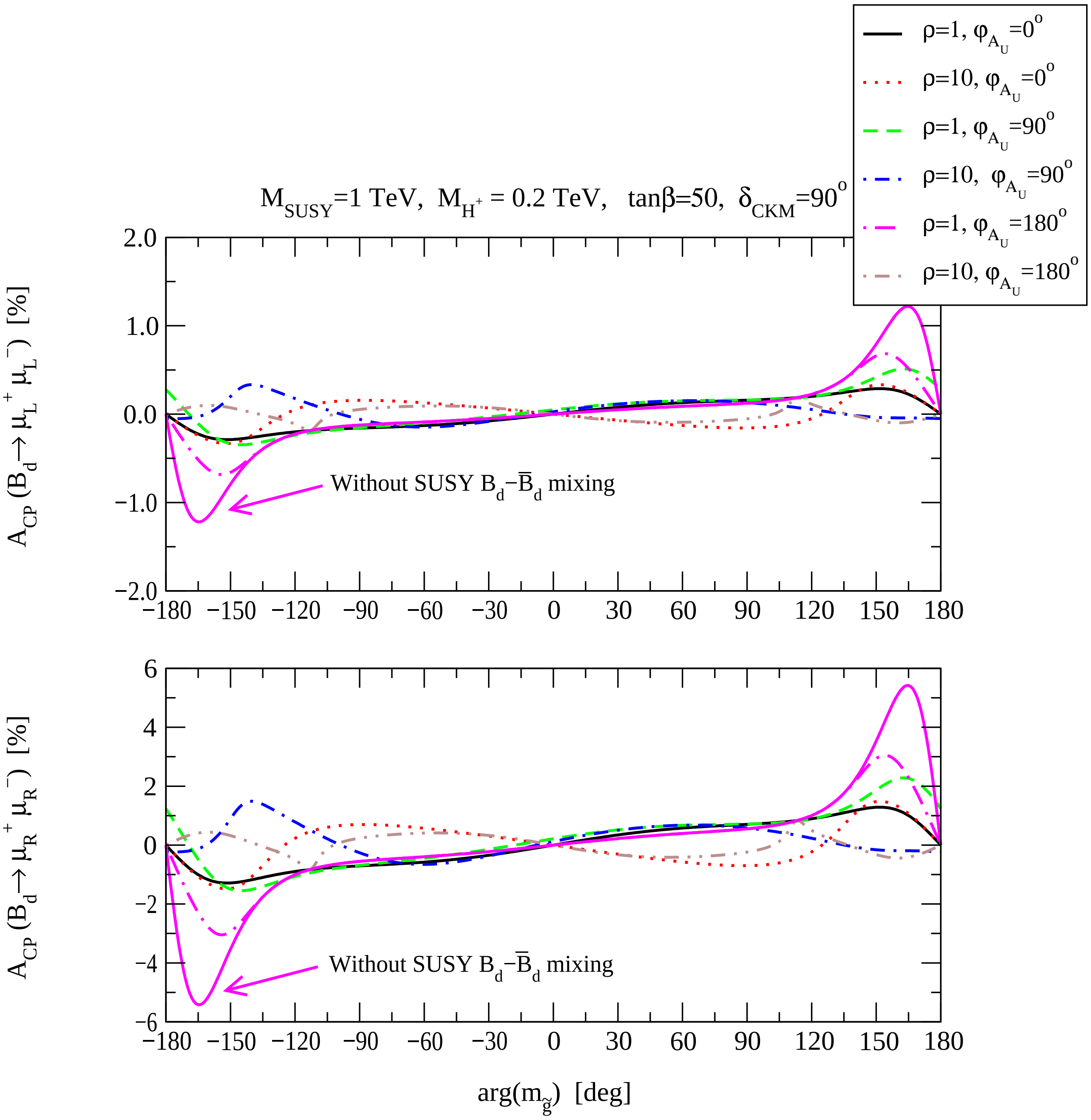

Figure 8: *Numerical values for the CP asymmetries* $\mathcal{A}_{\rm CP}^{(B_d^0\to\mu_L^+\mu_L^-)}$ *and* $\mathcal{A}_{\rm CP}^{(B_d^0\to\mu_R^+\mu_R^-)}$ *as functions of the gluino phase* $\arg(m_{\tilde{g}})$, *for* $M_{\rm SUSY} = 1$ *TeV*, $M_{H^+} = 0.2$ *TeV*, $\tan\beta = 50$, *and* $\delta_{\rm CKM} = 90°$, *where* ρ *and* ϕ_{A_U} *are varied discretely. Also shown is the prediction for the CP asymmetries without including* B_d^0-$\bar{B}_d^0$ *mixing.*

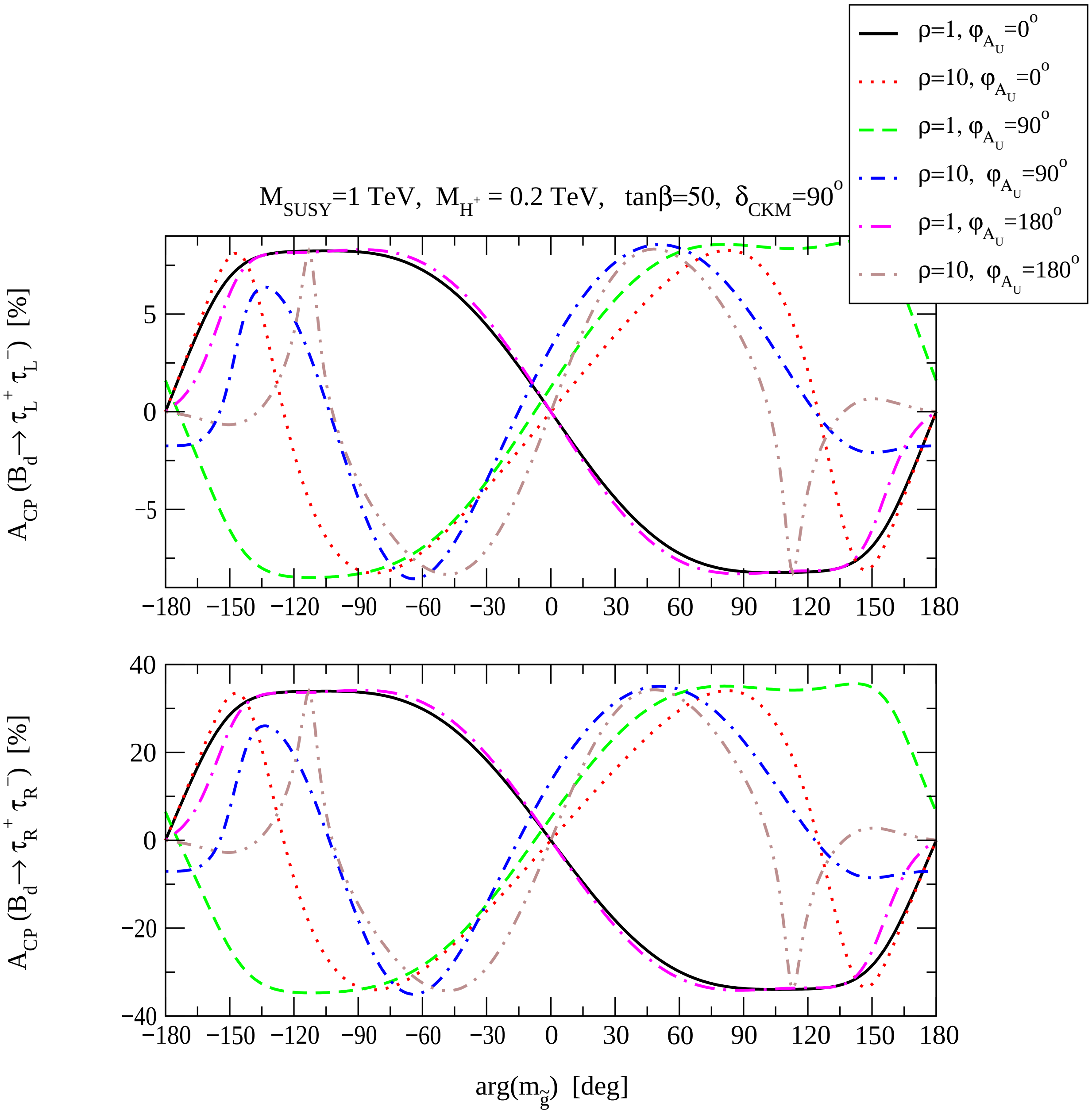

Figure 9: *Numerical estimates of the CP asymmetries* $\mathcal{A}_{\rm CP}^{(B_d^0\to\tau_L^+\tau_L^-)}$ *and* $\mathcal{A}_{\rm CP}^{(B_d^0\to\tau_R^+\tau_R^-)}$ *versus the gluino phase* $\arg(m_{\tilde g})$*, for* $M_{\rm SUSY}=1$ *TeV,* $M_{H^+}=0.2$ *TeV,* $\tan\beta=50$*, and* $\delta_{\rm CKM}=90^\circ$*, where* ρ *and* ϕ_{A_U} *take discrete values as shown above.*