Correlation of $B_s \to \mu^+\mu^-$ and $(g-2)_\mu$ in Minimal Supergravity

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We analyse the rare decay mode $B_s \to \mu^+\mu^-$ in the minimal supergravity scenario (mSUGRA). We find a strong correlation with the muon anomalous magnetic moment $(g-2)_\mu$. An interpretation of the recently measured excess in $(g-2)_\mu$ in terms of mSUGRA corrections implies a substantial supersymmetric enhancement of the branching ratio $\mathcal{B}(B_s \to \mu^+\mu^-)$: if $(g-2)_\mu$ exceeds the Standard Model prediction by $3 \cdot 10^{-9}$, $\mathcal{B}(B_s \to \mu^+\mu^-)$ is larger by a factor of **XXX** — **YYY** than in the Standard Model and well within reach of Run-II of the Tevatron. **a remark on Higgs masses or SO(10)?**

Supersymmetry (SUSY) is an attractive and widely studied extension of the Standard Model (SM). The minimal supergravity model (mSUGRA) [1] relates all supersymmetric parameters to just 5 real quantities: the universal scalar and gaugino masses M_0 and $M_{1/2}$, the trilinear term A_0, the ratio $\tan\beta$ of the two Higgs vacuum expectation values, and $\mathrm{sgn}\,\mu$, where μ is the Higgsino mass parameter. These quantities are defined at a high, grand unified energy scale and determine the physical parameters, which are then evolved to low energies by the renormalisation group equations. Precision observables, which are affected by SUSY corrections through loop effects, play an important role in constraining the supersymmetric parameter space. The small number of parameters makes mSUGRA sufficiently predictive to be significantly tested by low energy precision measurements.

Recently the Brookhaven National Laboratory (BNL) reported an excess of the muon anomalous magnetic moment $a_\mu = (g-2)_\mu/2$ over its SM value [2]. The difference $\delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (43 \pm 16) \cdot 10^{-10}$ corresponds to a 2.6σ deviation from the SM. An mSUGRA interpretation of this anomaly implies $\mu > 0$ (in the sign convention with $M_{1/2} > 0$ and equal signs of the diagonal elements of the chargino mass matrix) [3–15]. It further invites a large $\tan\beta \gtrsim 10$. The discrepancy in the case of a_μ is by itself not significant enough to justify the claim of new physics. Especially since the calculation of a_μ^{SM} involves two hadronic quantities: the hadronic contributions to the photon self-energy, which must be obtained from other experiments, and the (smaller) light-by-light scattering contribution, which can only be estimated with hadronic models. A more conservative estimate of the latter would reduce the BNL anomaly to a 2σ effect [2]. Hence in order to resolve the possible ambiguity between mSUGRA and alternative explanations of δa_μ one ideally wishes to study other observables whose sensitivity to supersymmetric loop corrections is correlated with δa_μ.

SUSY modifies B meson observables if $\tan\beta$ is large, because the b Yukawa coupling becomes sizable. Especially sensitive are quantities which involve a chirality flip in the b quark line like the branching ratio $\mathcal{B}(B \to X_s\gamma)$ and $\mathcal{B}(B \to \ell^+\ell^-)$. In mSUGRA the low energy value for the trilinear term A_t is dominated by $M_{1/2}$ with $A_t < 0$ for $M_{1/2} > 0$ [16]. Then $\mu > 0$ implies that the charged-Higgs-top loop and the chargino-stop loop tend to cancel in $\mathcal{B}(B \to X_s\gamma)$, so that the sensitivity to mSUGRA corrections is weakened. A further disadvantage of this decay mode is that experimentally it requires a cut on the photon energy, which introduces some hadronic uncertainty.

In [16] the possible impact of *flavour-blind* SUSY on other B physics observables, in particular those which enter the fit of the unitarity triangle, were studied and only small effects were found. By contrast, the branching ratio $\mathcal{B}(B \to \ell^+\ell^-)$ grows like $\tan^6\beta$ [17–19], with a possible several orders of magnitude enhancement. Since $\mathcal{B}(B \to \ell^+\ell^-) \propto m_\ell^2$, the branching ratio is largest for $\ell = \tau$. Yet τ's are hard to detect, so that the prime experimental focus at hadron colliders is on the search for $B_s \to \mu^+\mu^-$. B factories running on the $\Upsilon(4S)$ resonance produce no B_s mesons. Leptonic branching ratios of B_d mesons are smaller by a factor of $|V_{td}/V_{ts}|^2$. Since in B factories the boost of the B_d meson is known and the considered leptonic decay rates are substantially enhanced over their SM values in SUSY, we encourage our colleagues at BaBar and BELLE to look for $B_d \to \tau^+\tau^-$ decays as well. From now on we restrict ourselves to the decay mode $B_s \to \mu^+\mu^-$.

In [17–19] the SUSY corrections to $\mathcal{B}(B_s \to \mu^+\mu^-)$ were calculated at the one-loop level. For large $\tan\beta$, however, higher order corrections can be large, eventually of order 1. In [20] $\tan\beta$-enhanced supersymmetric QCD corrections have been summed to all orders in perturbation theory. These dominant higher-order terms can be easily incorporated by replacing the b Yukawa coupling $h_b \propto m_b \tan\beta$ with $h_b^{\mathrm{eff}} = h_b/(1+\Delta m_b)$, where $\Delta m_b \propto \mu \tan\beta$ depends on the gluino and sbottom masses and can be found in [20]. Δm_b is positive for $\mu > 0$. The dominant contribution to $\mathcal{B}(B_s \to \mu^+\mu^-)$ is proportional to $h_b^{\mathrm{eff}\,4}$, so that the inclusion of Δm_b tempers the large-$\tan\beta$ behaviour.

The considered branching ratio can be expressed as

$$\mathcal{B}(B_s \to \mu^+\mu^-) =$$
$$6.0 \cdot 10^{-7} \left(\frac{|V_{ts}|}{0.040}\right)^2 \left(\frac{f_{B_s}}{230\,\mathrm{MeV}}\right)^2 \frac{m_\mu^2}{m_{B_s}^2}\sqrt{1-\frac{4m_\mu^2}{m_{B_s}^2}}$$
$$\left\{\left(1-\frac{4m_\mu^2}{M_{B_s}^2}\right)\left|\frac{m_{B_s}^2 C_S}{m_\mu}\right|^2 + \left|\frac{m_{B_s}^2 C_P}{m_\mu} - 2C_A\right|^2\right\}. \quad (1)$$

Here $|V_{ts}| = 0.040 \pm 0.002$ is the relevant element of the Cabibbo-Kobayashi-Maskawa matrix and $f_{B_s} = (230 \pm 30)$ MeV [21] is the decay constant of the B_s meson. In (1) we have kept the dependence on the lepton mass m_μ, so that the generalisation to $B_d \to \tau^+\tau^-$ is straightforward. The Wilson coefficients C_S, C_P and C_A, which contain the short-distance physics, are normalised as in [22]. The coefficients c_S, c_P and c_{10} defined in [19] are related to ours by $C_S = -2c_S \sin^2\theta_W$, $C_A = -2c_{10}\sin^2\theta_W$ and $C_P = 2c_P \sin^2\theta_W$. Within the SM, C_S and C_P are negligibly small and the next-to-leading order result for C_A can be well approximated by $C_A = 1.99(\overline{m}_t/166.6\ \mathrm{GeV})^{1.55}$ [23]. Here $\overline{m}_t \equiv \overline{m}_t(m_t)$ is the top quark mass in the $\overline{\mathrm{MS}}$ scheme. $\overline{m}_t = 166.6$ GeV corresponds to a pole mass of $m_t = 175$ GeV. The SM prediction is then given by $\mathcal{B}(B_s \to \mu^+\mu^-) = (3.7 \pm 1.2) \cdot 10^{-9}$, with the uncertainty dominated f_{B_s}.

Within mSUGRA the dominant coefficients are $C_{S,P}$ since they are proportional to $\tan^3\beta$. An approximate formula, which is good to within 50% for $\tan\beta > 45$ and $M_{1/2} < 600$ GeV is given by $|C_{S,P}| \approx (m_\mu F \tan^3\beta)/(4M_W M_{1/2})$. F is a loop factor which can be approximated by 1. Here we have made use of the fact that within mSUGRA the low-energy values of both μ and the squark masses are dominated by $M_{1/2}$ through the renormalization group equations. For large values of $\tan\beta$ the supersymmetric contribution will dominate in $\mathcal{B}(B_s \to \mu^+\mu^-)$, and we expect it to scale as $|C_{S,P}|^2 \propto \tan^6\beta/M_{1/2}^2$. A similar estimate of the supersymmetric contribution to a_μ yields $\delta a_\mu \propto \tan\beta/M_{1/2}^2$. Thus within mSUGRA we expect a strong correlation between $\mathcal{B}(B_s \to \mu^+\mu^-)$ and δa_μ. Furthermore, if an experimental discrepancy in a_μ requires a supersymmetric explanantion with large $\tan\beta$ then we would expect $\mathcal{B}(B_s \to \mu^+\mu^-)$ to be strongly enhanced. If however the supersymmetric explanation of δa_μ requires small $M_{1/2}$ and moderate to small values of $\tan\beta$ then we would expect the relative enhancement of $\mathcal{B}(B_s \to \mu^+\mu^-)$ to be much smaller.

During Run-I of the Tevatron CDF found the upper bound [24]

$$\mathcal{B}(B_s \to \mu^+\mu^-) < 2.6 \times 10^{-6}, \qquad \text{at } 95\%\ \text{C.L.} \quad (2)$$

The single event sensitivity of CDF at Run-IIa is estimated to be $1.0 \cdot 10^{-8}$ for an integrated luminosity of 2 fb^{-1} [25]. Thus if mSUGRA corrections enhance $\mathcal{B}(B_s \to \mu^+\mu^-)$ to e.g. $5 \cdot 10^{-7}$, one will see 50 events in Run-IIa. Run-IIb may collect 10-20 fb^{-1} of integrated luminosity, which will imply 250-500 events in this example.

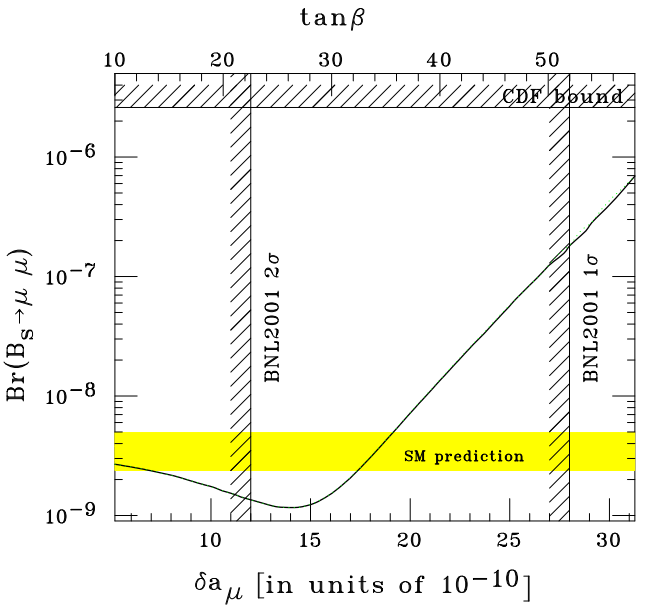

FIG. 1. The SUSY muon anomalous magnetic δa_μ (bottom), versus the predicted branching ratio $B_s \to \mu^+\mu^-$ (left) for the input values of $\tan\beta$ (top) in an indicative mSUGRA point [$M_{1/2} = 450, M_0 = 350, A_0 = 0, \mu > 0, m_t = 175$ GeV]. Shown also, the SM prediction, the present bound by CDF [24], on $\mathcal{B}(B_s \to \mu^+\mu^-)$ aswell as the present 1σ and 2σ bound on δa_μ from BNL [2]. f_{B_s} has been chosen to be 230 MeV.

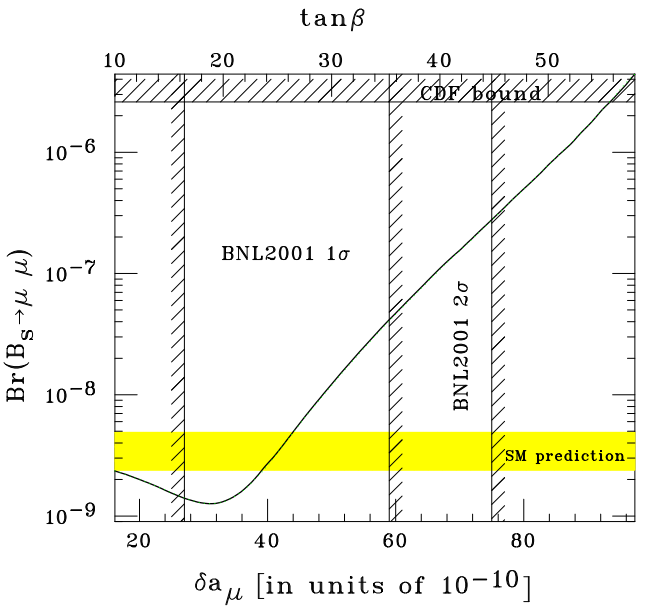

FIG. 2. Same as in Fig.1 but with $M_{1/2} = 250, M_0 = 200$ GeV.

In Fig.1 we plot the mSUGRA contribution to the anomalous magnetic moment of the muon against

$\mathcal{B}(B_s \to \mu^+\mu^-)$ for $10 < \tan\beta < 58$ * shown on the top of the figure. We have chosen one specific point[$M_{1/2} = 450$ GeV, $M_0 = 350$ GeV, $A_0 = 0, \mu > 0$ and $m_t = 175$ GeV]† of the mSUGRA parameter space. The SM prediction and the CDF bound from (2) for $\mathcal{B}(B_s \to \mu^+\mu^-)$ are also shown in the figure. The solid (dashed) curve represents the $\mathcal{B}(B_s \to \mu^+\mu^-)$ result with (without) resummation of the $\tan\beta$-enhanced SUSY-QCD corrections. In this example the resummation suppresses $\mathcal{B}(B_s \to \mu^+\mu^-)$ by 50% for $\tan\beta = 50$. Hence in order for mSUGRA to account for δa_μ within 1σ of the current BNL measurement [2] we need a large value of $\tan\beta \gtrsim 50$. Due to the strong correlation we have shown within mSUGRA we then predict $\mathcal{B}(B_s \to \mu^+\mu^-) \gtrsim 10^{-7}$, which should be observed by CDF at Run IIa.

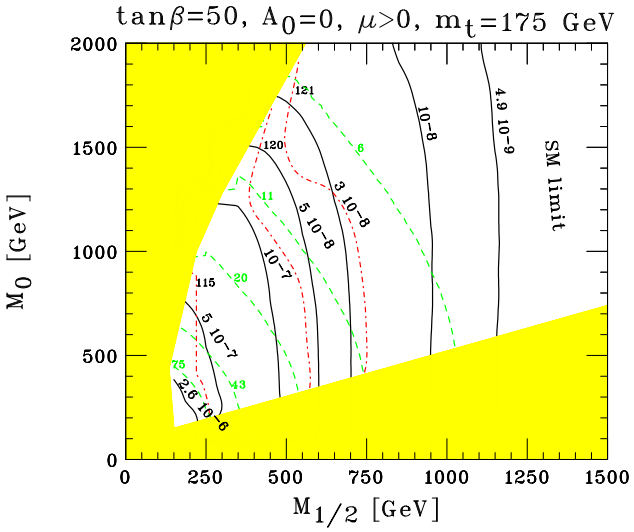

FIG. 3. Contour plots of the $\mathcal{B}(B_s \to \mu^+\mu^-)$ (solid) on the plane $M_0, M_{1/2}$ for $\tan\beta = 50, A_0 = 0, \mu > 0, m_t = 175$ GeV. The fully shaded region is excluded from both theoretical [REWSB(left-upper corner),not neutralino LSP(down)] and experimental [susy and higgs searches (mainly left strip till 1 TeV M_0 approximately)] facts. Contours of the light Higgs boson mass (red dot-dash line) and of the δa_μ [in units 10^{-10}] (dash) are also depicted.

Another interesting feature in Fig.1 is the dip in the curve of $\mathcal{B}(B_s \to \mu^+\mu^-)$ for moderate values of $20 \lesssim \tan\beta \lesssim 30$. This is the result of a destructive interference of the pure higgs boson contributions (2HDM contributions) and the SUSY box sneutrino-squark diagrams (which are similar in this case) on the one side with the SM contribution. This effect has been described in detail within the context of the 2HDM in [22]. The uncertainty in Fig.1 is mainly given by f_{B_s}. Changing it in the range 200-260 MeV changes $\mathcal{B}(B_s \to \mu^+\mu^-)$ by roughly $\mp 25\%$ with respect to the values shown in Fig.1.

In Fig.3 we show the dependence of $\mathcal{B}(B_s \to \mu^+\mu^-)$ on the mSUGRA parameters $M_{1/2}$ and M_0 for fixed $\tan\beta = 50$, $A_0 = 0$ GeV, and $\mu > 0$. The solid contours represent $\mathcal{B}(B_s \to \mu^+\mu^-)$ with resummed SUSY-QCD corrections. The lower left-hand corner in the figure is excluded by the CDF limit given in Eq.(2) and indicated in the figure. If as expected, CDF can probe down to $\mathcal{B}(B_s \to \mu^+\mu^-) \gtrsim 2 \cdot 10^{-7}$ at RUN IIa, we see from Fig.3 that this corresponds to a sensitivity of $M_{1/2} \lesssim 500\,\text{GeV}$ and $M_0 \lesssim 1.2$ TeV, respectively. The shaded-dotted region represents the 2σ range ($11 < \delta a_\mu < 75$) of the BNL experiment. It nicely reveals the correlation with $\mathcal{B}(B_s \to \mu^+\mu^-)$: if the BNL anomaly persists and is due to mSUGRA corrections, CDF will certainly confirm this evidence through the observation of $B_s \to \mu^+\mu^-$.

Hence while CDF is not able to see squark masses directly up to 1.5 TeV [corresponding to $M_{1/2} = 500$ GeV and $M_0 = 1.2$ TeV], it will nevertheless be able to prepare the ground for LHC by observing the $B_s \to \mu^+\mu^-$mode. Even better, after 10fb^{-1} CDF will probe $M_{1/2} = 750$ GeV and 1.8 TeV (for $\tan\beta = 50$) which in mSUGRA corresponds to masses for the heaviest superpartners of 2.4 TeV. We conclude the discussion of Fig.3 with the light Higgs boson prediction (dot-dashed line) for $\tan\beta = 50$ in the mSUGRA scenario. We find that Higgs masses less than 119 GeV are correlated with an enhancement of $\mathcal{B}(B_s \to \mu^+\mu^-)$ over the SM value by two orders of magnitude. The recent "hint" by LEP [29] of a mass around 115.6 GeV results in $\mathcal{B}(B_s \to \mu^+\mu^-) \gtrsim 5 \times 10^{-7}$ which would be measured before the Higgs boson is discovered.

Other contraints here : dark matter b to s gamma e.t.c I think the paper by Roszckowski et.al [14] is enough here Some words also on the recent paper [30]. Herbi could you please write this here??

The large values of $\tan\beta$ we have been considering are theoretically very well motivated within supersymmetric SO(10) Yukawa unification. There it has recently been found [31,32] that a narrow parameter region can explain the observed δa_μ while still being consistent with the constraint from $b \to s\gamma$. This is *not* within the context of mSUGRA. However, in this parameter region both μ and $M_{1/2}$ are light, while the CP-odd Higgs boson mass is less than 300 GeV, and $\tan\beta \approx 50$. Therefore the $B_s \to \mu^+\mu^-$ decay is strongly enhanced and the bound from CDF in Eq.(2) further restricts the allowed parameter space. As an example we determine $\mathcal{B}(B_s \to \mu^+\mu^-)$ for the best fitted points found in [32]: $M_A = 110$ GeV, $m_{\tilde{\chi}_1} \lesssim 250$ GeV, $A_t \lesssim 5$ TeV, $m_{\tilde{t}} \lesssim 1$ TeV and $\tan\beta \simeq 50$. We find that within the hadronic uncertainties $\mathcal{B}(B_s \to \mu^+\mu^-) \gtrsim 10^{-5}$ which is already excluded by CDF [24]. Thus the SO(10) models should

*The upper limit comes from the REWSB requirement.
†This point coincides with the point L quoted in [28].

be reconsidered in the light of the experimental bound on $\mathcal{B}(B_s \to \mu^+\mu^-)$. The only possible escape we can see for these models is to have $M_A \gtrsim 200$ GeV, or smaller values of $\tan\beta < 45$. That in turn means that one has to relax the Yukawa unification assumption by relying on large GUT threshold corrections. Or, turning it around, if an SO(10) GUT model is the correct description of nature then the decay $B_s \to \mu^+\mu^-$must be just around the corner.

In conclusion we have found a striking correlation between the muon anomalous magnetic moment a_μ and the decay $B_s \to \mu^+\mu^-$ in mSUGRA scenarios. If the reported excess of $\mathcal{O}(4 \cdot 10^{-9})$ in the measured value of a_μ [2] is caused by mSUGRA corrections, one faces a huge, order-of-magnitude enhancement of $\mathcal{B}(B_s \to \mu^+\mu^-)$ over its SM value. The enhanced branching ratio is within reach of Run-II of the Tevatron. A measurement of $\mathcal{B}(B_s \to \mu^+\mu^-)$ will further constrain the mass of the lightest Higgs bosons. SO(10) SUSY explanations of the measured a_μ are hardly compatible with the present upper bound on $\mathcal{B}(B_s \to \mu^+\mu^-)$.

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