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Can a light Higgs boson explain the recent BNL measurement of the muon anomalous magnetic moment ?

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Abstract: A light Higgs boson with mass of arround $\sim$ 10 GeV, could explain the recent BNL measurement on muon $g-2$ anomalous magnetic moment, in the framework of the minimal Standard Model with two Higgs doublets. Such a light Higgs boson should have a mass bounded from below from the Upsilon mass and from above from the bottonium mass.

Keywords: Beyond Standard Model.

Contents

1. Introduction

The recent reported 2.6σ [1] deviation

$$a_\mu^{exp} - a_\mu^{SM} = 426 \pm 165 \times 10^{-11} , \tag{1.1}$$

of the experimental value of the muon anomalous magnetic moment versus the Standard Model (SM) predicted one, is very exciting. It has also been reported [2] that at 90% CL a "New Physics" contribution of

$$215 \times 10^{-11} \lesssim \delta a_\mu(\text{New Physics}) \lesssim 637 \times 10^{-11} , \tag{1.2}$$

beyond the SM, is needed. This contribution is positive, and of the order of the electroweak corrections to the muon anomalous magnetic moment. More precisely, the "New Physics" needed, has to be of the order of $\frac{G_F m_\mu^2}{4\sqrt{2}\pi^2}$, where $G_F = 1.16637(1) \times 10^{-5}$ GeV^{-2} is the Fermi constant and $m_\mu = 0.105$ GeV is the mass of the muon. Furthermore, one could think that these corrections could directly be originated by the perturbative electroweak symmetry breaking mechanism or in other words by the Higgs Boson particle. However, in the SM, the Higgs boson contribution (in units of $\frac{G_F m_\mu^2}{4\sqrt{2}\pi^2}$) to the muon anomalous magnetic moment is suppressed by a factor of m_μ^2/M_h^2. Regarding the smallness of the mass of the muon and the largeness of the Higgs mass ($\gtrsim 115$ GeV) the SM Higgs contribution to δa_μ is negligible.

The above picture is not in general valid in a straightforward extension of the SM with two Higgs doublets. In this framework, δa_μ could be enhanced in two ways

: the first one is that δa_μ is proportional[1] to $\tan^2\beta$ in the case where $\sin(\beta-\alpha)=0$, and the second, it is in that case where the light Higgs boson can be as light as 10 GeV without conflict with experiment. In terms of $\frac{G_F m_\mu^2}{4\sqrt{2}\pi^2}$, the overall enhancement is of order

$$-\frac{m_\mu^2}{M_h^2}\times\tan^2\beta\times F(\frac{m_\mu^2}{M_h^2})\simeq 1-10\ , \tag{1.3}$$

where $F(x)$ is a loop factor which involves logarithms of the form $\ln(\frac{m_\mu^2}{M_h^2})$ of order $\mathcal{O}(-10)$. Such a light Higgs boson with mass $M_h\simeq 10$ GeV and $30\lesssim\tan\beta\lesssim 44$, predicts a muon anomalous magnetic moment in agreement with the current BNL experimental value.

This idea cannot take place in the MSSM simply because the Higgs bosons are constraint from certain mass sum rules[2]. Although in the exact Supersymmetric limit there are no contributions [3] to the muon $g-2$, this is not the case for the softly broken N=1 supersymmetric Standard model where the neutralino/chargino contributions are proportional to $\frac{m_\mu^2 M_2\tan\beta}{M_{SUSY}^4}F$ in units of $\frac{G_F m_\mu^2}{4\sqrt{2}\pi^2}$. Thus, for moderate and large $\tan\beta$ values one recovers the $g-2$ deviation (see the corresponding Refs in [1]). Although, the SUSY solution is quite elegant, we feel that the scenario we propose, is far more generic and minimal with respect to its field condent.

The two-doublet Higgs sector [4] contains eight scalar degrees of freedom: one complex $Y=-1$ doublet, $\boldsymbol{\Phi_d}=(\Phi_d^0,\Phi_d^-)$ and one complex $Y=+1$ doublet, $\boldsymbol{\Phi_u}=(\Phi_u^+,\Phi_u^0)$. The notation reflects the form of the MSSM Higgs sector coupling to fermions: Φ_d^0 [Φ_u^0] couples exclusively to down-type [up-type] fermion pairs. When the Higgs potential is minimized, the neutral components of the Higgs fields acquire vacuum expectation values:[3]

$$\langle\boldsymbol{\Phi_{\rm d}}\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}v_d\\0\end{pmatrix},\qquad\langle\boldsymbol{\Phi_{\rm u}}\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\v_u\end{pmatrix}, \tag{1.4}$$

where the normalization has been chosen such that $v^2\equiv v_d^2+v_u^2=4m_W^2/g^2=(246\ \text{GeV})^2$. Spontaneous electroweak symmetry breaking results in three Goldstone bosons, which are absorbed and become the longitudinal components of the $W^\pm$ and

[1] Definitions are given below.

[2] There is one exception here : It could be the case that the light Higgs boson in the MSSM decays "invisibly" into LSPs (ussually neutralinos). Although there are strict bounds on $\sin(\beta-\alpha)-M_h$ plane from LEP searches, an "invisible" Higgs boson with a mass of few GeV's and $\sin(\beta-\alpha)=0$ is allowed. Such a "supelight" higgs boson contributes to the $g-2$ muon anomalous magnetic, in exactly the same way as we describe here. However the Higgs boson should decay to very light particles and one has propably to consider R-parity violation or to depart from the minimal field content by adding a singlet superfield (NMSSM). This scenario would be highly fine-tuned.

[3] The phases of the Higgs fields can be chosen such that the vacuum expectation values are real and positive. That is, the tree-level MSSM Higgs sector conserves CP, which implies that the neutral Higgs mass eigenstates possess definite CP quantum numbers.

Z. The remaining five physical Higgs particles consist of a charged Higgs pair

$$H^{\pm} = \Phi_d^{\pm} \sin\beta + \Phi_u^{\pm} \cos\beta \,, \tag{1.5}$$

one CP-odd scalar

$$A = \sqrt{2}\left(\mathrm{Im}\,\Phi_d^0 \sin\beta + \mathrm{Im}\,\Phi_u^0 \cos\beta\right) \,, \tag{1.6}$$

and two CP-even scalars:

$$\begin{aligned} h &= -(\sqrt{2}\,\mathrm{Re}\,\Phi_d^0 - v_d)\sin\alpha + (\sqrt{2}\,\mathrm{Re}\,\Phi_u^0 - v_u)\cos\alpha \,, \\ H &= (\sqrt{2}\,\mathrm{Re}\,\Phi_d^0 - v_d)\cos\alpha + (\sqrt{2}\,\mathrm{Re}\,\Phi_u^0 - v_u)\sin\alpha \,, \end{aligned} \tag{1.7}$$

(with $m_h \leq m_H$). The angle α arises when the CP-even Higgs squared-mass matrix (in the Φ_d^0—Φ_u^0 basis) is diagonalized to obtain the physical CP-even Higgs states. The angle β is defined as $\tan^{-1}(\frac{v_u}{v_d})$.

The phenomenology of the Higgs sector depends in detail on the various couplings of the Higgs bosons to gauge bosons, Higgs bosons and fermions. The couplings of the two CP-even Higgs bosons to W and Z pairs are given in terms of the angles α and β by

$$\begin{aligned} g_{hVV} &= g_V m_V \sin(\beta - \alpha) \\ g_{HVV} &= g_V m_V \cos(\beta - \alpha) \,, \end{aligned} \tag{1.8}$$

where

$$g_V \equiv \begin{cases} g, & V = W \,, \\ g/\cos\theta_W, & V = Z \,. \end{cases} \tag{1.9}$$

There are no tree-level couplings of A or $H^{\pm}$ to VV. The couplings of one gauge boson to two neutral Higgs bosons:

$$\begin{aligned} g_{hAZ} &= \frac{g\cos(\beta-\alpha)}{2\cos\theta_W} \,, \\ g_{HAZ} &= \frac{-g\sin(\beta-\alpha)}{2\cos\theta_W} \,. \end{aligned} \tag{1.10}$$

There are two versions of the SM extended with two Higgs doublets called here Model I and Model II. The Model I is the case where quarks and leptons do not couple to the first Higgs doublet (denoted as $\mathbf{\Phi_d}$) but couple to the second doublet (denoted as $\mathbf{\Phi_u}$) in a manner analogous to the minimal Standard Model. The gauge invariant Model I Yukawa interactions for the muons can be read from

$$-\mathcal{L}_{\mathrm{Yukawa}} = h_\mu \left[\bar{\mu} P_L \mu \Phi_u^0 - \bar{\mu} P_L \nu_\mu \Phi_u^-\right] + \mathrm{h.c.} \,, \tag{1.11}$$

where $P_L \equiv \frac{1}{2}(1-\gamma_5)$ is the left-handed projection operator. In Model II, the Higgs tree-level couplings to fermions obey the following property: Φ_d^0 couples exclusively to down-type fermion pairs and Φ_u^0 couples exclusively to up-type fermion pairs. The gauge-invariant Type-II Yukawa interactions for the muons are given by:

$$-\mathcal{L}_{\text{Yukawa}} = h_\mu \left[\bar{\mu} P_L \mu \Phi_d^0 - \bar{\mu} P_L \nu_\mu \Phi_d^- \right] + \text{h.c.}\,, \tag{1.12}$$

Fermion masses are generated when the neutral Higgs components acquire vacuum expectation values. Inserting eq. (1.4) into eq. (1.12) yields a relation between the muon mass and its Yukawa coupling:

$$h_\mu = \frac{\sqrt{2}\, m_\mu}{v_u} = \frac{\sqrt{2}\, m_\mu}{v \sin\beta}\,, \qquad \text{Model I}\,, \tag{1.13}$$

$$h_\mu = \frac{\sqrt{2}\, m_\mu}{v_d} = \frac{\sqrt{2}\, m_\mu}{v \cos\beta}\,, \qquad \text{Model II}\,. \tag{1.14}$$

The couplings of the physical Higgs bosons to the muon is obtained from eqs. (1.11)–(1.12) by using eqs. (1.5)–(1.7). In particular, the couplings of the neutral Higgs bosons to $\mu\bar{\mu}$ relative to the Standard Model value, $g m_f / 2 m_W$, are given by the following Table :

	Model I	Model II
$g_{h\mu^+\mu^-}$	$-\frac{\cos\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\cos\beta}$
$g_{H\mu^+\mu^-}$	$-\frac{\sin\alpha}{\sin\beta}$	$-\frac{\cos\alpha}{\cos\beta}$
$g_{A\mu^+\mu^-}$	$-\gamma_5 \cot\beta$	$\gamma_5 \tan\beta$

(1.15)

(the γ_5 indicates a pseudoscalar coupling), and the charged Higgs boson couplings to muon pairs (with all particles pointing into the vertex) is given by

	Model I	Model II
$g_{H^-\mu^+\nu_\mu}$	$-\frac{g\cot\beta}{\sqrt{2}M_W} m_\mu P_L$	$\frac{g\tan\beta}{\sqrt{2}M_W} m_\mu P_L$

(1.16)

Some very useful trigonometric relations are in order :

$$\begin{aligned}
-\frac{\sin\alpha}{\cos\beta} &= \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)\,,\\
\frac{\cos\alpha}{\sin\beta} &= \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)\,,\\
\frac{\cos\alpha}{\cos\beta} &= \cos(\beta-\alpha) + \tan\beta\sin(\beta-\alpha)\,,\\
\frac{\sin\alpha}{\sin\beta} &= \cos(\beta-\alpha) - \cot\beta\sin(\beta-\alpha)\,.
\end{aligned} \tag{1.17}$$

We remind the reader, that we need the Higgs couplings to muons to be enhanced by $\tan\beta$ (or $\cot\beta$ for $\tan\beta < 1$) and a very light Higgs boson which does not couple to the Z-Gauge boson and thus almost undetectable at LEP. From the above tables and the trigonometric relations it is obvious that this is satisfied *if and only if*,

$$\sin(\beta-\alpha) = 0\,. \tag{1.18}$$

In that case from eq.(1.8) we obtain, $g_{hZZ} = 0$ and

$$g_{h\mu^+\mu^-} \quad , \quad g_{A\mu^+\mu^-} \quad , \quad g_{H^-\mu^+\nu_\mu} \propto \tan\beta \quad : \quad \text{Model I} \tag{1.19}$$
$$g_{h\mu^+\mu^-} \quad , \quad g_{A\mu^+\mu^-} \quad , \quad g_{H^-\mu^+\nu_\mu} \propto \cot\beta \quad : \quad \text{Model II} \tag{1.20}$$

and $|g_{H\mu^+\mu^-}| = 1$ in both models.

We would like to comment here on the other extreme limit where $\cos(\beta - \alpha) = 0$. In this case it is

$$g_{H\mu^+\mu^-} \quad , \quad g_{A\mu^+\mu^-} \quad , \quad g_{H^-\mu^+\nu_\mu} \propto \tan\beta \quad : \quad \text{Model I} \tag{1.21}$$
$$g_{H\mu^+\mu^-} \quad , \quad g_{A\mu^+\mu^-} \quad , \quad g_{H^-\mu^+\nu_\mu} \propto \cot\beta \quad : \quad \text{Model II} \tag{1.22}$$

and $|g_{h\mu^+\mu^-}| = 1$. An interesting aspect of both models is that as far as the higgs coupling to the muons is concerned, they are dual to one each other

$$\text{Model I} \quad \tan\beta \leftrightarrow \cot\beta \quad \text{Model II} \tag{1.23}$$

Thus, investigating one of the two models is enough. We follow the notation of Model II.

2. Higgs boson corrections to the muon $g-2$ anomalous magnetic moment

The first calculation of the electroweak corrections to the $g-2$ muon anomalous magnetic moment was done by Weinberg and Jackiw [5] and by Fujikawa, Lee and Sanda [6]. A very nice overview of these corrections in a general field theory framework can be found in Ref.[7]. In the Type II 2HDM framework these corrections can be read from [8]

$$\delta a_\mu^h = \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \left(\frac{\sin\alpha}{\cos\beta}\right)^2 R_h \; F_h(R_h) \tag{2.1}$$
$$\delta a_\mu^H = \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \left(\frac{\cos\alpha}{\cos\beta}\right)^2 R_H \; F_H(R_H) \tag{2.2}$$
$$\delta a_\mu^A = \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \; \tan^2\beta \; R_A \; F_A(R_A) \tag{2.3}$$
$$\delta a_\mu^{H^\pm} = \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \; \tan^2\beta \; R_{H^\pm} \; F_{H^\pm}(R_{H^\pm}, R_\nu) \tag{2.4}$$

where $R_{h,H,A,H^\pm} = \frac{m_\mu^2}{M^2_{h,H,A,H^\pm}}$, $R_\nu = \frac{m_\nu^2}{M^2_{H^\pm}}$ and

$$F_{h,H}(R_{h,H}) = \int_0^1 dx \frac{2x^2 - x^3}{R_{h,H}x^2 - x + 1} \; , \tag{2.5}$$

$$F_A(R_A) = \int_0^1 dx \frac{-x^3}{R_A x^2 - x + 1} , \tag{2.6}$$

$$F_{H^\pm}(R_{H^\pm}, R_\nu) = \int_0^1 dx \frac{x^2 - x^3}{R_{H^\pm} x^2 + (1 - R_{H^\pm} - R_\nu)x + R_\nu} , \tag{2.7}$$

Although we find approximate analytical solutions for the above integrals, we solve them numerically in the figures presented. The small neutrino mass does not affect our results and we set it zero for simplicity.

2.1 Non-Decoupling limit: $\sin(\beta - \alpha) = 0$

Expanding the integrals in powers of R, the leading contribution of the Higgs sector for $\sin(\beta - \alpha) = 0$ is given by

$$\delta a_\mu(\text{higgs}) = \delta a_\mu^h + \delta a_\mu^H + \delta a_\mu^A + \delta a_\mu^{H^\pm} =$$
$$-\frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \tan^2\beta \left\{ \frac{m_\mu^2}{M_h^2}\left[\frac{7}{6} + \ln(\frac{m_\mu^2}{M_h^2})\right] - \frac{m_\mu^2}{M_A^2}\left[\frac{11}{6} + \ln(\frac{m_\mu^2}{M_A^2})\right] + \frac{1}{6}\frac{m_\mu^2}{M_{H^\pm}^2} \right\} \tag{2.8}$$

Note the minus sign out of Eq.(2.8). Also note that the logarithms encountered are negative and always larger than the constant term when the Higgs masses are larger than 1 GeV. Having that in our mind, it is obvious that a possible light CP-Odd Higgs does not help since it gives negative δa_μ. The cituation of both A and h being light does not help either, since the logarithms associated with them cancel out. Furthermore, the charged Higgs boson enters[4] in Eq.(2.8) without a logarithm and a light charged Higgs boson less than 45 GeV is excluded by LEP I searches. The other CP-even Higgs does not give contributions proportional to $\tan\beta$ (see previous section) and thus its contribution to δa_μ is negligible.

The only possibility left is a very light Higgs boson which we advocate here. In that case, its contribution to the muon $g-2$ is given in a very good approximation by

$$\delta a_\mu(\text{higgs}) \simeq \delta a_\mu^h = -\frac{G_F m_\mu^2}{4\sqrt{2}\pi^2}(\frac{m_\mu^2}{M_h^2}) \tan^2\beta \left[\frac{7}{6} + \ln(\frac{m_\mu^2}{M_h^2})\right] . \tag{2.10}$$

One can easily check that a light Higgs boson with a mass of arround 10 GeV and with $\tan\beta = 35$ gives $\delta a_\mu(\text{higgs}) \simeq 280 \times 10^{-11}$ which is actually the new physics we need

[4] Actually Grifols and Pascua [9] have found that the two-loop corrections,

$$\delta a_\mu^{H^\pm} = \frac{1}{180}(\frac{\alpha}{\pi})^2(\frac{m_\mu}{m_{H^\pm}})^2 + \mathcal{O}\left[(\frac{m_\mu}{m_{H^\pm}})^4 \ln(\frac{m_\mu}{m_{H^\pm}})\right] , \tag{2.9}$$

are bigger than the 1-loop ones. But again a very light charged higgs of the order of few GeV's is required and this case is experimentally excluded.

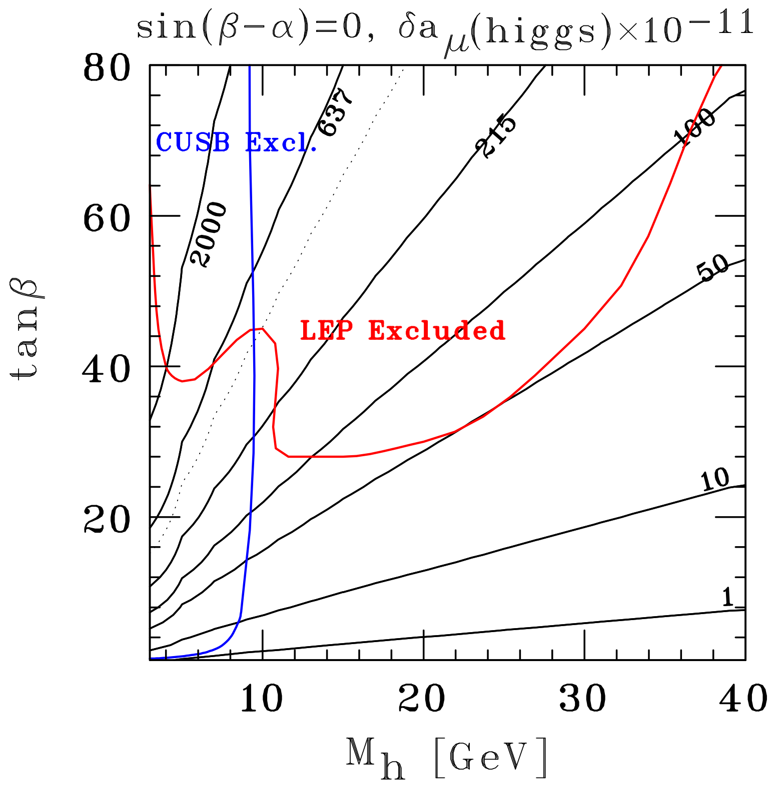

Figure 1: Contour plot of the predicted 1-loop Higgs contributions to the muon $g-2$ anomalous magnetic moment, $\delta\alpha_\mu$(higgs) (in units of 10^{-11}), on $M_h - \tan\beta$ plane, in the 2HDM(II) scenario. We present results with $\sin(\beta-\alpha) = 0$ as this is motivated in the text. The other Higgs masses have been set to $M_{H^\pm} = M_A = M_H$=200 GeV but their variation for above 100 GeV do not affect the results (see Eq.(2.8)). The dashed line represents the central difference between the experimental value and the SM predicted one, $\delta_{\alpha_\mu} = 426 \pm 165 \times 10^{-11}$, as reported from BNL [1]. Contours of 90% CL $215 \times 10^{-11} \leq$ (New Physics) $\leq 637 \times 10^{-11}$ [2] need to recover the current experimental value, are shown. The region on the left side and behind the blue line is excluded by CUSB Collaboration [10]. The region above the solid red line is experimentally excluded by DELPHI LEP I searches [11].

in Eq.(1.2). The full numerical 1-loop light Higgs boson contribution to muon $g-2$, δa_μ^h(higgs) (in units of 10^{-11}), is presented in contour lines on the plane $M_h - \tan\beta$ in Fig.1. The other Higgs masses have been set to $M_{H^\pm} = M_A = M_H$=200 GeV but their variation for above 100 GeV do not affect the results. The experimental bounds are also presented in the figure (see discussion below). As it is clear, a light Higgs boson of 10 GeV mass and $30 \lesssim \tan\beta \lesssim 44$ is capable to give the neccesary contributions to the currently measured value of the muon anomalous magnetic moment. We will turn back into this point in the last section where the mass region where such a light Higgs might exist, will be set.

2.2 Decoupling Limit: $\cos(\beta-\alpha)=0$

The couling of the Z-gauge boson to the light Higgs boson, g_{ZZh} is maximal and the lower LEP II bound[5] on the light Higgs mass, $M_h \gtrsim 115$ GeV, is applicable. The same approximate formula as in Eq.(2.8) is applied, with the light Higgs Boson h being replaced by the heavy, H, one. In the limit where all the other Higgses are equal, $M_A = M_H = M_{H^\pm}$, we obtain

$$\delta a_\mu(\text{higgs}) \simeq \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2}(\frac{m_\mu^2}{M_A^2})\tan^2\beta\left[\frac{1}{2}+2(\frac{m_\mu^2}{M_A^2})\ln(\frac{m_\mu^2}{M_A^2})\right] . \tag{2.11}$$

What is interesting here is that even though the Higgs masses are quite large, $M_A \to \infty$, there is still an enhancement of δa_μ with $\tan^2\beta$ (note that the logarithmic contribution is suppressed). The situation is even clearer if we look at Fig.2 where contour lines of the muon $g-2$ (in units of 10^{-14}) are plotted on a plane of $M_A - \tan\beta$. The conributions to δa_μ are three orders of magnitude smaller than those in the non-decoupling case and thus negligible (for reasonable values of $\tan\beta$). For fixed $\tan\beta$, δa_μ(higgs) dies out with a rate $\sim \frac{m_\mu^2}{M_A^2}$ and the decoupling is manifest, *i.e.,* we recover the SM-case.

3. CUSB and LEP I constraints on the light Higgs boson

As we have already discussed, the light higgs does not couple to the Z-boson in the case of $\sin(\beta-\alpha)=0$ and thus the SM LEP bound of 115 GeV does not apply. In principle, such a Higgs boson can be very light, or in other words : it has been escaped detection from the past Higgs searches. However, there are two major constraints on a possible Higgs mass, of arround 10 GeV, we quickly review here.

The first lower bound comes from CUSB Collaboration [10], where an extensive search for a monochromatic photon in radiative decays $\Upsilon(1S)\to h\gamma$ where

$$\frac{\Gamma(\Upsilon\to h\gamma)}{\Gamma(\Upsilon\to\mu^+\mu^-)} = \frac{G_F m_b^2\tan^2\beta(M_\Upsilon^2-M_h^2)}{4\sqrt{2}\pi\alpha}\left|\frac{T(\Upsilon\to h\gamma)}{T(\Gamma(\Upsilon\to\mu^+\mu^-)}\right|^2 \times\left(1-\frac{4\alpha_s}{3\pi}a_H(z)+\mathcal{O}(a_s^2)\right) \tag{3.1}$$

had led to a lower bound,

$$600\ \text{MeV} \lesssim M_h \lesssim 5\ \text{GeV} \quad , \quad \text{for} \quad \tan\beta = 1\ . \tag{3.2}$$

The first term [12, 13, 14, 15] in Eq.(3.1) contains the matrix elements T (six diagrams in time order perturbation theory [13]) and the second [16, 17] the QCD corrections

[5]For large $\tan\beta$ values this lower bound becomes weaker ($\gtrsim 91$ GeV) but it does not make any difference in the results since it is already too heavy and its contribution to δa_μ is not enhanced by $\tan^2\beta$.

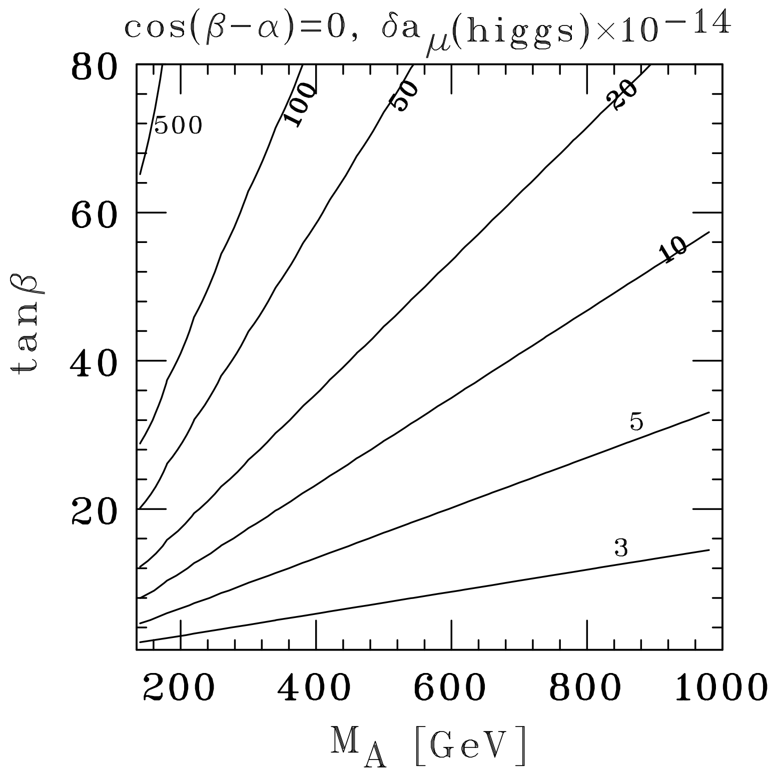

Figure 2: Contour plot of the Higgs 1-loop contributions to the muon $g-2$ anomalous magnetic moment δ_{α_μ}(higgs) (in units of 10^{-14}), on a plane $M_A - \tan\beta$, in the decoupling limit, *i.e.*, $\cos(\beta-\alpha) = 0.$. The light Higgs boson mass has been fixed to its current experimental bound, $M_h = 115$ GeV. The heavy CP-even and the charged Higgses are taken to be equal to M_A. Note the residual enhancement with the $\tan\beta$ even in the region of large M_A.

with $\alpha_H(z)$ being a loop factor with $z = 1 - \frac{M_h^2}{4m_b^2}$. The QCD corrections reduce the rate by 84% for $M_h << M_{\Upsilon(1S)}$. Some comments are in order here. In the first calculation, done by Wilczek [12], one of the two dominant diagrams was considered. In fact, as Faldt, Osland and Wu [13] had pointed out, strong cancellations can occur between the two diagrams leading to a suppresion factor to the rate of about 14 for $M_h = 8.5$ GeV (the suppression factor has a tendency of being increased while $M_h \to M_\Upsilon$). Their calculation is valid only when the Higgs mass is arround the Upsilon mass (7.5 GeV $\lesssim M_h \lesssim M_\Upsilon$). These supressions and the large QCD corrections and even problems with renormalization scheme dependencies had led to a cloudy situation, and no reliable SM bound on the Higgs mass had arised from CUSB. However, as the reader might have already noticed, in the 2HDM we have an extra huge enhancement proportional to $\tan^2\beta$. We feel like, whatever large theoretical uncertainties[6] involved in the calculations of the radiative Upsilon decays, the ratio (3.1) cannot be suppressed by a factor of $\tan^2\beta \sim 10^2 - 30^2$ (!), which is the case we are dealing with. We conclude that the lower bound on the Higgs boson mass is just at the Upsilon threshold

$$M_h \gtrsim M_{\Upsilon(1S)} = 9.46 \text{ GeV} \quad \text{for} \quad \tan\beta \gtrsim 10 \, . \tag{3.3}$$

This bound is denoted with the left area behind the yellow line in Fig.(1).

The second lower bound on the Higgs mass arises from (DELPHI) LEP I searches [11]. The only process which is capable to constraint our very light Higgs boson is the Yukawa process [7]. Preliminary limits on the Yukawa production ($Z \to f\bar{f}h$) of light neutral Higgs above the $2m_\tau$ threshold were set : $2m_\tau < M_h < 2m_B$ from 4τ (see left corner bounded from the red line in our fig.(1)) topologies and $M_h > 2m_B$ from 4b topologies (see right corner above the red line in fig.(1)). It turns out that LEP I might have already produced the higgs boson without seeing it. Unfortunately, LEP searches cannot ruled out or confirm the existence of such a light Higgs boson and only a bound,

$$\begin{aligned} M_h \gtrsim 2m_B = 10.6 \text{ GeV} \quad & \text{for } \tan\beta \gtrsim 28 \\ M_h \gtrsim m_\Upsilon \quad & \text{for } \tan\beta \gtrsim 44 \, , \end{aligned} \tag{3.4}$$

can be set.

4. Combined Results and Conclusions

If we combine the experimental searches for a light Higgs boson, (3.3) and (3.4) we conclude that the only explanation for the 2.6σ deviation of the BNL measurement in the minimal two Higgs doublet SM model (Model I or II) comes from a light Higgs boson in the range

$$m_\Upsilon \le M_h \le 2m_B \, . \tag{4.1}$$

with $\tan\beta$ being in the region $30 \lesssim \tan\beta \lesssim 44$. Such a Higgs could be confirmed or ruled out in the future colliders [18], or by the future improvement of the BNL [1] $g-2$ muon anomalous magnetic moment.

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[6] Unfortunately the calculations towards that direction had been stopped since collider searches took over.

[7] The pair production process is not kinematically allowed here since as we have already explained, all the other Higgses should be above the Z-threshold in order to obtain the correct δa_μ.

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