The Fayet-Iliopoulos term in the MSSM

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Abstract

Since the Standard Model gauge group is a non-semisimple gauge group and Supersymmetry is broken, the Fayet-Iliopoulos (FI) term is always generated at low energies. We study its effects, using two loop renormalization group equations and taking into account threshold corrections from the Supersymmetric spectrum. Even if the FI-term set to zero at the Grand Unification scale, its logarithmic running produces a non-zero value at the electroweak scale. If non-zero at high energies it affects the spectrum dramatically and phenomenologicaly is restricted to be of order of the soft SUSY breaking masses squared. Finally, non-universality assumptions at the GUT scale enhance the magnitude of the FI-term at low energies which in turn leads to different phenomenology.

1 Introduction

For Abelian supersymmetric gauge theories there is a D-term, known as Fayet-Iliopoulos (FI) term [1],

$$\mathcal{L}_{FI} \;=\; 2\xi \int d^4\theta V(x,\theta,\bar{\theta}) \stackrel{WZ}{=} \xi D(x) \; . \tag{1}$$

ξ is a real constant and $D(x)$ is gauge invariant and $\int d^4x D(x)$ supersymmetric. The first term on the right hand side of (1) is invariant under a special form of gauge transformation, known as Wess-Zumino gauge [2]. The existence of the FI-term in Supersymmetric theories that include U(1) gauge group is potentialy dangerous since it can be generated in perturbation theory (terms in the integrand $\int d^4\theta$ are not protected by the non-renormalization theorem [3]), it can lead to quadratic divergences (the FI-term has dimension of mass squared) and it can break supersymmetry spontaneously. It had been shown [4] that such a term cannot be generated in a semisimble grand unified gauge group G, simply because one cannot construct a G-invariant supersymmetric generalization of the D-term at the unification scale, $M_{\rm GUT}$. Moreover, it was shown in [5] that a FI-term is generated perturbatively at 1-loop level and is proportional to Tr(Y), the trace of the $U(1)_Y$ chiral multiplets hypercharge. This is [usually] zero in any realistic model building construction since one, in principle, demands cancelation of U(1)-U(1)-U(1) and U(1)-graviton-graviton anomalies. Furthermore in [5], the vanishing of the FI-term was proved to all orders in perturbation theory.

Matters are different when supersymmetry is softly broken and when the gauge group is non-semisimple. This is the case of the Minimal Supersymmetric Standard Model (MSSM) with the Standard Model gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. In this case, the FI-term will suffer logarithmic divergencies. There are two ways to implement this effect in studing the low energy spectrum of the MSSM i) we can eliminate the D-terms at the GUT scale and absorb the effect of the FI-term in the renormalization group running of the soft SUSY breaking masses ii) to eliminate the D-terms at low energies and "run" the FI-term as an independent parameter from the GUT scale down to M_Z. Results in both cases should be the same (for a proof see [8]) and in almost all the studies in the literature the case (i) has been realized. In this letter, we will follow the second path : we will set the FI-parameter, as a free parameter at the GUT scale and study its effects at the EW one. **the motivation here could be : a) to have a feeling for the magnitude of the FI-term b) to parameterize the naturalness of the MSSM c) to what masses M_0 we refer at the GUT scale???** In this case, the FI-term, even if zero at tree level, arises in perturbation by higher order corrections and its β-function is proportional to (at 1-loop)

$$\beta_\xi \propto \frac{g_1}{16\pi^2}\ \mathrm{Tr}\Big[m_i^2\ Y_i\ \theta(Q^2 - m_i^2)\Big]\ , \tag{2}$$

where m_i are the soft supersymmetry breaking masses, Q is the renormalization scale and $\theta(Q^2 - m_i^2)$ is the step function which takes care of the thresholds m_i. In the case of universality $m_i^2 = M_0^2$ at the GUT scale, and $\xi(M_{\rm GUT}) = 0$, $\xi(M_Z)$ is induced at the electroweak scale due to the spliting of the soft SUSY breaking masses,

$$\xi(M_Z) \propto \frac{g_1}{16\pi^2}\ \sum_{i=1}^{N}\ m_i^2\ Y_i\ \ln(\frac{m_i}{M_{\rm GUT}})\ , \tag{3}$$

where m_i are the soft SUSY breaking masses evaluated at the EW scale. Depending on how much the spliting between the SUSY soft breaking masses and the magnitude of M_0 is, the FI-parameter, $\xi(M_Z)$, can obtain a value from (few GeV)^2 to (100 GeV)^2. In the case of non-universal boundary conditions at the GUT scale (with $\xi(M_{\rm GUT}) = 0$) and with $\mathrm{Tr}(m_i^2 Y_i) \neq 0$ the induced FI-term at the electroweak scale turns out to be proporional to

$$\xi(M_Z) \propto \frac{g_1}{16\pi^2}\ \sum_{i=1}^{N}\ M_i^2\ Y_i\ \ln(\frac{M_{\rm GUT}}{M_Z})\ , \tag{4}$$

with M_i being the soft SUSY breaking masses at the GUT scale. In this case the magnitude of ξ at the Electroweak scale can be of the order of soft SUSY mass squared.

Once the FI-term is generated at the electroweak scale, it contributes to the scalar potential as [6] (after the elimination of the D-terms)

$$\mathcal{V}(\phi;\xi)\ =\ \sum_n \Big|\frac{\partial \mathcal{W}(\phi)}{\partial \phi_n}\Big|^2 + \frac{1}{2}\sum_A \Big(\xi_A + g_A \phi_n^* (t_A)_{nm}\phi_m\Big)^2 + \sum_n m_n^2 |\phi_n|^2 + \ldots\ , \tag{5}$$

where ξ_A are real constants which gauge invariance requires to vanish except where t_A is a U(1) generator, *i.e.,* the FI-parameter $\xi_1 \equiv \xi$, $g_A = (g, g_2, g_3)$ and $\mathcal{W}$ is the superpotential. With '...' we denote other soft SUSY breaking terms, like trilinear couplings, bilinear mass terms or gaugino masses. It is easy to see, that the potential of Eq.(5) is reduced to

$$\mathcal{V}(\phi;\xi) \;=\; \frac{1}{2}\xi^2 + \sum_n \left|\frac{\partial \mathcal{W}(\phi)}{\partial \phi_n}\right|^2 + \frac{1}{2}\sum_A \Big(g_A \phi_n^* (t_A)_{nm}\phi_m\Big)^2 + \sum_n \Big(m_n^2 + \xi g \frac{Y_n}{2}\Big)|\phi_n|^2 + \ldots \; . \quad (6)$$

So the effect of the FI-term in the scalar potential is firstly to shift the vacuum energy (or in other words the cosmological constant) and secondly the SUSY soft breaking masses,

$$m_i^2 \to \bar{m}_i^2 = m_i^2 + \frac{Y}{2} g\xi \; . \quad (7)$$

We will concentrate on the later effect on the mass spectrum of the MSSM. In the MSSM, the soft breaking masses are shifted as

$$\begin{aligned}
&\bar{m}^2_{\tilde{Q}} = m^2_{\tilde{Q}} + \frac{1}{6}g\xi \;\;,\;\; \bar{m}^2_{\tilde{U}^c} = m^2_{\tilde{U}^c} - \frac{2}{3}g\xi \;\;,\;\; \bar{m}^2_{\tilde{E}^c} = m^2_{\tilde{E}^c} + g\xi \;\;, \\
&\bar{m}^2_{\tilde{D}^c} = m^2_{\tilde{D}^c} + \frac{1}{3}g\xi \;\;,\;\; \bar{m}^2_{\tilde{L}} = m^2_{\tilde{L}} - \frac{1}{2}g\xi \;\;, \\
&\bar{m}^2_{1} = m^2_{1} - \frac{1}{2}g\xi \;\;,\;\; \bar{m}^2_{2} = m^2_{2} + \frac{1}{2}g\xi \;, \qquad (8)
\end{aligned}$$

where $m^2_{1,2} \equiv m^2_{H_{1,2}} + \mu^2$, and $m_{H_{1,2}}$, μ the higgs soft SUSY breaking masses and the superpotential mass parameter, respectively. From (8) we see that the FI-term contribution is flavour blind and (depending on its magnitude and its sign) alters the SUSY and Higgs spectrum in one or another way. Since it is generic in the MSSM, its effect has to be included in every analyses of the spectrum. The minimization conditions of the tree level potential can be read now as

$$\begin{aligned}
\mu^2 &= \frac{m^2_{H_1} - m^2_{H_2}\tan^2\beta}{\tan^2\beta - 1} + \frac{g\xi}{2\cos 2\beta} - \frac{1}{2}M_Z^2 \qquad (9) \\
B &= -\frac{(m_1^2 + m_2^2)\sin 2\beta}{2\mu} \; . \qquad (10)
\end{aligned}$$

Only Eq.(9) is explicitly affected from the presence of the FI-term. In addition, the requirement of Radiative Electroweak Symmetry Breaking (REWSB) now reads

$$\left[m^2_{H_1} - \frac{g\xi}{2}\right]\left[m^2_{H_2} + \frac{g\xi}{2}\right] - |\mu B|^2 \leq 0 \; , \quad (11)$$

where μB is soft SUSY bilinear breaking term. The equality (11) is understood at some scale $Q = Q_b$ nearby the electroweak one. REWSB is not satisfied at tree level when the relations (9,10, 11) are violated. Obviously, the FI-term destructs this requirement. The tree level Higgs masses remain unchanged apart from the implicit contribution through the Eq.(9).

Interestingly, it has been recently shown [7] that the renormalization of the FI-term in a softly broken SUSY gauge theory, is completely defined in a rigid or an unbroken theory. In addition, the β function for the FI-term running has been calculated up to three loops in [8]. In all our results here we use the two loop β function[1] (see also footnote 3 below) for the FI-term with the appropriate threshold corrections. We shall discuss the running of the FI-parameter in the next section, where we derive an approximate formula for its one loop running. Next we will present a numerical analysis of the effect of the FI-term on the MSSM spectrum and give some examples of physical interest where this term substantialy changes the low energy spectrum.

2 The "running" of the FI-term

The β-function of the FI-term, β_ξ has been calculated in [8] up to three loops. We display here the corresponding formulae up to two loops. The general formula for the β_ξ [8] can be separeted into two pieces : a part which is linear to the FI-parameter, ξ, which vanishes when $\xi(Q) = 0$ at some scale Q and a part which is independent on ξ and is always proportional to the $U(1)$ gauge coupling, $g_1 = \sqrt{5/3}g$. The later (denoted as $\hat{\beta}_\xi$ below) can be viewed as a tadpole contribution from the vertex $\bar{\Phi} V \Phi$ in Eq.(1), where Φ is a chiral superfield. Following the authors in [8] we have,

$$\beta_\xi \equiv Q\frac{d\xi}{dQ} \;=\; \frac{\beta_{g_1}}{g_1}\,\xi + \hat{\beta}_\xi \,, \tag{12}$$

where β_{g_1} are the one[2] and two loop beta functions which can be found for example in [12]). More specificly, for the ξ independent part we have[3],

$$\begin{aligned}\hat{\beta}_\xi^{(1)} \;&=\; \frac{2}{16\pi^2}\sqrt{\frac{3}{5}}\,g_1 \;\times \\ &\left[\mathrm{Tr}\left(m_{\tilde{Q}}^2\theta_{m_{\tilde{Q}}} - 2m_{\tilde{U}^c}^2\theta_{m_{\tilde{U}^c}} + m_{\tilde{E}^c}^2\theta_{m_{\tilde{E}^c}} + m_{\tilde{D}^c}^2\theta_{m_{\tilde{D}^c}} - m_{\tilde{L}}^2\theta_{m_{\tilde{L}}} - m_1^2\theta_{m_1} + m_2^2\theta_{m_2}\right)\right] ,\end{aligned} \tag{14}$$

[1] Consistently we employ two loop running for all the couplings and masses in the MSSM [9].

[2] We include the low threshold corrections by using the step function approach as in [9, 10, 11].

[3] All the masses here, m_i, are refering to the masses of the fields before the elimination of the D-terms. The masses after the elimination of the D-terms are denoted by $\bar{m}_i$ in (7). Using (7) is not difficult to prove that the β-function, $\beta_{\bar{m}^2}$, in the theory with eliminated D-terms is related to the β_{m^2}, with the unliminated D-terms as

$$\beta_{\bar{m}^2}(\bar{m}^2, ...) \;=\; \beta_{m^2}(\bar{m}^2, ...) + g\frac{Y}{2}\hat{\beta}_\xi(\bar{m}^2, ...) \,. \tag{13}$$

In the literature, it is (almost) always the shifted masses $\bar{m}^2$, which are cited, where their running include the effect of the FI-term but in a, somehow, hidden way. Here we run $\beta_{m^2}(m^2, ...)$ and β_ξ seperately. See [8, 12] for more details.

where the Tr should be understood as a sum over the flavours. We have abreviated also the θ-function, $\theta(Q^2 - m_i^2) \equiv \theta_{m_i}$. The two loop $\hat{\beta}_\xi$ is

$$\hat{\beta}_\xi^{(2)} = \frac{-4}{(16\pi^2)^2} \sqrt{\frac{3}{5}}\, g_1 \;\times$$
$$\left[\mathrm{Tr}\Big(m_{\tilde{Q}}^2 P_{\tilde{Q}} - 2m_{\tilde{U}^c}^2 P_{\tilde{U}^c} + m_{\tilde{E}^c}^2 P_{\tilde{E}^c} + m_{\tilde{D}^c}^2 P_{\tilde{D}^c} - m_{\tilde{L}}^2 P_{\tilde{L}^c} - m_1^2 P_1 + m_2^2 P_2\Big)\right], \quad (15)$$

where every mass term inside in Tr has to decouple below its mass as in (15). The third generation anomalous dimensions P_i in (15) are

$$\begin{aligned}
P_{\tilde{Q}_3} &= Y_t^2 + Y_b^2 - \left(\frac{8}{3}g_3^2 + \frac{3}{2}g_2^2 + \frac{1}{30}g_1^2\right), \\
P_{\tilde{U}_3^c} &= 2Y_t^2 - \left(\frac{8}{3}g_3^2 + \frac{8}{15}g_1^2\right), \\
P_{\tilde{E}_3^c} &= 2Y_\tau^2 - \left(\frac{6}{5}g_1^2\right), \\
P_{\tilde{D}_3^c} &= 2Y_b^2 - \left(\frac{8}{3}g_3^2 + \frac{2}{15}g_1^2\right), \\
P_{\tilde{L}_3} &= Y_\tau^2 - \left(\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right), \\
P_1 &= 3Y_b^2 + Y_\tau^2 - \left(\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right), \\
P_2 &= 3Y_t^2 - \left(\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right),
\end{aligned} \quad (16)$$

and the corresponding anomalous dimensions for the first and second generation where we set the Yukawa couplings to zero. An approximate solution of (12) at 1-loop gives

$$\xi(M_Z) = \frac{g(M_Z)}{g(M_{\mathrm{GUT}})}\xi(M_{\mathrm{GUT}}) - \frac{g(M_Z)}{32\pi^2}\mathrm{Tr}[M_i^2 Y_i]\ln(\frac{M_{\mathrm{GUT}}}{M_Z}) + \frac{g(M_Z)}{32\pi^2}\mathrm{Tr}[m_i^2 Y_i \ln(\frac{m_i}{M_Z})], \quad (17)$$

with (as we mentioned in the introduction) M_i and m_i the GUT and EW SUSY soft breaking masses respectively. The Tr's in second term of (17) is

$$\mathrm{Tr}[m_i^2 Y_i] \equiv \mathrm{Tr}\Big(m_{\tilde{Q}}^2 - 2m_{\tilde{U}^c}^2 + m_{\tilde{E}^c}^2 + m_{\tilde{D}^c}^2 - m_{\tilde{L}}^2 - m_1^2 + m_2^2\Big)\Big], \quad (18)$$

and anologously for the trace of the third term. Based on Eq.(17) we ditinguish three cases

- Universal boundary conditions at the GUT scale and $\xi(M_{\mathrm{GUT}}) \neq 0$.
- $\xi(M_{\mathrm{GUT}}) = 0$ and universal boundary conditions for the soft SUSY breaking scalar masses (or the gauge group at the GUT scale is semisimle)
- $\xi(M_{\mathrm{GUT}}) = 0$ and non universal boundary conditions. In this case the second term in (17) dominates the magnitude of the FI-term at the electroweak scale.

In the following we shall go through those cases one by one. investigating the effect of the FI-term at low energies. In particular, we choose boundary conditions such that are of physical interest (refer to a particular problem) at the electroweak scale.

3 The magnitude of the FI-term at low energies

We shall investigate the effect of the FI-term at low energies. In particular, we shall choose boundary conditions such that are of physical interest *i.e.,* they refer to a particular problem at the electroweak scale.

3.1 Universality and $\xi(M_{\rm GUT}) \neq 0$

In this case the first term in (17) is dominant with $\xi(M_Z) = 0.63\, \xi(M_{\rm GUT})$.

3.2 Universality and $\xi(M_{\rm GUT}) = 0$

In this case the third term in (17) is dominant.

3.3 Non-Universality and $\xi(M_{\rm GUT}) = 0$

In this case the second term in (17) dominates the magnitude of the FI-term at the electroweak scale.

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