Correlation of $B_s \to \mu^+\mu^-$ and $(g-2)_\mu$ in Minimal Supergravity

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We analyse the rare decay mode $B_s \to \mu^+\mu^-$ in the minimal supergravity scenario (mSUGRA). We find a strong correlation with the muon anomalous magnetic moment $(g-2)_\mu$. An interpretation of the recently measured excess in $(g-2)_\mu$ in terms of mSUGRA corrections implies a substantial supersymmetric enhancement of the branching ratio $\mathcal{B}(B_s \to \mu^+\mu^-)$: if $(g-2)_\mu$ exceeds the Standard Model prediction by $4 \cdot 10^{-9}$, $\mathcal{B}(B_s \to \mu^+\mu^-)$ is larger by a factor of 10–100 than in the Standard Model and within reach of Run-II of the Tevatron. Thus an experimental search for $B_s \to \mu^+\mu^-$ is a stringent test of the mSUGRA GUT scale boundary conditions. If the decay $B_s \to \mu^+\mu^-$ is observed at Run-II of the Tevatron, then we predict the mass of the lightest supersymmetric Higgs boson to be less than 120 GeV. The decay $B_s \to \mu^+\mu^-$ can also significantly probe the favoured parameter range in SO(10) SUSY GUT models.

Supersymmetry (SUSY) is an attractive and widely studied extension of the Standard Model (SM). The minimal supergravity model (mSUGRA) [1] relates all supersymmetric parameters to just 5 real quantities: the universal scalar and gaugino masses M_0 and $M_{1/2}$, the trilinear term A_0, the ratio $\tan\beta$ of the two Higgs vacuum expectation values, and $\operatorname{sgn}\mu$, where μ is the Higgsino mass parameter. The first three quantities are defined at a high, grand unified energy scale and the others at the electroweak scale. They are the boundary conditions for the renormalization group equations, which determine the physical parameters at our low scale. Precision observables, which are affected by SUSY corrections through loop effects, play an important role in constraining the supersymmetric parameter space. The small number of parameters makes mSUGRA highly predictive so it can be significantly tested by low energy precision measurements. In this letter we show that the decay $B_s \to \mu^+\mu^-$ is a stringent test of the mSUGRA scenario, in particular when correlated with $(g-2)_\mu$.

Recently the Brookhaven National Laboratory (BNL) reported an excess of the muon anomalous magnetic moment $a_\mu = (g-2)_\mu/2$ over its SM value [2]. The difference $\delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (43 \pm 16) \cdot 10^{-10}$ corresponds to a 2.6σ deviation from the SM. An mSUGRA interpretation of this anomaly implies $\mu > 0$ (in the sign convention with $M_{1/2} > 0$ and equal signs of the diagonal elements of the chargino mass matrix) [3]. It further invites a large $\tan\beta \gtrsim 10$ [4]. The discrepancy in the case of a_μ is by itself not significant enough to justify the claim of new physics, especially since the calculation of a_μ^{SM} involves two hadronic quantities: the hadronic contributions to the photon self-energy, which must be obtained from other experiments, and the (smaller) light-by-light scattering contribution, which can only be estimated with hadronic models. A more conservative estimate of the latter would reduce the BNL anomaly to a 2σ effect [2]. Hence in order to resolve the possible ambiguity between mSUGRA and alternative explanations of δa_μ one ideally wishes to study other observables whose sensitivity to supersymmetric loop corrections is correlated with δa_μ. It is our purpose here to show the strong correlation between $\mathcal{B}(B_s \to \mu^+\mu^-)$ and δa_μ in mSUGRA.

SUSY modifies B meson observables if $\tan\beta$ is large, because the b Yukawa coupling becomes sizable. Especially sensitive are quantities with a b quark chirality flip like the branching ratios $\mathcal{B}(B \to X_s\gamma)$ and $\mathcal{B}(B \to \ell^+\ell^-)$. In mSUGRA the low energy value for the trilinear term A_t is dominated by $M_{1/2}$ with $A_t < 0$ for $M_{1/2} > 0$ [5]. Then $\mu > 0$ implies that the charged-Higgs-top loop and the chargino-stop loop tend to cancel in $\mathcal{B}(B \to X_s\gamma)$, so that the sensitivity to mSUGRA corrections is weakened. A further disadvantage of this decay mode is that it requires an experimental cut on the photon energy, which introduces some hadronic uncertainty.

In [5] the possible impact of flavour-blind SUSY on other B physics observables, in particular those which enter the fit of the unitarity triangle, were studied and only small effects were found. This did not include the decay $B_s \to \mu^+\mu^-$. In contrast to the observables in [5], the branching ratio $\mathcal{B}(B_s \to \mu^+\mu^-)$ grows like $\tan^6\beta$ [6–8], with a possible several orders of magnitude enhancement. We here go beyond this work to study $B_s \to \mu^+\mu^-$ in the mSUGRA model. Since $\mathcal{B}(B \to \ell^+\ell^-) \propto m_\ell^2$, the branching ratio is largest for $\ell = \tau$. Yet τ's are hard to detect at hadron colliders, so that the prime experimental focus is on the search for $B_s \to \mu^+\mu^-$. B factories running on the $\Upsilon(4S)$ resonance produce no B_s mesons. Leptonic branching ratios of B_d mesons are smaller by a factor of $|V_{td}/V_{ts}|^2 \lesssim 0.06$. Since in B factories the boost of the B_d meson is known and the considered leptonic decay rates can be substantially enhanced over their SM values in SUSY, we encourage our colleagues at BaBar and BELLE to look for $B_d \to \tau^+\tau^-$ decays, as well. From now on we restrict ourselves to the decay mode $B_s \to \mu^+\mu^-$.

In [6–8] the SUSY corrections to $\mathcal{B}(B_s \to \mu^+\mu^-)$ were calculated at the one-loop level. For large $\tan\beta$, higher order corrections can be large, eventually of order 1. In [9] $\tan\beta$-enhanced supersymmetric QCD corrections have been summed to all orders in perturbation theory. We have incorporated these dominant higher order corrections by replacing the b Yukawa coupling $h_b \propto m_b \tan\beta$ with $h_b^{\rm eff} = h_b/(1+\Delta m_b)$, where $\Delta m_b \propto \mu \tan\beta$ depends on the gluino and sbottom masses and can be found in [9]. Δm_b is positive for $\mu > 0$. The dominant contribution to $\mathcal{B}(B_s \to \mu^+\mu^-)$ is proportional to $h_b^{\rm eff\,4}$, so that the inclusion of Δm_b tempers the large-$\tan\beta$ behaviour.

The considered branching ratio can be expressed as

$$\mathcal{B}(B_s \to \mu^+\mu^-) = 6.0 \cdot 10^{-7} \left(\frac{|V_{ts}|}{0.040}\right)^2 \left(\frac{f_{B_s}}{230\,\mathrm{MeV}}\right)^2 \frac{m_\mu^2}{m_{B_s}^2} \sqrt{1-\frac{4m_\mu^2}{m_{B_s}^2}} \left\{ \left(1-\frac{4m_\mu^2}{M_{B_s}^2}\right) \left|\frac{m_{B_s}^2 C_S}{m_\mu}\right|^2 + \left|\frac{m_{B_s}^2 C_P}{m_\mu} - 2C_A\right|^2 \right\}. \quad (1)$$

Here $|V_{ts}| = 0.040 \pm 0.002$ is the relevant CKM matrix element and $f_{B_s} = (230 \pm 30)$ MeV [10] is the B_s decay constant. In (1) we have kept the dependence on the lepton mass m_μ, so that the generalisation to $B_d \to \tau^+\tau^-$ is straightforward. The Wilson coefficients C_S, C_P and C_A, which contain the short-distance physics, are normalised as in [11]. The coefficients c_S, c_P and c_{10} defined in [8] are related to ours by $C_S = -2c_S \sin^2\theta_W$, $C_A = -2c_{10}\sin^2\theta_W$ and $C_P = 2c_P \sin^2\theta_W$. Within the SM, C_S and C_P are negligibly small and the NLO result for C_A can be well approximated by $C_A = 2.01(\overline{m}_t/167\ \mathrm{GeV})^{1.55}$ [12]. Here $\overline{m}_t \equiv \overline{m}_t(m_t)$ is the top quark mass in the $\overline{\rm MS}$ scheme. $\overline{m}_t = 167$ GeV corresponds to a pole mass of $m_t = 175$ GeV. The SM prediction is given by $\mathcal{B}(B_s \to \mu^+\mu^-) = (3.7 \pm 1.2)\cdot 10^{-9}$, with the uncertainty ($\pm 25\%$) dominated by f_{B_s}. This is also the main hadronic uncertainty in the SUSY calculation.

During Run-I of the Tevatron, CDF determined [13]

$$\mathcal{B}(B_s \to \mu^+\mu^-) < 2.6 \times 10^{-6}, \qquad \text{at 95\% C.L.} \quad (2)$$

The single event sensitivity of CDF at Run-IIa is estimated to be $1.0 \cdot 10^{-8}$, for an integrated luminosity of 2 fb^{-1} [14]. Thus if mSUGRA corrections enhance $\mathcal{B}(B_s \to \mu^+\mu^-)$ to e.g. $5 \cdot 10^{-7}$, one will see 50 events in Run-IIa. Run-IIb may collect 10-20 fb^{-1} of integrated luminosity, which implies 250-500 events in this example.

In SUSY, the dominant coefficients are $C_{S,P}$ since they are proportional to $\tan^3\beta$. We would like to understand the effect of the restricted mSUGRA parameters on $C_{S,P}$ and thus on $\mathcal{B}(B_s \to \mu^+\mu^-)$. In mSUGRA the low-energy values of both μ and the squark masses are dominated by the (GUT scale) value of $M_{1/2}$ through the renormalization group equations. For not-too-large $M_0, M_{1/2} \lesssim 500$ GeV and $A_0 \simeq 0$ GeV we can derive the approximate formula $\mathcal{B}(B_s \to \mu^+\mu^-) \approx 10^{-6} \tan^6\beta\, M_{1/2}^2 \mathrm{GeV}^4/(M_{1/2}^2 + M_0^2)^3$ showing that $\mathcal{B}(B_s \to \mu^+\mu^-)$ scales like $\mathcal{B}(B_s \to \mu^+\mu^-) \propto \tan^6\beta/M_{1/2}^4$ for $M_{1/2} \gg M_0$. In the vicinity of the maximum (near $M_{1/2} = 0.4\,M_0$) the approximate formula is not accurate. A similar estimate of the supersymmetric contribution to a_μ yields $(\delta a_\mu)_{SUSY} \propto \tan\beta f(M_0)/M_{1/2}^2$. $(\delta a_\mu)_{SUSY}$ depends on slepton masses, which are less sensitive to $M_{1/2}$ than squark masses; they are dominated by M_0. We have encoded the M_0 dependence in the slowly varying function $f(M_0)$. Hence both $\mathcal{B}(B_s \to \mu^+\mu^-)$ and $(\delta a_\mu)_{SUSY}$ grow with $\tan\beta$ and decrease with increasing $M_{1/2}$. For this it is essential that we have made the assumption of the mSUGRA GUT scale boundary conditions. Thus within mSUGRA we expect a strong correlation between $\mathcal{B}(B_s \to \mu^+\mu^-)$ and $(\delta a_\mu)_{SUSY}$: if $\delta a_\mu \neq 0$ requires a SUSY explanation with large $\tan\beta$ then we would expect $\mathcal{B}(B_s \to \mu^+\mu^-)$ to be strongly enhanced. If, however, the supersymmetric explanation of δa_μ requires small $M_{1/2}$ and a moderate value of $\tan\beta$ then we would expect only a moderate enhancement of $\mathcal{B}(B_s \to \mu^+\mu^-)$.

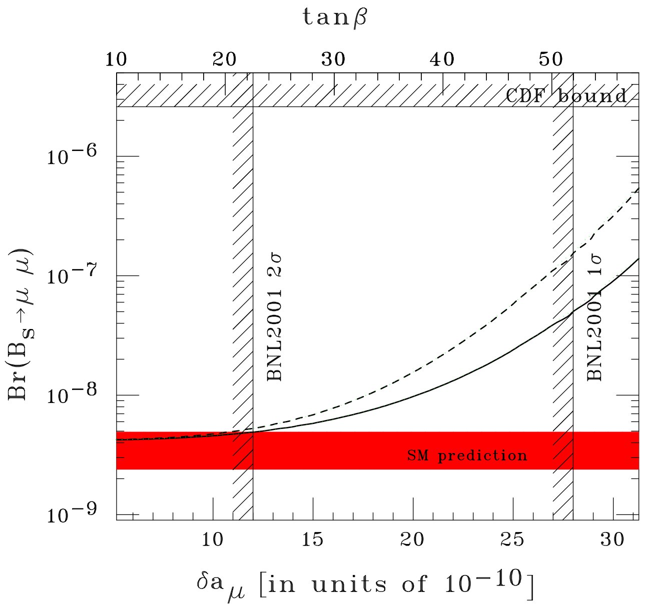

FIG. 1. $(\delta a_\mu)_{SUSY}$, versus $\mathcal{B}(B_s \to \mu^+\mu^-)$ for $\tan\beta$ (top) and $M_{1/2}$=450, $M_0 = 350, A_0 = 0, \mu > 0, m_t = 175$ GeV. Shown also, the SM prediction, the present bound by CDF [13], on $\mathcal{B}(B_s \to \mu^+\mu^-)$ as well as the present 1σ and 2σ bound on δa_μ from BNL [2]. We used $f_{B_s} = 230$ MeV.

We now study these effects quantitatively. For this we use the full computation of Eq.(1) including the resummed SUSY QCD corrections, and restricting ourselves to the mSUGRA parameters. In Fig.1, we show the direct correlation between $\mathcal{B}(B_s \to \mu^+\mu^-)$ and $(\delta a_\mu)_{SUSY}$ for the fixed parameters: $M_{1/2} = 450$ GeV$, M_0 = 350$ GeV$, A_0 = 0, \mu > 0$ and $m_t = 175$ GeV. On the upper edge we show the $\tan\beta$ dependence. We restrict ourselves to $\tan\beta < 58$ in order to guarantee radiative electroweak symmetry breaking (REWSB). We have included the SM prediction and the CDF bound from Eq.(2). The solid (dashed) curve represents the

$\mathcal{B}(B_s \to \mu^+\mu^-)$ result with (without) resummation of the $\tan\beta$-enhanced SUSY-QCD corrections. In this example, the resummation suppresses $\mathcal{B}(B_s \to \mu^+\mu^-)$ by 75% for $\tan\beta \gtrsim 50$. In order for mSUGRA to account for δa_μ within 1σ of the current BNL measurement at this parameter point, we see that we need a large value of $\tan\beta \gtrsim 50$. Due to the strong correlation within mSUGRA we then predict $\mathcal{B}(B_s \to \mu^+\mu^-) \gtrsim 5 \cdot 10^{-8}$, which is observable by CDF at Run II.

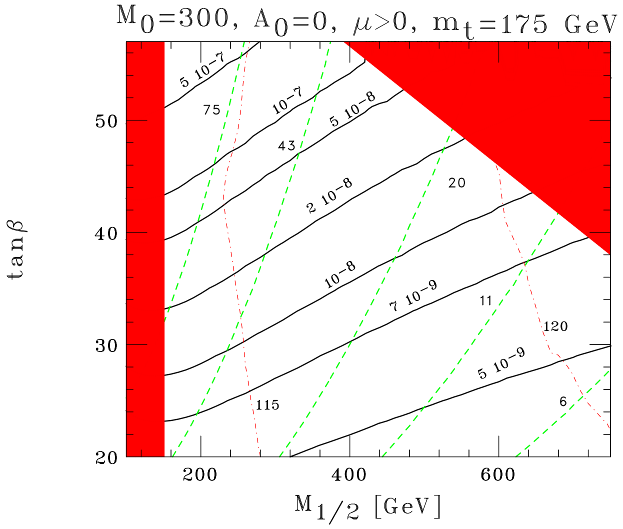

FIG. 2. Contours of $\mathcal{B}(B_s \to \mu^+\mu^-)$ (solid) and $(\delta a_\mu)_{SUSY}$ (in units 10^{-10}) (dashed) in the $M_{1/2}$-$\tan\beta$ plane. The lightest neutral CP-even Higgs mass is shown as well (dot-dashed). The shaded regions are excluded, as described in the text. The mSUGRA parameters are given at the top.

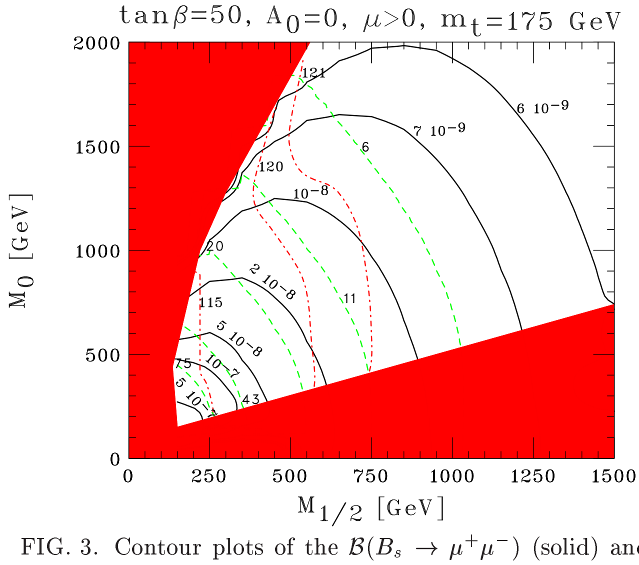

FIG. 3. Contour plots of the $\mathcal{B}(B_s \to \mu^+\mu^-)$ (solid) and on $(\delta a_\mu)_{SUSY}$ (dashed) in the $(M_0, M_{1/2})$-plane for mSUGRA parameter values as shown. The shaded regions are excluded as described in the text. Contours of the light Higgs boson mass (dot-dash line) are also shown.

As we discussed above, we expect $\mathcal{B}(B_s \to \mu^+\mu^-)$ to dominantly depend on the mSUGRA parameters $M_{1/2}$ and $\tan\beta$. In Fig.2 we show the $\mathcal{B}(B_s \to \mu^+\mu^-)$ (solid) and the $(\delta a_\mu)_{SUSY}$ (dashed) contours in this plane. We have fixed: $M_0 = 300$ GeV, $A_0 = 0$, $\mu > 0$ and $m_t = 175$ GeV. The 2σ contours for δa_μ (11,75) are explicitly given. The left vertical shaded region is theoretically excluded since it does not allow for REWSB or violates the LEP chargino bound. The upper right triangular shaded region is excluded, since the LSP is not neutral. If as expected, CDF can probe down to $\mathcal{B}(B_s \to \mu^+\mu^-) \gtrsim 2{\cdot}10^{-7}$ at RUN IIa, this corresponds to a sensitivity of $(M_{1/2}, \tan\beta)$ ranging from (160 GeV, 47) to (450 GeV, 57). The qualitative discussion of before is now nicely reproduced. $\mathcal{B}(B_s \to \mu^+\mu^-)$ decreases with increasing $M_{1/2}$ and rapidly increases with $\tan\beta$. Fig.2 also nicely shows the cross-correlation between $(\delta a_\mu)_{SUSY}$ and $\mathcal{B}(B_s \to \mu^+\mu^-)$. If both $\mathcal{B}(B_s \to \mu^+\mu^-)$ and $(\delta a_\mu)_{SUSY}$ are found in disagreement with the SM and are measured with a 50% and 20% accuracy, respectively, then for given M_0, this fixes $\tan\beta$ to better than 20% and $M_{1/2}$ to better than 30%.

It is conventional to discuss mSUGRA physics in the $(M_{1/2}, M_0)$-plane. In Fig.3 we show the contours of $\mathcal{B}(B_s \to \mu^+\mu^-)$ (solid) and $(\delta a_\mu)_{SUSY}$ (dashed) in this plane, for $\tan\beta = 50$, $A_0 = 0$, $\mu > 0$ and $m_t = 175$ GeV. Again we include the CDF bound Eq.(2) and the Higgs mass contours. The left shaded region is excluded through the requirement of REWSB or the chargino bound. The lower right shaded region is excluded through the requirement of a neutral LSP. A sensitivity of $\mathcal{B}(B_s \to \mu^+\mu^-) \gtrsim 2 \cdot 10^{-7}$ at CDF now corresponds to a sensitivity of $M_{1/2} \lesssim 280$ GeV and $M_0 \lesssim 400$ GeV, respectively.

While CDF is not able to see squark masses directly up to 0.7 TeV (corresponding to $M_{1/2} = M_0 \simeq 300$ GeV,), it will nevertheless be able to prepare the ground for LHC by observing the $B_s \to \mu^+\mu^-$ mode. Even better, after 10 fb^{-1} CDF will probe $M_{1/2} \lesssim 450$ GeV and $M_0 \lesssim 600$ GeV (for $\tan\beta = 50$) which in mSUGRA corresponds to masses for the heaviest superpartners of 1 TeV. We conclude the discussion of Fig.3 with the prediction of the light Higgs boson mass M_h (dot-dashed line) for $\tan\beta = 50$ in the mSUGRA scenario [15]. Any measurement of $\mathcal{B}(B_s \to \mu^+\mu^-)$ by itself implies a useful *upper* bound on M_h. The simultaneous information of $\mathcal{B}(B_s \to \mu^+\mu^-)$ and δa_μ fixes M_h in most regions of the $(M_{1/2}, M_0)$-plane. A Higgs mass around 115.6 GeV results in $10^{-8} \lesssim \mathcal{B}(B_s \to \mu^+\mu^-) \lesssim 3 \cdot 10^{-7}$ which would most likely be measured before the Higgs boson is discovered.

In Figs. 1-3 we have chosen $A_0 = 0$. A non-zero A_0 changes the value of A_t at low energies. This parameter plays a crucial role for the GIM cancellations among the contributions of different squarks to $\mathcal{B}(B_s \to \mu^+\mu^-)$. Changing A_0 to -500 GeV in the scenario of Fig. 1 enhances $\mathcal{B}(B_s \to \mu^+\mu^-)$ by up to a factor of 6 compared to the case with $A_0 = 0$. For $A_0 = +500$ GeV $\mathcal{B}(B_s \to \mu^+\mu^-)$ is slightly decreased.

In our figures we have omitted further constraints on the mSUGRA parameter space, in order to clearly show the correlation between $\mathcal{B}(B_s \to \mu^+\mu^-)$ and $(\delta a_\mu)_{SUSY}$. The most significant further constraint comes from the measurement of $\mathcal{B}(B \to X_s\gamma)$ [16], whose prediction is less certain in the large $\tan\beta$ region [17,5]. If we take the conservative approach of [18], then we can exclude values of $M_{1/2} \lesssim 250\,\mathrm{GeV}$ in Fig.2 for $\tan\beta \gtrsim 25$. In the scenario of Fig.3 this implies $\mathcal{B}(B_s \to \mu^+\mu^-) \lesssim 5 \cdot 10^{-7}$. For a discussion of the constraints from supersymmetric dark matter see for example [4,18] and references therein.

The large values of $\tan\beta$ we have been considering are theoretically well motivated within SUSY SO(10) Yukawa unification. There a narrow parameter region can explain the observed δa_μ while still being consistent with the constraint from $b \to s\gamma$ [19,20]. This is *not* within the context of mSUGRA. However, in this parameter region both μ and $M_{1/2}$ are light, while the CP-odd Higgs boson mass is less than 300 GeV, and $\tan\beta \approx 50$. Therefore we expect $\mathcal{B}(B_s \to \mu^+\mu^-)$ to be strongly enhanced. As an example we determine $\mathcal{B}(B_s \to \mu^+\mu^-)$ for the best fit points found in [20]: $M_A = 110$ GeV, $m_{\tilde{\chi}_1} \lesssim 250$ GeV, $|A_t| \gtrsim 1$ TeV, $m_{\tilde{t}} \lesssim 1$ TeV and $\tan\beta \simeq 50$. Within the hadronic uncertainties $\mathcal{B}(B_s \to \mu^+\mu^-) \gtrsim 10^{-5}$ which is already excluded by CDF [13]. Thus the SO(10) models should be reconsidered in the light of $\mathcal{B}(B_s \to \mu^+\mu^-)$. Turning it around, if an SO(10) GUT model is the correct description of nature then the decay $B_s \to \mu^+\mu^-$ must be just around the corner.

In conclusion, we have found a striking correlation between the muon anomalous magnetic moment a_μ and the branching ratio $\mathcal{B}(B_s \to \mu^+\mu^-)$ in mSUGRA scenarios. If the reported excess in a_μ [2] is caused by mSUGRA corrections with large $\tan\beta$, one faces more than an order of magnitude enhancement of $\mathcal{B}(B_s \to \mu^+\mu^-)$ over its SM value. This is within reach of Run-II of the Tevatron. The combined measurements significantly constrain the mSUGRA parameters, allowing a determination of $\tan\beta$ and $M_{1/2}$. A measurement of $\mathcal{B}(B_s \to \mu^+\mu^-)$ will further constrain the mass of the lightest Higgs bosons. SO(10) SUSY explanations of the measured a_μ are barely compatible with the present upper bound on $\mathcal{B}(B_s \to \mu^+\mu^-)$.

We thank B. Dutta, G. Isidori, K. Mizukoshi and X. Tata for cross-checking our numerical results. A.D. acknowledges financial support from the Network RTN European Program HPRN-CT-2000-00148.

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