Unifying the Strengths of Forces in Higher Dimensions

A. Dedes[1] and P. Kanti[2]

[1] *Physikalisches Institut der Universität Bonn, Nußallee 12, D-53115 Bonn, Germany*
[2] *Scuola Normale Superiore and INFN, Piazza dei Cavalieri 7, I-56126 Pisa, Italy*

Abstract

We consider the embedding of the Standard Model fields in a $(4+d)$-dimensional theory while gravitons may propagate in d' extra, compact dimensions. We study the modification of strengths of the gravitational and gauge interactions and, for various values of d and d', we determine the energy scale at which these strengths are unified. We demonstrate that our "unification" constraint can be reduced to the Type I string theory or M-theory constraint and, therefore, the scale m_U where the unification of the strengths of forces takes place should be interpreted as the true unification scale in the framework of a more fundamental theory. We show that our analysis succeeds in reproducing all known results coming from string theory and in providing a simple framework for the study of alternative models. PACS : 11.10.Kk, 04.50.+h

It is widely believed that any fundamental theory capable of describing our world at higher energy scales always predicts the existence of extra, spatial, compact dimensions. Being motivated by attempts to lower the string scale at the gauge unification scale [1] or the TeV scale [2], the concept of large [3] or small [4] extra dimensions, that are being felt only by gravitons, has been used in order to attack the hierarchy problem. On the other hand, the existence of extra dimensions, where gauge bosons can propagate, was proposed in an attempt to explain the size of the supersymmetry breaking scale [5] or to lower the unification scale of the gauge interactions [6]. In both cases, the Kaluza-Klein excitations of either gravitons or Standard Model particles appear in the framework of the 4-dimensional, effective theory and, in principle, may modify the low-energy physics. The accuracy with which the electroweak and strong interactions have been probed at low energies demands that any contributions to SM processes coming from the KK excitations be extremely small which, in turn, places a lower bound on the energy scale associated with the extra dimensions.

In this letter, we assume the existence of two different sets of extra dimensions: one being felt only by gravitons and one where gauge bosons can also propagate. The gravitational sector of the theory is described by a higher-dimensional version of Einstein's gravity while a higher-dimensional analog of the Standard Model describes the self- and cross-interactions of bosons, fermions and Higgs-like particles. Both theories should be considered as effective theories valid from the decompactification scales of the two sets of extra dimensions up to a higher scale where a more fundamental theory (string or M-theory) takes over. The problem of unification of forces, in a higher-dimensional framework, has been studied before from the string theory point of view [6]. Here, we address the same problem by using an alternative point of view, that of the higher-dimensional effective theories for the gravitons and Standard Model particles. It is well known that the presence of the extra dimensions modifies both the strength of the gravitational forces [3] and Standard Model couplings [7]. We take this idea one step further and we calculate the strength of the electromagnetic (gauge) interactions in the presence of extra dimensions. We search for specific combinations of the numbers and sizes of the extra dimensions, that lead to the unification of strengths of forces, in the framework of the higher-dimensional theory, and determine the corresponding pattern of decompactification and unification scales. We demonstrate that the apparently *ad hoc* demand of the unification of strengths of forces, in the higher-dimensional framework, is justified by the fact that our unification constraint straightforwadly reduces to either the Type I string constraint [6] or the M-theory constraint [1], and, therefore, our results are relevant to the true unification of forces in the context of a more fundamental theory. We conclude our paper by demonstrating that results presented previously in the literature can be easily recovered in the context of our analysis and that alternative models with new patterns of energy scales can be easily determined.

We start our analysis by considering a higher-dimensional formulation of an $SU(N)$ Yang-Mills theory with matter. We assume that the vector and scalar particles, i.e. the gauge bosons, $\hat{A}^{\alpha}_M$, and the Higgs field, $\hat{\phi}$, may propagate in $4+d$ dimensions, while all the fermionic particles, Ψ, are localized on the 4-dimensional boundary*. The $(4+d)$-dimensional action functional, that describes the above theory and preserves Lorentz invariance, may be written

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*Our results apply also in the case where the fermions live in the bulk.

as [7]

$$S_{4+d} = \int d^4x\, d^dz \left\{ -\frac{1}{4} \left(\partial_M \hat{A}^\alpha_N - \partial_N \hat{A}^\alpha_M + \frac{\hat{g}}{\sqrt{\Lambda^d}} C_{\alpha\beta\gamma} \hat{A}^\beta_M \hat{A}^\gamma_N \right)^2 - \bar{\Psi}_{L,R} \gamma^\mu \left(\partial_\mu - \frac{i\,\hat{g}}{\sqrt{\Lambda^d}} t^\alpha \hat{A}^\alpha_\mu \right) \Psi_{L,R}\, \delta(\vec{z}) \right.$$
$$\left. - \left| \left(\partial_M - \frac{i\,\hat{g}}{\sqrt{\Lambda^d}} t^\alpha \hat{A}^\alpha_M \right) \hat{\phi} \right|^2 - \hat{\mu}^2 \hat{\phi}^* \hat{\phi} - \frac{\hat{\lambda}}{2\Lambda^d} (\hat{\phi}^* \hat{\phi})^2 - \left(\frac{\hat{Y}_1}{\sqrt{\Lambda^d}} \bar{\Psi}_L\, \hat{\phi}\, \Psi_R + \frac{\hat{Y}_2}{\sqrt{\Lambda^d}} \bar{\Psi}_L\, \hat{\phi}_c\, \Psi_R + h.c. \right) \delta(\vec{z}) \right\}, \qquad (1)$$

where $\hat{g}$, $C_{\alpha\beta\gamma}$ and t^a are the coupling constant, the structure constants and the generators, respectively, of the $SU(N)$ gauge group while $\hat{\mu}^2$ and $\hat{\lambda}$ are the mass and coupling constant of the Higgs field. In order to render the coupling constants of the theory dimensionless in $(4+d)$-dimensions, an arbitrary energy scale Λ has been introduced. Note that, in the above, $M, N = \{t, x_1, x_2, x_3, z_1, z_2, ..., z_d\}$, $\mu, \nu = \{t, x_1, x_2, x_3\}$ and the hat denotes $(4+d)$-dimensional quantities.

Next, we assume that the extra d dimensions are compactified over an internal manifold with the size of every compact dimension being $2L$. Then, we can Fourier expand the $(4+d)$-dimensional vector and scalar fields along the compact dimensions in the following way

$$\hat{\Phi}(x,z) = \hat{\Phi}^{(0)}(x) + \sum_{\vec{n}=1}^{\infty} \frac{\hat{\Phi}^{(\vec{n})}(x)}{\sqrt{2}} \left(e^{i\frac{\pi\vec{n}}{L}\vec{z}} + e^{-i\frac{\pi\vec{n}}{L}\vec{z}} \right), \qquad (2)$$

where $\vec{n} = \{n_1, n_2, ..., n_d\}$. By performing a Kaluza-Klein compactification, i.e by using the above field expansion and integrating over the extra dimensions, the action (1) reduces to an effective 4-dimensional theory. A prominent feature of this effective theory is its complexity due to extra terms involving the massive Kaluza-Klein (KK) excitations, $\hat{\Phi}^{(\vec{n})}(x)$, of all the fields propagating in the extra dimensions, apart from the usual, massless zero-modes, $\hat{\Phi}^{(0)}(x)$. Here, we are only interested in the part of the effective theory that contains the zero modes of the various fields and, more specifically, in the relations that hold between the $(4+d)$ and 4-dimensional couplings and masses. These are found to be

$$g = \frac{\hat{g}}{(2L\Lambda)^{d/2}}, \qquad Y_{1,2} = \frac{\hat{Y}_{1,2}}{(2L\Lambda)^{d/2}}, \qquad (3)$$

$$\mu^2 = \hat{\mu}^2, \qquad \lambda = \frac{\hat{\lambda}}{(2L\Lambda)^d}, \qquad (4)$$

where we have used the following field redefinitions

$$A^a_\mu = (2L)^{d/2} \hat{A}^a_\mu, \qquad \phi = (2L)^{d/2} \hat{\phi} \qquad (5)$$

in order to obtain canonical kinetic terms in 4-dimensions. Note that the fermionic fields being always localized on the 4-dimensional boundary remain unchanged.

In the framework of the 4-dimensional effective theory, the gauge bosons and fermions acquire mass through the spontaneous symmetry breaking of the gauge group. Their masses are given in terms of the vacuum expectation value of the Higgs field which is also redefined according to Eq. (5). The question, then, arises whether the mass spectrum of the 4-dimensional effective theory corresponds to a different one in the framework of the original (4+d)-dimensional theory, a point which is going to be of significant importance in our subsequent analysis. To answer this question, we consider the masses of the fermions which are given by the following expression

$$m_{\Psi_{L,R}} = Y_{1,2} \frac{v}{\sqrt{2}} = \frac{\hat{Y}_{1,2}}{\Lambda^{d/2}} \frac{\hat{v}}{\sqrt{2}} = \hat{m}_{\Psi_{L,R}}. \qquad (6)$$

Similar relations can also be written for the masses of the gauge bosons and the Higgs field. In all cases, the tree-level masses, that are generated via the Higgs mechanism in the 4-dimensional theory, remain unaltered when one embeds this theory in a higher-dimensional one.

As it is well known [7], this is not the case with the coupling constant $\hat{g}$ that eventually determines the strength of the gauge interactions of the theory. The $SU(N)$ gauge group of the $(4+d)$-dimensional theory (1) could be replaced by the $U(1) \times SU(2) \times SU(3)$ group with the gauge field $\hat{A}_M^\alpha$, the generators t^α and the coupling constant $\hat{g}$ standing for each one of the corresponding quantities of the Standard Model group, i.e. $(\hat{B}_M, \hat{W}_M^a, \hat{G}_M^\alpha)$, $(Y/2, \tau^a/2, \lambda^\alpha/2)$ and $(\hat{g}_1, \hat{g}_2, \hat{g}_3)$, respectively. In that case, each coupling constant and gauge boson of the SM group is redefined according to Eqs. (3) and (5), respectively.

By making use of the redefinition (3) for the gauge coupling constants, we find that the electric charge e changes as follows

$$\hat{e} = \hat{g}_1 \cos\hat{\theta}_W = (2L\Lambda)^{d/2} g_1 \cos\theta_W = (2L\Lambda)^{d/2}\, e\,, \tag{7}$$

where we have used the fact that the Weinberg angle θ_W remains unaltered since its tangent is given by the ratio of the gauge couplings g_1 and g_2. The above rescaling of the electric charge inevitably affects the strength of the electromagnetic interactions in higher dimensions. The strength of a force is classically defined [8] as the potential energy of the corresponding interaction between two identical particles, separated by a distance equal to the particle's Compton wavelength, $\lambda\!\!\!^{-} = \hbar/mc$, compared to the energy of the rest mass of each particle. This is indeed the only way of providing a dimensionless measure of the strength of a force that depends on the distance between the interacting particles. As is well known, in 4-dimensions and for two particles with mass m and charge e, the above rule gives the result

$$\alpha_{EM} = \frac{E_{int}}{E_m} = \frac{1}{mc^2}\,\frac{e^2}{4\pi\epsilon_0\,(\hbar/mc)} = \frac{e^2}{4\pi\epsilon_0} \simeq \frac{1}{137}\,, \tag{8}$$

where we have set $c = \hbar = 1$. Generalizing the above definition of the strength of a force in $(4+d)$ dimensions, in the limit $\lambda\!\!\!^{-} < 2L$, i.e. for energies larger than the decompactification scale, we obtain

$$\hat{\alpha}_{EM} = \frac{\hat{E}_{int}}{\hat{E}_m} = \frac{1}{\hat{m}}\,\frac{(\hat{e}^2/\Lambda^d)}{4\pi\epsilon_0\,(1/\hat{m})^{1+d}} = \alpha_{EM}\left(\frac{m}{M_x}\right)^d, \tag{9}$$

where we have used the fact that the masses of the particles do not change, according to Eq. (6), and where we have defined $M_x \equiv (2L)^{-1}$. It is straightforward to check that, in the limit, $\lambda\!\!\!^{-} > 2L$, the above formula reduces to Eq. (8) as expected. The above result clearly reveals that the strength of the electromagnetic force changes as the gauge bosons start "feeling" the extra dimensions. Moreover, the strength of the force strongly depends not only on the number and size of extra dimensions but also on the mass of the test particle, i.e. on the energy scale where the measurement takes place †. It is also worth noting that the auxiliary energy scale Λ does not appear in the electromagnetic strength formula (9).

So far, we have not considered the gravitational interactions. We now assume that gravitons may propagate, apart from the usual 4 dimensions, in $d' = \delta + d$ extra dimensions, where δ is the number of transverse dimensions, with size $2L'$, felt only by gravitons. In that case, the $(4+d')$-dimensional action functional

$$S_{4+d'} = -\frac{M_{GR}^{2+d'}}{16\pi}\int d^4x\, d^{d'}z\,\sqrt{G_{4+d'}}\,R_{4+d'} \tag{10}$$

reduces to an effective, 4-dimensional Einstein's theory of gravity only when the following relation between the sizes of the extra dimensions and the energy scales of gravity, M_P and M_{GR}, in 4 and $(4+d')$ dimensions, respectively,

$$(2L')^\delta\,(2L)^d\,M_{GR}^{2+d'} = M_P^2 \tag{11}$$

holds. Here, we restrict ourselves in the case of product spacetimes where the 4-dimensional subspace does not depend on the geometry of the internal, compact manifolds. In this case, the strength of the gravitational interactions also changes when one introduces extra dimensions for the gravitons [3]. Using the same rule as above, the strength of the gravitational interaction between two particles with mass m, in $4+d'$ dimensions, is given by the expression

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†A similar dependence of the strength of electromagnetic forces on the mass of the test particle arises also in lower-dimensional problems, i.e. in the case where the total number of dimensions is lower than four. We thank M. Shifman for pointing this out.

$$\hat{\alpha}_{GR} = \frac{\hat{E}_{int}}{\hat{E}_m} = \frac{1}{\hat{m}} \frac{\hat{G}_N \, \hat{m}^2}{(1/\hat{m})^{1+d'}} = \left(\frac{m}{M_{GR}}\right)^{2+d'} \quad , \tag{12}$$

where $\hat{G}_N = 1/M_{GR}^{2+d'}$. By choosing appropriately the sizes of the transverse and longitudinal, extra dimensions, the higher-dimensional gravity scale, M_{GR}, could be much lower than the 4-dimensional one, according to Eq. (11), a fact which, subsequently, changes the strength of gravity.

The question we would like to address is the following: can these two effective theories, described by Eqs. (1) and (10), provide useful information on the problem of unification of forces? Note that, here, we are interested in bridging the gap between the gravitational and electromagnetic forces. The effective theory (1) being non-renormalizable can not address the problem of the unification of gauge couplings. This problem can be studied independently, through the power-law mechanism [6,10], in the framework of the 4-dimensional, effective theory. Here, we ignore the small differences in the strengths of electromagnetic, weak and strong interactions and we focus on the study of the unification of gravitational and gauge interactions, the latter being represented by the electromagnetic interactions[‡]. Having derived the strengths of these forces, in the presence of extra dimensions, it seems natural to assume that there is a scale m_U where the strengths of gravitational and electromagnetic forces become equal. Combining Eqs. (9) and (12), we obtain the "unification" constraint

$$\hat{\alpha}_{EM} = \hat{\alpha}_{GR} \Rightarrow \alpha_{EM} \left(\frac{m_U}{M_x}\right)^d = \left(\frac{m_U}{M_{GR}}\right)^{2+d'} \quad . \tag{13}$$

A priori, there is no reason to believe that such a demand is indeed relevant to the true unification of forces in the framework of a more fundamental theory, like string or M-theory. One could argue that demanding unification of the relevant mass scales, instead of unification of couplings, would be more robust. Moreover, any combination of numbers and sizes of the two sets of extra dimensions, that would lead to the unification of gravitational and gauge forces, should be chosen in such a way that they would also lower the scale of the fundamental theory, in a consistent way, down to m_U [§]. We will now demonstrate that this is indeed the case with the numbers and sizes of the extra dimensions that follow from our "unification" constraint, Eq. (13). As we will see, the above relation reduces to the fundamental constraint of Type I string theory or M-theory, according to our assumptions, and that the scale of the unification of strengths of the forces m_U can be interpreted as the string scale M_s or the M-theory scale M_M, respectively. To this end, we rewrite Eq. (13), combined with Eq. (11), in the form

$$\left(\frac{m_U}{M_P}\right)^2 = \hat{\alpha}_{EM} \, (m_U \, 2L')^{-\delta} \, (m_U \, 2L)^{-d} \, , \tag{14}$$

which reduces to the Type I string constraint $M_s^2 \sim (\alpha_{gauge} M_P \sqrt{V})^{-1}$ [6] if we identify m_U with M_s, and $\hat{\alpha}_{EM}$ with α_{gauge}^2. The total compactified volume is now $V = (2L)^d (2L')^\delta$ and a T-duality transformation $m_U \, r \leftrightarrow (m_U \, r)^{-1}$ needs to be performed, where $r \equiv 2L, 2L'$. Alternatively, for $\delta = 1$ and $d = 0$, Eq. (13) reduces to the relation

$$\frac{1}{M_c} = \hat{\alpha}_{EM} \, \frac{M_P^2}{m_U^3} \tag{15}$$

which is nothing else than the M-theory constraint $R_{11} = \alpha_{gauge}^2 \, (l_M^3 / l_P^2)$ [1], that relates the size of the extra dimension for the gravity with the scale of the M-theory and the Planck length, if we identify m_U with M_M and R_{11} with M_c^{-1}. We may therefore conclude that the constraint (13) not only is the right constraint for the unification of forces, in the framework of the higher-dimensional effective theories (1) and (10), but also any combination of compactification radii is directly embedded into a finite framework such as string or M-theory.

We will now study the four distinct categories, that are formed according to the values[**] of the numbers δ and d of the extra dimensions, in order to demonstrate the simplicity with which results, presented previously in the literature,

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[‡]Under the assumption that the electromagnetic, weak and strong forces "feel" the same number d of extra dimensions, we may assume that their strengths remain comparable at every scale even in $(4+d)$-dimensions.

[§]We are grateful to Keith Dienes for a fruitful discussion on this point.

[**]Motivated by M-theory, we assume that $0 \leq \delta, d \leq 7$.

arise in the framework of our analysis, but also the fact that new patterns of energy scales, associated with the extra dimensions, can easily be determined. Hereafter, for simplicity, we will take $\alpha_{EM} \simeq 10^{-2}$.

(*i*) $\delta = d = 0$. In this case, $M_{GR} \equiv M_P$ and *only* at energies $m_U \simeq 10^{18}$ GeV, i.e. close to the Planck scale, the strengths of the forces become equal, as expected.

(*ii*) $\delta \neq 0$, $d = 0$. Here, we assume that only the gravitational fields can feel extra δ dimensions. Then, from Eq. (13, we may easily conclude that the unification of strengths takes place at a scale $m_U = (0.2 - 0.6)M_{GR}$, for $1 \leq \delta \leq 7$. This means that the strength of the gravitational force becomes comparable to that of the other forces only when the measurement takes place near the gravity scale, independently of where M_{GR} lies. The results associated with M-theory [1] can be recovered for $\delta = 1$ and $M_c \equiv (2L')^{-1} = 10^{12}$ GeV. Then, the unification takes place at $m_U \simeq M_{GUT} \simeq M_{GR} \sim 10^{16}$ GeV. The low-scale gravity model of Ref. [3], with $M_{GR} \sim 1$ TeV can be also reproduced by choosing appropriately the value of δ and the size of the transverse dimensions M_c. In the framework of our analysis, we may easily see that the new hierarchy problem, between the compactification scale M_c and the electroweak scale M_W, can be resolved by increasing the number δ of extra dimensions accompanied with a small increase in the value of the higher-dimensional gravity scale M_{GR}. For example, allowing the compactification scale, M_c, to be close to the electroweak one, i.e. $M_c = 10^{\pm 1} M_W$, the gravity scale lies in the range $M_{GR} = (10^5 - 10^7)$ GeV, for $\delta \geq 6$. In this case, there is no energy gap between M_c and M_W while the gravity scale is still close to the electroweak scale (the ratio M_{GR}/M_W, in this case, is the same as the one of the top quark mass over the down quark mass, $m_t/m_d \sim 10^4$).

(*iii*) $\delta = 0$, $d \neq 0$. In this case, it is the d extra, longitudinal dimensions that "open up" for the gauge and scalar fields, as well as for gravitons, at the scale M_x. Substituting Eq. (11) into Eq. (13) and solving for m_U, we find that the unification of strengths takes place only near the string scale, i.e. $m_U = M_s \simeq 10^{18}$ GeV, independently of the number d of extra dimensions. If we take the new gravity scale, M_{GR}, to be the string scale, then, from Eq. (11), we are led to the following relation between M_x and M_{GR}:

$$M_x = 10^{-2/d} M_{GR} = (0.01 - 0.5) M_{GR} \ , \tag{16}$$

for $1 \leq d \leq 7$. In this case, the compactification scale M_x is 1 or 2 orders of magnitude smaller than the gravity scale. A quite interesting result arises for $d = 1$. Then, Eq. (16) gives the result $M_x \simeq 10^{16}$ GeV, which coincides with the scale of the unification of gauge couplings, M_{GUT}, in the minimal supersymmetric standard model (according to the power-law mechanism for the unification of gauge couplings [6,10], the presence of the extra dimensions does not affect the unification scale M_{GUT} when $M_x \simeq M_{GUT}$). The compactification scale, M_x, being very close to M_s does not introduce any new energy scale in the theory and is sufficiently high to suppress the "dangerous" baryon number violating operators. From that point of view, the above arrangement of energy scales resembles the one that arises in the framework of string unification [9] where the compactification scale, M_x, is very close to the string scale, M_s. This arrangement is indeed necessary in order to prevent the gauge coupling α_{EM} from acquiring an unacceptably small value near the unification scale, a goal which is also accomplished in our case. Although the hierarchy between the electroweak scale, M_W, and the gravity scale, M_{GR}, still remains, the existence of an extra, longitudinal dimension that "opens up" just above M_{GUT} may provide a natural explanation for the energy gap between M_{GUT} and M_s.

(*iv*) $\delta \neq 0$, $d \neq 0$. This is the most general case where both gravitational and gauge fields feel a number of extra dimensions. According to Eq. (13), the unification of strengths, now, takes place at the energy scale

$$m_U^{2+\delta} = \alpha_{EM}\, M_P^2\, M_c^\delta \ . \tag{17}$$

We make the natural assumption that the ultimate unification of strengths occurs near the new gravity scale, i.e. $m_U = M_{GR}$. Then, the combination of Eqs. (11) and (17) leads to the same relation (16) between the compactification scale M_x and the gravity scale M_{GR}. On the other hand, the scale M_c, associated with the size of the transverse dimensions felt only by gravitons, can be found from the following expression:

$$M_c^\delta = \alpha_{EM}^{-1} \frac{M_{GR}^{2+\delta}}{M_P^2} \ . \tag{18}$$

Note that it is *only* the number of transverse dimensions δ that determines the compactification scale M_c, in terms of M_P and M_{GR}, exactly in the same way that it is *only* the number of longitudinal dimensions d that determines the corresponding expression of M_x. If we, now, choose $M_{GR} = M_s \simeq 10^{18}$ GeV and $d = 1$, the interesting result $M_x \simeq M_{GUT}$ of case (*iii*) arises once again. However, in order for this picture to be viable in the existence of extra, transverse dimensions for gravitons, the compactification scale M_c should lie very close to the string scale for every value of δ. Since $M_c, M_x \geq M_{GUT}$, the unification pattern of gauge couplings is not affected by the presence of the extra dimensions, either transverse or longitudinal, and the ultimate unification of all forces takes place exactly at the string scale. Moreover, the compactification scale M_c, being very close to M_s, does not introduce any new energy scale in the theory. The only other case where this problem is also resolved is when $M_c = 10^{\pm 1} M_W$, according to the analysis presented in case (*ii*). For $\delta = 6$ and $M_c = 10$ GeV, the gravity scale lies at the energy scale 2×10^5 GeV. Then, for $d = 1$, Eq. (16) shows that the unification of strengths takes place only if M_x is exactly two orders of magnitude smaller than M_{GR}, i.e. $M_x = 2$ TeV. Remarkably, this is in perfect agreement with the proposal of the existence of a single extra dimension for the gauge bosons with size at the TeV scale necessary for the explanation of the supersymmetry breaking scale [5]. The existence of an extra dimension of this size would lead to the unification of gauge couplings, through the power-law mechanism [6], at an energy scale which is, approximately, one order of magnitude larger than M_x [6,10]. The extra hypothesis, that gravitons may feel 6 extra, compact dimensions, brings the scale of gravity down to 200 TeV, thus, completing the picture of unification of forces and removing the hierarchy problem. It is worth noting that, by changing appropriately the values of δ and M_c, we may easily recover the results of Ref. [6], where $M_{GUT} = M_{GR} = M_s = 10$ TeV, and which were derived with a completely different method.

In conclusion, in this letter, we have demonstrated that the addition of extra, compact dimensions for gravitons and gauge bosons modifies the strengths of gravitational and gauge interactions. We have formulated a "unification" constraint under the assumption that the strengths of gravitational and gauge interactions become equal at a scale m_U. This scale is uniquely defined in terms of the numbers and sizes of the two sets of extra dimensions. We have demonstrated that the aforementioned "unification" constraint reduces either to the Type I string theory or the M-theory constraint and, therefore, m_U should be interpreted as the string theory scale M_s or the M-theory scale M_M, respectively. As a result, the scale of the unification of strengths of forces, in the framework of the higher-dimensional effective theory for gravitons and gauge bosons, signifies the true unification of forces in the framework of a more fundamental theory. We have shown that previously derived results can be easily recovered in the framework of our analysis and that new features of the unification and patterns of compactification radii can be easily derived. For example, we have found that, when gravitons propagate in $4 + \delta$ dimensions, the unification always takes place near the new gravity scale, M_{GR}, whereas, when gauge bosons and gravitons feel the same number d of extra dimensions, the unification occurs only near the string scale, M_s. In the latter case, the existence of only one extra dimension, that "opens up" just above M_{GUT}, can justify the energy gap between M_{GUT} and M_s. In the more interesting case where gravitons and gauge bosons feel a different number of extra dimensions, a low-scale unification pattern arises: 6 extra dimensions "open up" only for gravitons at $M_c = 10$ GeV while one extra dimension "opens up" for the gauge bosons, as well, at $M_x = 2$ TeV, a value that could explain the size of the supersymmetry breaking scale. The unification of all forces takes place at the new gravity scale of the theory, i.e. at $M_{GR} = 200$ TeV, thus rendering this 11-dimensional theory free of any hierarchy problem. It is worth noticing that our analysis can lead to many more alternative models, possibly phenomenologically viable, and only few of them have been presented above as illustrating examples.

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