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Implications of different soft SUSY-breaking scenarios on the Higgs boson sector

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Abstract

We investigate the Higgs boson sector of the Minimal Supersymmetric Standard Model (MSSM) in the three most prominent soft SUSY-breaking scenarios mSUGRA, (m)GMSB and AMSB. For each scenario we combine the evaluation of the parameters at the electroweak scale obtained from the set of input variables at higher energy scales (depending on the specific scenario) with the todays most accurate evaluation of Higgs boson properties in the Feynman-diagrammatic approach. Results are obtained for the maximum value of the mass of the lightest neutral $\mathcal{CP}$-even MSSM Higgs boson, m_h. Restrictions on $\tan\beta$ and on the set of higher-energy scale parameters are derived from the lower limit on m_h obtained at LEP2, as well as from the value of m_h "indicated" by recent measurements at LEP2, $m_h = 115^{+1.3}_{-0.9}$ GeV. The case that this mass corresponds to the heavier neutral $\mathcal{CP}$-even Higgs boson is also considered.

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1 Introduction

The search for the light neutral Higgs boson is a crucial test of Supersymmetry (SUSY) that can be performed with the present and the next generation of high-energy colliders. The prediction of a relatively light Higgs boson is common to all supersymmetric models whose couplings remain in the perturbative regime up to a very high energy scale [1]. Finding the Higgs boson is thus one of the main goals of todays high-energy physics. The running of LEP at $\sqrt{s} \gtrsim 206$ GeV in the year 2000 has for the first time resulted in data that could be interpreted as evidence of the Higgs boson [2]. The "indicated" value of the Higgs boson mass is

$$m_h = 115^{+1.3}_{-0.9} \text{ GeV}. \tag{1}$$

This mass value lies outside the theoretically allowed region for a Standard Model (SM) Higgs boson, requiering perturbativity up to scales $\Lambda \gtrsim 10^5$ GeV [3]. However, in the Minimal Supersymmetric Standard Model (MSSM) the mass of the lightest Higgs boson, m_h, is only bounded from above by $m_h \lesssim 135$ GeV [4], including radiative corrections up to two-loop order [4–11]. Therefore the evidence found hat LEP could be interpreted as a sign of the lightest MSSM Higgs boson.

In the MSSM no restrictions are placed on the low energy parameters, i.e. all parameters can be varied independently. However, the huge number of more than 100 free parameters makes this model somewhat less attractive. On the other hand, models that define all the low energy parameters in terms of a few parameters at the GUT scale, employing a special soft SUSY-breaking scenario, are much more predictive. The most prominent scenarios are minimal Supergravity (mSUGRA) [12, 13], (minimal) Gauge Mediated SUSY Breaking ((m)GMSB) [14] and Anomaly Mediated SUSY Breaking (AMSB) [15–17]. Analyses of the Higgs sector in these scenarios have been performed [18–20], mostly focusing only on the maximum value of m_h. More recently also the LEP results have been taken into account for special mSUGRA realizations [21].

In this paper the three soft SUSY-breaking scenarios are analyzed in view of their implications for Higgs physics in the following, similar way. From the GUT scale parameters, using renormalization group equations (RGEs) at the two-loop level, the low energy parameters are derived. After transforming them into the corresponding on-shell parameters [22,23], they are used as input for the most precise evaluation of the neutral $\mathcal{CP}$-even Higgs boson sector, based on the Feynman-diagrammatic (FD) approach [4, 11, 24, 25]. Further restrictions such as from precision observables and the non-observation of SUSY particles are also taken into account. Using this precise prediction of m_h we obtain the maximum value of m_h for each soft SUSY-breaking scenario. Combining the prediction of m_h with the exclusion regions obtained at LEP [26] or with the evidence found at LEP [2] (see eq. (1)), we derive limits on the low energy parameter $\tan\beta$ and on the GUT scale parameters of the corresponding scenario.

The rest of the paper is organized as follows. In Sect. 2 the three soft SUSY-breaking scenarios as well as the evaluation of the $\mathcal{CP}$-even Higgs boson sector of the MSSM are reviewed. Details about the combination of RGE and FD calculation are given and the parameter restrictions used are listed. The description of our data sets and the numerical analyses for the three scenarios is given in Sect. 3. The conclusions can be found in Sect. 4.

2 The Higgs sector in soft SUSY-breaking scenarios

The fact that supersymmetry is not observed at low energies requires the introduction of extra supersymmetry breaking interactions. This is achieved by adding to the Lagrangian density (defined by the given $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry and the superpotential W) some extra interaction terms that respect the gauge symmetry but break supersymmetry. This breaking however should be such that no quadratic divergences appear and the technical "solution" to the hierarchy problem is not spoiled. Such terms are generally called "soft SUSY-breaking" terms. The most general supersymmetry breaking interaction Lagrangian resulting from spontaneously broken Supergravity in the at flat limit ($M_P \to \infty$) contains just four types of soft SUSY-breaking terms [27], i.e. gaugino masses, $\Phi^*\Phi$-scalar masses, $\Phi\Phi\Phi$-scalar cubic superpotential interactions and $\Phi\Phi$-scalar quadratic superpotential interactions. Assuming that R-parity is conserved, which we do in this paper for all SUSY breaking scenarios, reduces the amount of new soft terms allowed in the Lagrangian. Choosing a particular soft SUSY-breaking pattern allows further reduction of the number of free parameters and the construction of predictive models.

In this section, we first explain how we employ the principle of radiative electroweak symmetry breaking (REWSB). Then we present the three most attractive soft SUSY-breaking scenarios and describe the general method used to derive the low-energy Higgs sector, which applies to all scenarios.

2.1 Radiative electroweak symmetry breaking

As in the numerous existing analyses of radiative electroweak symmetry breaking in the MSSM [28], in the present article we employ the RGEs. The Higgs boson "running" mass-squared matrix, although positive definite at large energy scales of the order of Grand Unification scale M_{GUT}, yields a negative eigenvalue at low energies causing the spontaneous breakdown of the electroweak (EW) symmetry. The running of mass parameters from high to low scales is equivalent to computing the leading logarithmic radiative corrections to the soft Higgs mass parameters. Although this scenario depends on the values of few (3 or 4) free parameters, one could interpret it as leading to the prediction of M_Z in terms of $M_{\rm GUT}$ (or the Planck mass) and the top quark Yukawa coupling. Another way to interpret the predictions of this model is to consider M_Z determined by the experiment and deriving the absolute value of the μ-parameter as well the value of the bilinear soft-SUSY breaking parameter B at a scale in the vicinity of the EW scale, from the minimization conditions of the effective potential,

$$\mu^2(Q) = \frac{\bar{m}_{H_1}^2 - \bar{m}_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}M_Z^2(Q) \ , \tag{2}$$

$$B(Q) = -\frac{(\bar{m}_1^2 + \bar{m}_2^2)\sin 2\beta}{2\mu(Q)} \ , \tag{3}$$

where Q is derived from the scalar fermion sector. It is usually chosen such that radiative corrections to the effective potential are rather small compared to other scales. In eqs. (2),(3) $\tan\beta$ is the ratio of the two vacuum expectation values of the Higgs fields H_2 and H_1 responsible in giving masses to the up-type and down-type quarks, respectively.

In eqs. (2),(3), $\tan\beta$ is evaluated at the scale Q, from the scale M_Z, where it is considered as an input parameter[1]. By $\bar{m}^2_{H_i}$ in eqs. (2),(3) we denote the radiatively corrected "running " Higgs soft-SUSY breaking masses and

$$\bar{m}^2_i = m^2_{H_i} + \mu^2 + \Sigma_{v_i} \quad (i = 1, 2) \ , \tag{4}$$

where Σ_{v_i}, $v_i = (v_1, v_2)$ are the one loop corrections based on the 1-loop Coleman-Weinberg effective potential ΔV, $\Sigma_{v_i} = \frac{1}{2v_i}\frac{\partial \Delta V}{\partial v_i}$,

$$\Sigma_{v_i} = \frac{1}{64\pi^2}\sum_a (-)^{2J_a}(2J_a+1)C_a\Omega_a \frac{M^2_a}{v_i}\frac{\partial M^2_a}{\partial v_i}\left[\ln\frac{M^2_a}{Q^2} - 1\right] \ . \tag{5}$$

Here J_a is the spin of the a-particle, C_a are the color degrees of freedom, and $\Omega_a = 1(2)$ for real scalar (complex scalar), $\Omega_a = 1(2)$ for Majorana (Dirac) fermions. Q is the energy scale and the M_a are the field dependent mass matrices. Explicitly formulas of the Σ's are given in the Appendices of Refs. [30, 31]. In our analysis contributions from all SUSY particles at one-loop level are incorporated. With M^2_Z here we denote the tree level "running" Z boson mass, $M^2_Z(Q) = \frac{1}{2}(g^2 + g^2_2)v^2$ extracted at the scale Q from its physical pole mass $M_Z = 91.187$ GeV. The Radiative Electroweak Symmetry Breaking (REWSB) is fulfilled, see Sect. 2.6.3, if and only if, there is a solution to the eqs. (2),(3)[2].

2.2 mSUGRA

A dramatic simplification of the structure of the SUSY breaking interactions is provided either by Grand Unification assumptions or by Superstrings. For example, SU(5) unification implies at tree level equality relations between the scalar soft-SUSY breaking masses $m_{\tilde{Q}} = m_{\tilde{U}^c} = m_{\tilde{E}^c}$, and $m_{\tilde{L}} = m_{\tilde{D}^c}$, equality between the soft breaking gaugino masses $M_1 = M_2 = M_3$ and for the two of the trilinear soft breaking couplings, $A_d = A_e$. Furthermore, SO(10) unification implies equality of all scalar particle masses, equality of Higgs masses and equality of the three types of cubic couplings. The simplest possible choice at tree level is to take all scalar particle and Higgs masses equal to a common mass parameter M_0, all gaugino masses are chosen to be equal to the parameter $M_{1/2}$ and all cubic couplings flavor blind and equal to A_0. This situation is common in the effective Supergravity theories resulting from Superstrings but there exist more complicated alternatives. Interestingly, the contribution of the family–anomalous $U(1)$ universal D-term to the scalar quark masses may be intra-family non-universal, and may differ from the usual (universal) boundary conditions assumed in the MSSM [33]. Another alternative are for example Superstrings with massless string modes of different modular weights that lead to different scalar particle masses at tree level [34]. Thus, it seems that in almost all the "realistic" models motivated by Grand Unified Theories (GUTs) or Superstrings the universality assumption is broken and certainly one should address each model separately in order to study its phenomenology at low energies. On the other hand, one should

[1] See for example the discussion in the Appendix of Ref. [29].

[2] Sometimes in the literature, the requirement of the REWSB is described by the inequality $m^2_1(Q)m^2_2(Q) - |\mu(Q)B(Q)|^2 < 0$. This relation is automatically satisfied here from eqs. (2),(3) and from the fact that the physical squared Higgs masses must be positive.

note that such non-minimal alternatives like flavor dependent scalar particle masses are constrained by limits on Flavor Changing Neutral Currents (FCNC) processes. In what follows, we shall therefore consider the simplest case (which is considered as a rather simplification assumption here) of the three parameters at the GUT scale, namely M_0, $M_{1/2}$ and A_0, which is usually called the mSUGRA scenario.

In order to solve eqs. (2),(3), i.e. in order to impose the constraint of REWSB, one needs furthermore to "decide" about the $sign(\mu)$ and the input value of $\tan\beta(M_Z)$. The running soft SUSY-breaking Higgs mass parameters, m_{H_1} and m_{H_2}, are defined at the EW scale after their evolution from the GUT scale where we assume that they have a common value, M_0. In addition the radiative corrections Σ_{v_i} to the minimization conditions 5 are defined from the low energy SUSY spectrum and the masses of the SM particles, which in turn means knowledge of M_0, $M_{1/2}$ and A_0 at the GUT scale. Thus, apart from the SM masses provided by the experimental data [32], $4\frac{1}{2}$ parameters are required to define the mSUGRA scenario:

$$\{\ M_0\ ,\ M_{1/2}\ ,\ A_0\ ,\ \tan\beta\ ,\ sign(\mu)\ \}\ . \tag{6}$$

2.3 (m)GMSB

A very promising alternative to (m)SUGRA is based on the hypothesis that the SUSY breaking (SSB) occurs at relatively low energy scales and it is mediated mainly by gauge interactions (GMSB) [14,42,43]. This scheme provides a natural, automatic suppression of the SUSY contributions to flavor-changing neutral currents and $\mathcal{CP}$-violating processes. Furthermore, in the simplest versions of GMSB (denoted hereafter with mGMSB), the MSSM spectrum and other (not all) observables depend on just $4\frac{1}{2}$ parameters,

$$\{\ M_{\rm mess},\ N_{\rm mess},\ \Lambda,\ \tan\beta,\ {\rm sign}(\mu)\ \}\ , \tag{7}$$

where $M_{\rm mess}$ is the overall messenger scale; $N_{\rm mess}$ is the so-called messenger index, parameterizing the structure of the messenger sector; Λ is the universal soft SUSY breaking scale felt by the low-energy sector; $\tan\beta$ is the ratio of the vacuum expectation values of the two Higgs doublets; sign(μ) is the ambiguity left for the SUSY higgsino mass after imposing a correct REWSB (see Sect. 2.1 and e.g. Refs. [44–48]).

The phenomenology of GMSB (and more in general of any theory with low-energy SSB) is characterized by the presence of a very light gravitino $\tilde{G}$ with mass [49] given by $m_{3/2} = m_{\tilde{G}} = \frac{F}{\sqrt{3}M'_P} \simeq \left(\frac{\sqrt{F}}{100\ {\rm TeV}}\right)^2 2.37$ eV, where $\sqrt{F}$ is the fundamental scale of SSB and $M'_P = 2.44 \times 10^{18}$ GeV is the reduced Planck mass. Since $\sqrt{F}$ is typically of order 100 TeV, the $\tilde{G}$ is always the lightest SUSY particle (LSP) in these theories. Hence, if R-parity is conserved, any MSSM particle will decay into the gravitino. Depending on $\sqrt{F}$, the interactions of the gravitino, although much weaker than gauge and Yukawa interactions, can still be strong enough to be of relevance for collider physics. In most cases, the last step of any SUSY decay chain is the decay of the next-to-lightest SUSY particle (NLSP), which can occur either outside or inside a typical detector, possibly close to the interaction point. The nature of the NLSP – or, better, of the scalar particle(s) having a large branching ratio for decaying into the gravitino and the relevant SM partner – determines four main scenarios giving rise to qualitatively different phenomenology [46].

The parameter sets used in this model have been created by using the program *SUSYFIRE* [50] and adopting the phenomenological approach of Refs. [46–48], see also Ref. [19]. The origin of μ is not specified, nor the assumption $B\mu = 0$ is made at the messenger scale. Instead, correct REWSB is imposed to trade μ and $B\mu$ for M_Z and $\tan\beta$, leaving the sign of μ undetermined, see Sect. 2.1. However, note that to build a satisfactory GMSB model, one should also find a more fundamental solution to the latter problem, perhaps providing a dynamical mechanism to generate μ and $B\mu$, possibly with values of the same order of magnitude. This might be accomplished radiatively through some new interaction. However, in this case, the other soft terms in the Higgs potential, namely $m^2_{H_{1,2}}$, will be also affected and this will in turn change the values of $|\mu|$ and $B\mu$ coming from REWSB conditions.

non-minimal models?

To determine the MSSM spectrum and low-energy parameters, the RGE evolution is solved with boundary conditions at the $M_{\rm mess}$ scale, where

$$\begin{aligned} M_a &= N_{\rm mess}\Lambda g\left(\frac{\Lambda}{M_{\rm mess}}\right)\alpha_a, \quad (a=1,2,3) \\ \tilde{m}^2 &= 2N_{\rm mess}\Lambda^2 f\left(\frac{\Lambda}{M_{\rm mess}}\right)\sum_a\left(\frac{\alpha_a}{4\pi}\right)^2 C_a, \end{aligned} \tag{8}$$

respectively for the gaugino and the scalar masses. The exact expressions for g and f at the one and two-loop level can be found, e.g., in Ref. [46], and C_a are the quadratic Casimir invariants for the scalar fields. As usual, the scalar trilinear couplings A_f are assumed to vanish at the messenger scale, as suggested by the fact that they (and not their square) are generated via gauge interactions with the messenger fields at the two loop-level only.

The interesting region of the GMSB parameter space is selected as follows. Barring the case where a neutralino is the NLSP and decays outside the detector (large $\sqrt{F}$), the GMSB signatures are very spectacular and the SM background is generally negligible or easily subtractable. Therefore, also in accordance with negative LEP2 searches [51], only models where the NLSP mass is larger than 100 GeV are considered, assuming that searches at LEP and Tevatron, if unsuccessful, will at the end exclude a softer spectrum in most cases. Other requirements are: $M_{\rm mess} > 1.01\Lambda$, to prevent an excess of fine-tuning of the messenger masses; the mass of the lightest messenger scalar be at least 10 TeV; $M_{\rm mess} > M_{\rm GUT}\times\exp(-125/N_{\rm mess})$, to ensure the perturbativity of gauge interactions up to the GUT scale; $M_{\rm mess} \lesssim 10^5\Lambda$, for simplicity. As a result of this and other constraints, the messenger index $N_{\rm mess}$, which is assumed to be an integer independent of the gauge group, cannot be larger than 8. To prevent the top Yukawa coupling from blowing up below the GUT scale, $\tan\beta > 1.2$ is required. Models with $\tan\beta \gtrsim 55$ (with a mild dependence on Λ) are forbidden by the REWSB requirement, see Sect. 2.6.3, and typically fail to give $m_A^2 > 0$.

Ok, Sandro?

The model samples are generated using *SUSYFIRE* with logarithmic steps for Λ (between about 45 TeV$/N_{\rm mess}$ and about 220 TeV$/\sqrt{N_{\rm mess}}$, which corresponds to excluding models with scalar particle masses above $\sim$ 4 TeV), $M_{\rm mess}/\Lambda$ (between about 1.01 and 10^5) and $\tan\beta$ (between 1.2 and about 60), subject to the constraints described above. *SUSYFIRE* starts from the values of particle masses and gauge couplings at the weak scale and then evolves them up to the messenger scale through RGEs. At the messenger

scale, it imposes the boundary conditions (8) for the soft scalar particle masses and then evolves the RGEs back to the weak scale. The decoupling of each scalar particle at the proper threshold is taken into account. Two-loop RGEs are used for gauge couplings, third generation Yukawa couplings and gaugino soft masses. The other RGEs are taken at the one-loop level. At the scale Q, derived from the scalar fermion sector, REWSB conditions are imposed by means of the one-loop effective potential approach. For the Σ_{v_i} in eq. (5) all dominant corrections from the stop, sbottom and stau sector are included. The program then evolves up again to $M_{\rm mess}$ and so on. Three or four iterations are usually enough to get a good approximation for the MSSM spectrum.

2.4 AMSB

The newly proposed Anomaly Mediated SUSY Breaking (AMSB) scenario [15,16] provides an alternate way to give mass to all the SUSY particles. In this model, SUSY breaking happens on a separate brane and is communicated to the visible world via the super-Weyl anomaly. The overall scale of scalar particle masses are set by $m_{\rm aux}$, which is the VEV of the auxiliary field in the supergravity multiplet. In the AMSB scenario, the low energy soft supersymmetry breaking parameters M_i (gaugino masses, i=1−3), $m^2_{\rm scalar}$ and A_y at the GUT scale are given by [15, 17]

$$M_{\rm i} = \frac{\beta_{g_i}}{g_{\rm i}} m_{\rm aux}, \tag{9}$$

$$m^2_{\rm scalar} = -\frac{1}{4}\left(\frac{\partial\gamma}{\partial g}\beta_g + \frac{\partial\gamma}{\partial y}\beta_y\right) m^2_{\rm aux} + m_0^2, \tag{10}$$

$$A_y = -\frac{\beta_y}{y} m_{\rm aux}. \tag{11}$$

Notice that the slepton squared-masses would be negative if m_0 is absent. There have been several proposals to solve this tachyonic slepton problem: bulk contributions [15], non-decoupling effects of ultra-heavy vectorlike matter fields [52], coupling extra Higgs doublets to the leptons [53], contributions from the R-parity violating couplings in the eq. (10) (with $m_0 = 0$) [54], and the heavy mass threshold contribution at higher orders [55]. Here we have adopted a phenomenological approach and have introduced an additional mass scale m_0 at the GUT scale in order to keep the slepton masses positive [17]. For simplification, we choose m_0 to be the same for all the super scalar particles. Therefore, in the minimal case, the particle spectrum can be determined by $3\frac{1}{2}$ parameters:

$$\{m_{\rm aux},\ m_0,\ \tan\beta,\ {\rm sign}(\mu)\}. \tag{12}$$

Eq. (9), (10) and (11) would be true at all scales if m_0 is absent. However, once m_0 is introduced at the GUT scale, the above definitions of M_i, $m^2_{\rm scalar}$ and A_y set the boundary conditions and the entire SUSY spectrum can be obtained via the running of supersymmetric RGEs down to a lower scale.

Once we cross the squark threshold, the squarks decouple and we are left with an

effective field theory with two Higgses and all the standard model particles[3]. The two unknown parameters $|\mu|$ and B are determined by the minimization of the Higgs effective potential as explained in Sect. 2.1, where details used for the AMSB scenario can be found in Ref. [56]. Therefore, the low-energy spectrum is fixed once the values of $m_{\rm aux}$, m_0 $\tan\beta$ and the sign of μ are known.

2.5 Evaluating the Higgs boson sector in the MSSM

As described in Sects. 2.2 – 2.4 the low-energy parameters are derived in each of the soft SUSY-breaking scenarios. The most relevant parameters for Higgs boson phenomenology are the mass of the $\mathcal{CP}$-odd Higgs boson, M_A, the ratio of the two vacuum expectation values, $\tan\beta$, the scalar top masses and mixing angle, $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}$ (and for large $\tan\beta$ also the scalar bottom masses and mixing angle, $m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$), the Higgs mixing parameter, μ, the gluino mass, $m_{\tilde{g}}$, and the soft SUSY-breaking parameters of the gaugino sector, M_1 and M_2. Since these parameters are derived from RGEs, they are $\overline{\rm MS}$ parameters.

The MSSM Higgs sector is evaluated in the Feynman-diagrammatic approach, see below. Therefore the $\overline{\rm MS}$ parameters obtained from the soft-SUSY-breaking scenarios have to be transformed into on-shell parameters [22]. It has been shown that this is most relevant for the parameters of the $\tilde{t}$ sector. Therefore we transform the $\tilde{t}$ masses and mixing angle to their on-shell value, taking into account the full one-loop contribution presented in Refs. [22, 23].

In the FD approach the masses of the two $\mathcal{CP}$-even Higgs bosons, m_h and m_H, are derived beyond tree level by determining the poles of the $h-H$-propagator matrix whose inverse is given by

$$(\Delta_{\rm Higgs})^{-1} = -i \begin{pmatrix} q^2 - m^2_{H,\rm tree} + \hat{\Sigma}_H(q^2) & \hat{\Sigma}_{hH}(q^2) \\ \hat{\Sigma}_{hH}(q^2) & q^2 - m^2_{h,\rm tree} + \hat{\Sigma}_h(q^2) \end{pmatrix}, \tag{13}$$

where the $\hat{\Sigma}$ denote the renormalized Higgs boson self-energies. Determining the poles of the matrix $\Delta_{\rm Higgs}$ in eq. (13) is equivalent to solving the equation

$$\left[q^2 - m^2_{h,\rm tree} + \hat{\Sigma}_{hh}(q^2)\right]\left[q^2 - m^2_{H,\rm tree} + \hat{\Sigma}_{HH}(q^2)\right] - \left[\hat{\Sigma}_{hH}(q^2)\right]^2 = 0. \tag{14}$$

In our calculation the complete one-loop result for the Higgs boson self-energies in the on-shell scheme [25] is combined with the dominant two-loop contributions of $\mathcal{O}(\alpha\alpha_s)$ [4, 11] and further subdominant corrections [6,7], see Ref. [4] for details. The matrix eq. (13) therefore contains the renormalized Higgs boson self-energies

$$\hat{\Sigma}_s(q^2) = \hat{\Sigma}_s^{(1)}(q^2) + \hat{\Sigma}_s^{(2)}(0), \quad s = h, H, hH, \tag{15}$$

where the momentum dependence is neglected only in the two-loop contribution.

[3]Gluinos are also decoupled since their masses are close to the squark masses. The contributions of Bino and Winos to the Higgs sector can be neglected since the $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)$ gauge couplings are small.

Neglecting the momentum dependence also in the one-loop part of the renormalized self-energies eq. (15), the Higgs boson masses m_h and m_H are obtained by diagonalizing the higher order corrected mass matrix with the mixing angle $\alpha_{\rm eff}$ (see e.g. Ref. [57]):

$$\alpha_{\rm eff} = \arctan\left[\frac{-(M_A^2+M_Z^2)\sin\beta\,\cos\beta\, - \hat{\Sigma}_{\phi_1\phi_2}}{M_Z^2\cos^2\beta\, + M_A^2\sin^2\beta\, - \hat{\Sigma}_{\phi_1} - m_h^2}\right] \quad , \quad -\frac{\pi}{2} < \alpha_{\rm eff} < \frac{\pi}{2}\ . \tag{16}$$

Inserting $\alpha_{\rm eff}$ in the tree-level formulas for Higgs production and decay, the higher-order dominant contributions are automatically included [58, 59].

The complete one-loop calculation together with the leading two-loop corrections and the other corrections beyond $\mathcal{O}(\alpha\alpha_s)$ have been implemented into the Fortran code *FeynHiggs* [24]. The transition from $\overline{\rm MS}$ to on-shell parameters is also included in the program. *FeynHiggs* is available via its WWW page `www.feynhiggs.de`.

The evaluation of the Higgs boson sector implemented in *FeynHiggs* has been combined with the program *SUITY* [60], used for the evaluation of the low-energy spectrum of the mSUGRA scenario. This new combined code, *FeynSSG* [61], will be available at `www-itp.physik.uni-karlsruhe.de/feynssg/`. The results presented for the mSUGRA scenario below are derived from the *FeynSSG* output.

2.6 Low-energy constraints

2.6.1 Precision observables

The whole set of low-energy SUSY particles contributes to electroweak precision observables. The most prominent role plays the ρ-parameter [62], which gives the main contribution to other electroweak precision observables, such as the W boson mass, M_W, and the effective leptonic mixing angle, $\sin^2\theta_{\rm eff}$. Here we take into account the corrections arising from $\tilde{t}/\tilde{b}$ loops up to $\mathcal{O}(\alpha\alpha_s)$ [63]. A value of $\Delta\rho$ outside the experimentally preferred region of $\Delta\rho^{\rm SUSY} \lesssim 2\times10^{-3}$ [32] indicates experimentally disfavored $\tilde{t}$ and $\tilde{b}$ masses. The evaluation of $\Delta\rho^{\rm SUSY}$ is implemented into *FeynHiggs*.

2.6.2 Experimental bounds on SUSY particle masses

The search for SUSY particles has been one of the main tasks pursued at the Tevatron and at LEP. The searches all turned out to be negative, thus lower limits on the SUSY particle masses have been set. In order to restrict the allowed parameter space in the three soft SUSY-breaking scenarios we employed the following constraints on their low-energy mass spectrum [32, 51, 64–67]:

$$\begin{aligned}
m_{\tilde{e}} &> 95\ \text{GeV}\\
m_{\tilde{\mu}} &> 85\ \text{GeV}\\
m_{\tilde{\tau}} &> 71\ \text{GeV}\\
m_{\tilde{\nu}} &> 43\ \text{GeV}\\
m_{\tilde{t}} &> 95\ \text{GeV}\\
m_{\tilde{b}} &> 85\ \text{GeV}
\end{aligned}$$

$$
\begin{aligned}
m_{\tilde{g}} &> 190 \text{ GeV} \\
m_{\tilde{\chi}^\pm} &> 103 \text{ GeV (mSUGRA}, m_{\tilde{\nu}} > 300 \text{ GeV)} \\
m_{\tilde{\chi}^\pm} &\gtrsim 84.6 \text{ GeV (mSUGRA}, m_{\tilde{\nu}} < 300 \text{ GeV)} \\
m_{\tilde{\chi}^\pm} &> \text{??? (GMSB)} \\
m_{\tilde{\chi}^\pm} &\gtrsim 50 \text{ GeV (AMSB)} \\
m_{\tilde{\chi}_1^0} &\gtrsim 36 \text{ GeV (mSUGRA, AMSB(??))} \\
m_{\text{NLSP}} &> 100 \text{ GeV (GMSB)}
\end{aligned}
$$

Although it has recently been shown that the case for a light $\tilde{b}$ with $m_{\tilde{b}} \approx m_b$ is not ruled out [68] and that such a light $\tilde{b}$ is hard to detect [69], we do not take this possibility into account. We furthermore demanded that the lightest SUSY particle is uncolored and uncharged, see also the next section.

2.6.3 Other phenomenological restrictions

Besides constraints from precision observables and from unsuccessful direct search for SUSY particles, we also take into account the following restrictions (if not indicated otherwise):

- For the top-quark mass most part of the analysis is based on the value $m_t = 175$ GeV. However, implications of deviations by ± 5 GeV, i.e. $m_t = 170, 180$ GeV are also investigated. As a rule of thumb, a change in m_t by 1 GeV also results in a change in m_h of about 1 GeV.

- The GUT scale parameters are taken to be real, no SUSY $\mathcal{CP}$-violating phases are assumed.

- In all models under consideration the R-parity symmetry is taken to be conserved.

- Parameter sets that do not fulfill the condition of radiative electroweak symmetry breaking (REWSB), i.e. they do the one-loop minimization conditions of eqs. (2),(3), are discarded. Although these conditions have been specified for the mSUGRA scenario, similar conditions apply in all soft SUSY-breaking scenarios considered here.

  Within all soft SUSY-breaking scenarios considered here, the condition of REWSB leads to restrictions on $\tan\beta$. For example, it is almost impossible (or a huge fine tuning is required) to find very large values of $\tan\beta$, $\tan\beta \gtrsim 60$ which pass this constraint. This is because in that region both the RGEs of the Higgs soft breaking masses receive large corrections not only from the top Yukawa coupling but also from the bottom and the tau Yukawa couplings. This drives the μ^2 in the numerator of eq. (2) negative, thus excluding this parameter set from our analysis. Another possibility of not fulfilling the REWSB condition is a very heavy soft SUSY-breaking spectrum, so that $m_{H_2}^2$ does not reach negative values and thus not triggering REWSB.

- Parameter sets that do not fulfill the CCB constraints are discarded, i.e. models for which charge or color breaking minima occur.

  This corresponds to cases where the squared scalar quark or lepton masses are becoming negative at the scale Q, where Q is the energy scale at which the low-energy parameters are decoupled. However, it could be possible for values of the scale Q greater than few TeV that the lifetime of a color or charge breaking minimum is longer than the present age of the universe, and thus are in practice not dangerous [70–73]. Within the mSUGRA analysis e.g. the scale Q is taken to be smaller than a few TeV. In this way these CCB bounds are taken into account in the present analysis. A detailed analysis in the whole mSUGRA parameter space is rather complicated and beyond the aim of this article.

  How about that?

- Within the mSUGRA and AMSB scenario, the lightest SUSY particle (LSP) is required to be the neutralino. Parameter sets that result in a different LSP are excluded.

  LSP correct, Shufang?

- We do not take into account the constraints coming from $b \to s\gamma$, since the uncertainty is still relatively large and the parameter space where the obtained results are valid are still quite restricted [74].

3 Numerical analyses

3.1 Experimental bounds from the MSSM Higgs sector

The running of LEP in the year 2000 has excluded a large part of the MSSM parameter space. On the other hand a few events have been found, which can be interpreted as evidence of the Higgs boson [2]. For our numerical analysis we will focus on three different cases implying different restrictions on the MSSM parameter space:

(I) The MSSM Higgs boson have been searched in three different benchmark scenarios [75]. From the non-observation of a Higgs boson lower bounds of $m_h \gtrsim 90$ GeV and $M_A \gtrsim 90$ GeV have been obtained [26], depending for both searches on $\tan\beta$ and for m_h also on the assumed M_A value. These results are used to infer bounds on the parameters of the three soft SUSY-breaking scenarios. More precisely, we made use of the analysis presented in Ref. [76]. In this reference the excluded Higgs boson mass is presented with respect to the reduced ZZh coupling squared (i.e. $\sin^2(\beta - \alpha_{\rm eff})$, see eq. (16)) at 95% CL for $m_t = 175$ GeV. It automatically puts bounds also on the $\mathcal{CP}$-odd Higgs boson mass since the channel $e^+e^- \to Ah$ is included in the analysis. The theoretical prediction obtained at the two-loop level of m_h and $\sin^2(\beta - \alpha_{\rm eff})$ for each parameter set is then compared with the excluded region given in Ref. [76]. This allows to set stricter bounds on the Higgs boson masses and on the $\tan\beta$, compared to those derived by directly applying the m_h–$\tan\beta$ exclusion plots for the unconstrained MSSM.

  Another important constraint on m_h comes from the searches in $p\bar{p}$ collisions at CDF [77]. No evidence of a signal of a type $p\bar{p} \to b\bar{b}h \to b\bar{b}b\bar{b}$ has been found.

This sets lower mass limits for the light Higgs boson in the region of large $\tan\beta$, $\tan\beta \gtrsim 35$. This bound is taken into account in our analysis (see Fig. 2 in Ref. [77]).

(II) The data collected at $\sqrt{s} \gtrsim 206$ GeV resulted in a 3σ excess [2] for a SM-like Higgs boson with a "measured" mass of

$$m_h = 115 \pm 2 \text{ GeV}. \tag{17}$$

This "measurement" interpreted as the mass of the lightest MSSM Higgs boson is used to derive constraints on the parameter space of the three soft SUSY-breaking scenarios.

(III) Finally it is also possible to interpret the evidence of the Higgs as a measurement of the heavy $\mathcal{CP}$-even Higgs boson mass. Therefore it is necessary that H has SM-like couplings, i.e. $\cos^2(\beta - \alpha_{\text{eff}}) \gtrsim 0.8$. Two interpretations are possible:

(IIIA) There exists also a 3σ excess in the m_h-M_A plane at $m_h \approx M_A \approx 95$ GeV [26]. Although it is likely that at least at one place in this plane a 3σ excess occurs, we interpret this as an indication of the two Higgs boson masses (with errors of ± 2 GeV)

(IIIB) For $\cos^2(\beta - \alpha_{\text{eff}}) \gtrsim 0.8$ the h production via Higgs strahlung, $e^+e^- \to Zh$, is highly suppressed, whereas the associated production $e^+e^- \to Ah$ could be beyond the kinematic reach of LEP. In this scenario a bound of $m_h + M_A > 206$ GeV is assumed.

3.2 mSUGRA

In our numerical procedure we follow a two loop renormalization group analysis for all parameters involved, i.e couplings and dimensionful parameters, v.e.v's included. We start with the $\overline{MS}$ values for the gauge couplings at M_Z giving a trial input value for the strong coupling constant α_s in the vicinity of 0.120, which are then converted to their $\overline{DR}$ values. These are run down to $M_b = 4.9$ GeV, $M_\tau = 1.777$ GeV with the $SU_c(3) \times U_{em}(1)$ RGE's [78] to know the running bottom and tau masses extracted from their pole masses including SUSY at 1-loop and QCD at 2-loop corrections as given in [31]. From these and by making use of the RGE's for the running masses m_b, m_τ at 2-loops we run upwards to know their $\overline{MS}$ values at M_Z which are subsequently converted to their corresponding $\overline{DR}$ values. This provides us with the bottom and tau Yukawa couplings at the scale M_Z. The top Yukawa coupling is known from the input top quark pole mass $M_t = 175$ GeV which is subsequently converted to the $\overline{DR}$, $m_t(M_t)$ value where the top Yukawa coupling is defined. The evolution of all couplings from M_Z running upwards to high energies determines the unification scale M_{GUT} and the value of the unification coupling α_{GUT} by

$$\alpha_1(M_{GUT})|_{\overline{DR}} = \alpha_2(M_{GUT})|_{\overline{DR}} = \alpha_{GUT} \ . \tag{18}$$

At the GUT scale we set the boundary conditions for the soft SUSY breaking parameters in the region,

$$\begin{aligned} 50 \text{ GeV} \leq \quad & M_0 \quad \leq 1 \text{ TeV} , \\ 50 \text{ GeV} \leq \quad & M_{1/2} \quad \leq 1 \text{ TeV} , \\ -3 \text{ TeV} \leq \quad & A_0 \quad \leq 3 \text{ TeV} . \end{aligned} \tag{19}$$

Running down from M_{GUT} to M_Z the trial input value for α_s has now changed. We also vary the $\tan\beta$ in the region,

$$1.4 \leq \quad \tan\beta \quad \leq 60 , \tag{20}$$

As we have already mentioned earlier the parameters $|\mu|$ and B are calculated from the minimization conditions (2) and (3), respectively. The sign of the μ-parameter is input and for every data point in (19,20) we scann for both the signs of μ. For the calculation of the soft breaking masses at the Electroweak scale we use the so called step function approximation [41]. Thus, if the equation at hand is the Renormalization Group equation for a particular running mass $m(Q)$, then Q_0 is the corresponding physical mass determined by the condition $m(Q_0) = Q_0$. This procedure is iterated several times until convergence is reached.

All the information relevant to the light Higgs boson mass in the mSUGRA scenario has been collected in our Figures.(3.3, 3.3,3.3). This fugures show the variation of the light Higgs boson mass with respect to $\tan\beta$ for the various cases of interest in the paragraph (3.1) for up to 50000 models scanned in the region of (19,20). From the fig.(3.3) we obtain that the general behaviour of m_h^{max} upper bound is to be increased as $\tan\beta \leq 12$ is increasing. For larger $\tan\beta$ values m_h^{max} takes on rather constant values until $\tan\beta \simeq 50$ where it starts declining. Larger than $\tan\beta > 60$ values are not allowed die to REWSB constraint. The upper bound on the light Higgs boson mass in the mSUGRA scenario is[4]

$$m_h^{max} \lesssim 122 \text{ GeV} \quad (\text{mSUGRA}) . \tag{21}$$

This upper limit on m_h takes place on the region $20 < \tan\beta < 30$.

- $\tan\beta$ is excluded up to $\tan\beta = 2.4$ for all the allowed Higgs masses.
- Light Higgs boson mass is excluded up to 107.7 GeV for values of $\tan\beta$ excluded up to $\tan\beta \simeq 32$. As we see this is *not* the same as in the unconstraint MSSM.
- Upper bound on the Higgs mass , $m_h \simeq 125$ GeV.
- Case (II) excludes values of $\tan\beta$ up to 3.1 for $m_h = 113$ GeV and up to 4.5 for $m_h = 117$ GeV.For all the other values of $\tan\beta \leq 60$ we find solutions for the $m_h = 115 \pm 2$ GeV.

[4]This bound is 5 GeV lower than that obtained in Ref. [18]. This is mainly because of two reasons : a) The bound of $\Delta\rho$ we apply here and b) the sort of "naturalness" bound on the squark masses to be less than 2.5 TeV which is also applied here but not in [18]. We also have to admit that several upgrades in both programs `SUITY` and `FeynHiggs` had been made since then which might (in principle) couse such a discrepancy.

- Case (IIIA) happens for large values of $\tan\beta \geq 30$.
- Case (IIIB) indeed happens only for very large values of $\tan\beta \simeq 54$.

  The signal is quite robust with respect to the variation of the other parameters. These points are denoted with 'magenta' color triangle up symbols in our plots.

Sakis, pleas explain in more detail

Fig.2

We present only points which are allowed (denoted in black circles) from the experimental searches on the Higgs boson and the SUSY particles.

- The mass of the $\mathcal{CP}$-Odd Higgs ($M_A \leq 180$ GeV) boson is excluded for all the values of $\tan\beta \leq 32$.
- Values of the $\mathcal{CP}$-Odd Higgs mass ($M_A \leq 91.6$ GeV) are excluded for values of $\tan\beta \geq 32$.
- If the Higgs has a mass of $m_h = 115 \pm 2$ GeV then that excludes the possibility of a $\mathcal{CP}$-Odd Higgs with mass up to 180 GeV with values of $\tan\beta$ lower than 43. Smaller $\mathcal{CP}$-Odd Higgs masses are reached only for larger $\tan\beta$ values.
- Cases (IIIA or IIIB) valid only for small masses of M_A in the region of 92-120 GeV.

Fig.3,4

- For the case (II) we conclude that $M_A \geq 112$ GeV and $M_H \geq 117$ GeV.
- For the case (IIIA) we obtain : $M_H = (100 - 120)$ GeV.
- The most interesting case is the case (IIIB). We find a robust prediction for the heavy $\mathcal{CP}$-even Higgs of 115 ± 2 GeV which is the Higgs seeing by LEP while the other two neutral Higgses vary in the region $m_h = (92-113)$ GeV and $M_A = 93-122$ GeV. For the region of $m_h \simeq m_A \geq 104$ GeV the hA process is kinematically not allowed and we see a heavy $\mathcal{CP}$-Even Higgs with a mass of 115 ± 2 GeV.

3.3 GMSB

↓

For this paper, about XXXXX GMSB models were generated under well defined hypotheses described in Sect. 2.3, using the program *SUSYFIRE* [50] and adopting the phenomenological approach of Refs. [46–48], see also Ref. [19].

↑

3.4 AMSB

↓

According to the description presented in Sect. 2.4 about $\sim YYYYY$ models have been created. The GUT scale parameters have been varied in the ranges

$$\begin{aligned} 20 \text{ TeV} \leq \quad & m_{\text{aux}} \quad \leq 100 \text{ TeV} \\ 0 \leq \quad & M_0 \quad \leq 2 \text{ TeV} \\ 1.2 \leq \quad & \tan\beta \quad \leq 60, \end{aligned} \tag{22}$$

and both signs of μ.

↑

4 Conclusions

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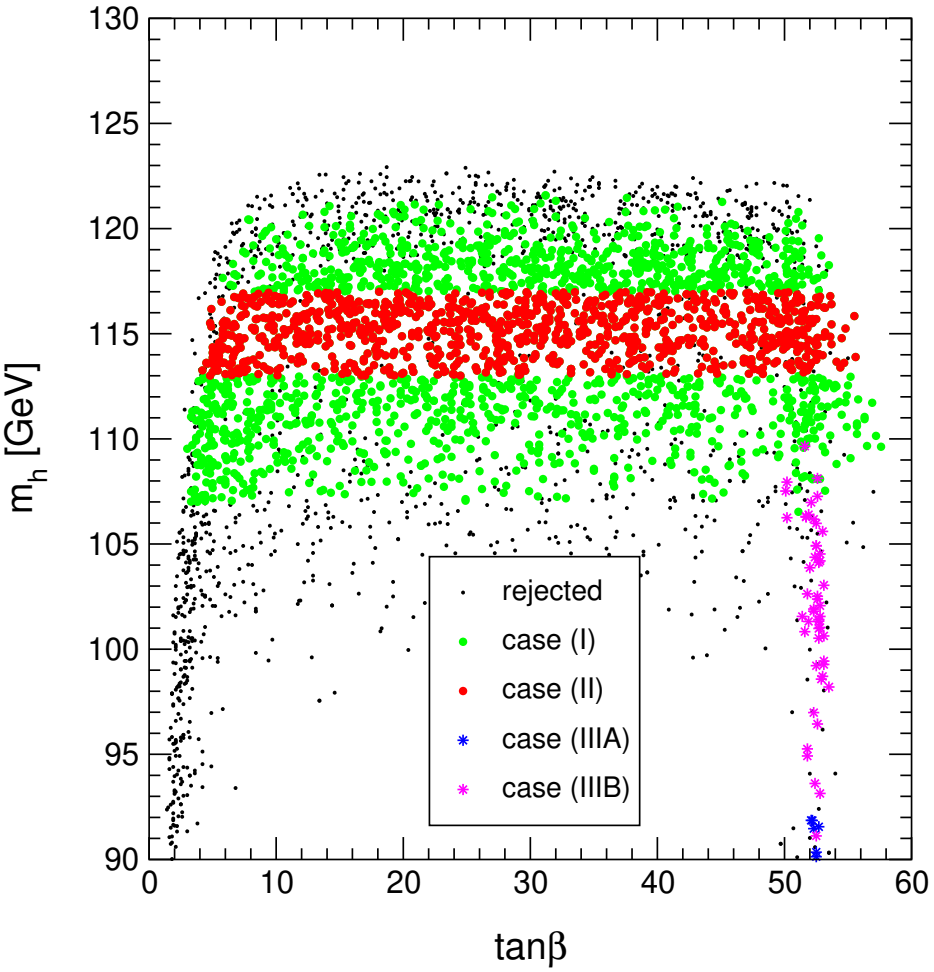

Figure 1: The light Higgs boson mass m_h as a function of $\tan\beta$ in the mSUGRA scenario.The various cases discussed in the paragraph (3.1) are displayed. All the black small dots are excluded either by direct Higgs searches or by SUSY experimental ones. The mass of the top quark has been taken to be fixed at 175 GeV. The CDF bound on the Higgs mass has not been applied here (see the discussion on the text).

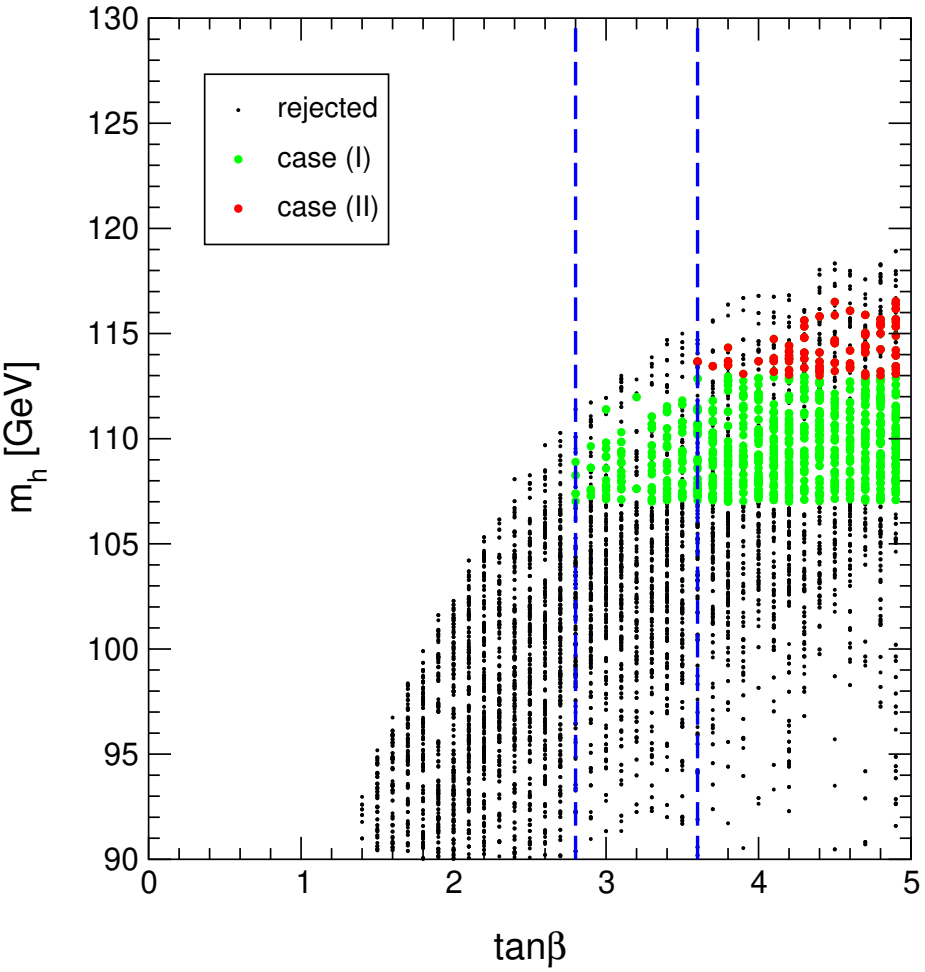

Figure 2: As in Fig.(3.3) but focusing in the excluded region of the $\tan\beta$.

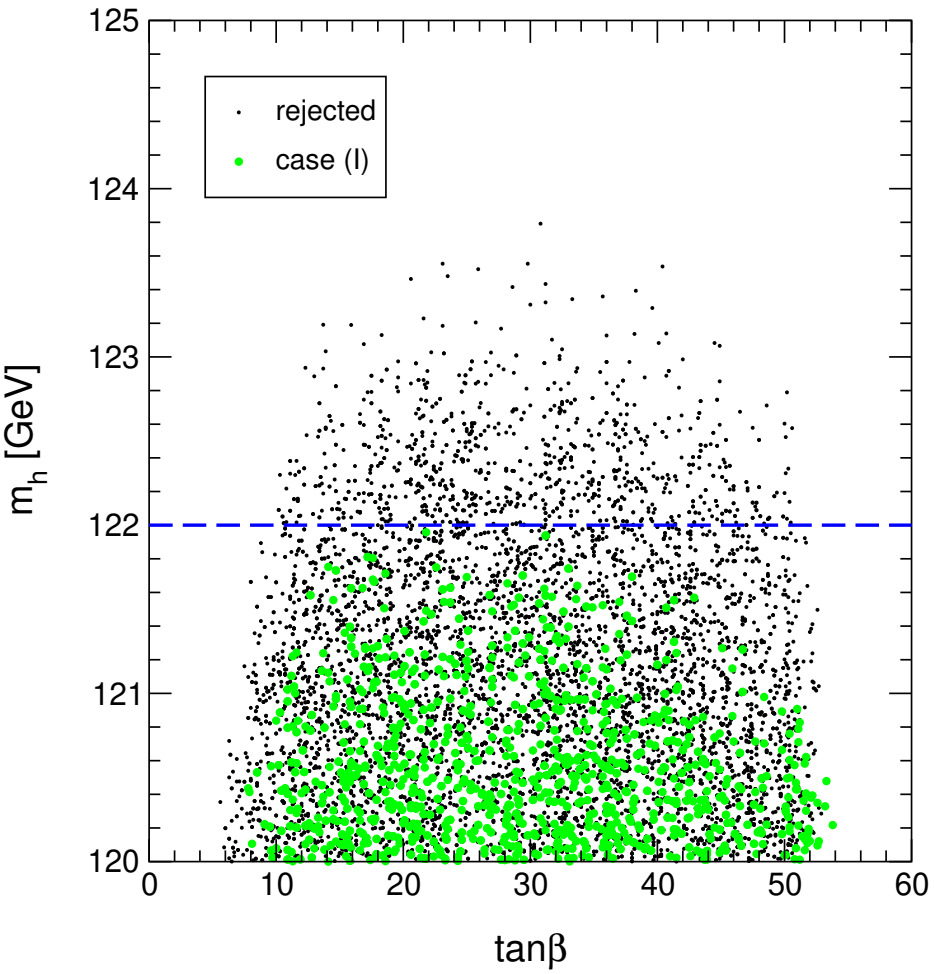

Figure 3: As in Fig.(3.3) but focusing on the upper limit of the light Higgs boson mass.

Figure 4: The $\mathcal{CP}$-Odd Higgs boson mass m_A as a function of $\tan\beta$ in the mSUGRA scenario. The grey points are not accessible by the theory.

Figure 5: The light Higgs boson mass m_h as a function of the $\mathcal{CP}$-ODD m_A Higgs mass in the mSUGRA scenario. The grey points are not accessible by the theory.

Figure 6: The light Higgs boson mass m_h as a function of the $\mathcal{CP}$-Even m_H Higgs boson mass in the mSUGRA scenario. The grey points are not accessible by the theory.