A Light Higgs Boson Explanation for the $g-2$ Crisis [a]

A. Dedes[1] and H. E. Haber[2]
[1] *Physikalisches Institut der Universität Bonn, Nußallee 12, D-53115 Bonn, Germany*

[2] *Santa Cruz Institute for Particle Physics University of California, Santa Cruz, CA, 95064 USA*

We have shown in hep-ph/0102297 that a light CP-even Higgs boson with a mass of around 10 GeV could explain the recent BNL measurement of the muon anomalous magnetic moment. This observation is based on a general two Higgs doublet extension of the Standard Model with neither CP violating nor tree level flavour changing neutral current couplings. The Higgs mass is constraint from CESR and LEP experiments to be less than twice the lightest B-meson mass and greater than the Upsilon mass. LEP and CLEO would be able to exclude or discover such a possibility by using their existing data.

1 A possibility for New physics from the $g-2$ measurement at BNL E821

Based on Davier-Höcker calculation for the hadronic contributions to the muon anomalous magnetic moment, BNL [1] reports a 2.6σ deviation from the Standard Model(SM) prediction. There are four possibilities to interpret this discrepancy [2] :

- a statistical fluctuation with 0.9% probability.
- something is "wrong" with the experiment : this is going to be resolved soon (by Christmas [2]) since there is four times more data to be analyzed, and the measurement on μ^- which is in progress at the moment.
- something is "wrong" with the hadronic contributions to the muon anomalous magnetic moment, a_μ^{had}. However, there is a lot more data from Novorsibirsk and BES to appear and there is an intense effort from both theory and experimental groups [4,6,5,7] towards to that direction.
- the by far most exciting possibility : there is a New Physics beyond the SM, $\delta a_\mu^{\rm NP}$. At 90% CL this is [3,4,7]

$$215\times 10^{-11} \lesssim \delta a_\mu^{\rm NP} \lesssim 637\times 10^{-11} \quad [\text{Davier}-\text{Hocker}(98)]\,, \tag{1}$$

$$159\times 10^{-11} \lesssim \delta a_\mu^{\rm NP} \lesssim 599\times 10^{-11} \quad [\text{Narison}(01)] \tag{2}$$

  The above different bounds are based on the most recent independent evaluations of the hadronic vacuum polarization contributions to a_μ.

In what follows, we assume the fourth possibility as an explanation to the discrepancy under discussion. Two remarks on (1,2) are in order [3] : *i)* the contribution from the New Physics is positive, $\delta a_\mu^{\rm NP} > 0$, and *ii)*, is of the order of the Electroweak (EW) contributions, *i.e.*, $\delta a_\mu^{\rm NP} \propto \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}}$.

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1.1 *Why not from the Higgs sector?*

In the SM, the 1-loop Higgs boson contribution to the muon anomalous magnetic moment is suppressed by a factor of m_μ^2/m_h^2, since the SM Higgs mass is bounded from LEP to be greater than 113.5 GeV. The situation is different if we extend the SM by adding an extra Higgs doublet [8]. The idea that a light Higgs boson could enhanced the predicted value of a_μ has been recently employed in Ref[9,10]. Here we strictly follow the discussion of Ref [9] [b]. The enhancement arises from the Higgs sector is twofold: *i)* the coupling of the muon to the light CP-even Higgs (h), CP-Odd Higgs (A) and charged higgs ($H^\pm$) is proportional to $\tan\beta$ (the ratio of the two vacuum expectation values), and hence $\delta a_\mu^{\rm NP} \propto \tan^2\beta$ *ii)* the higgs coupling to the Z-boson can be set to zero, *i.e.,* $\sin(\beta-\alpha)=0$, (α is the mixing angle between the two CP-even Higgs bosons) and thus LEP constraints from $e^+e^- \to hZ$ cannot be applied [12]. In this case the Higgs mass can be as light as ~ 10 GeV.

2 Model II Higgs boson corrections to a_μ

The first calculation of the one-loop electroweak corrections to the muon anomalous magnetic moment was presented by Weinberg and Jackiw [13] and by Fujikawa, Lee and Sanda [14]. A very useful compendium of formulae for the one-loop corrections to $g-2$ in a general electroweak model was given in [15], and applied to the 2HDM in [16]. In the 2HDM, both neutral and charged Higgs bosons contribute to $g-2$. One can derive approximate results by expanding the loop integrals in terms of the parameters $R_{h,H,A,H^\pm} \equiv m_\mu^2/m_{h,H,A,H^\pm}^2$, $R_\nu \equiv m_\nu^2/m_{H^\pm}^2$ (here we neglect the neutrino mass).

2.1 *Non-decoupling limit:* $\sin(\beta-\alpha)=0$

In section 1, we argued that the most significant Higgs contribution to $\delta a_\mu^{\rm NP}$ (consistent with the LEP SM Higgs search) arises in the parameter regime in which $\sin(\beta-\alpha)\simeq 0$ and $\tan\beta \gg 1$. Setting $\sin(\beta-\alpha)=0$ and keeping only the leading terms in R when evaluating the above integrals, the total Higgs sector contribution to a_μ at 1-loop[c] is given by:

$$\begin{aligned}\delta a_\mu^{\rm Higgs} &= \delta a_\mu^h + \delta a_\mu^H + \delta a_\mu^A + \delta a_\mu^{H^\pm} \\ &\simeq \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}}\tan^2\beta\left\{\frac{m_\mu^2}{m_h^2}\left[\ln\left(\frac{m_h^2}{m_\mu^2}\right)-\frac{7}{6}\right]-\frac{m_\mu^2}{m_A^2}\left[\ln\left(\frac{m_A^2}{m_\mu^2}\right)-\frac{11}{6}\right]-\frac{m_\mu^2}{6m_{H^\pm}^2}\right\}. \qquad (3)\end{aligned}$$

Note that the logarithms appearing in (3) always dominate the corresponding constant terms when the Higgs masses are larger than 1 GeV. It is then clear that A and $H^\pm$ exchange contribute a negative value to $\delta a_\mu^{\rm NP}$ and thus cannot explain the BNL $g-2$ measurement which suggests a positive value for $\delta a_\mu^{\rm NP}$. In addition, we should take m_A and $m_{H^\pm}$ large (masses above 100 GeV are sufficient) in order that the corresponding A and $H^\pm$ negative contributions are neglibly small. If $\delta a_\mu^{\rm NP}$ is to be a consequence of the Higgs sector, it must be entirely due to the contribution of the light CP-even Higgs boson. Note that the heavier CP-even Higgs, H, does not give a contribution proportional to $\tan\beta$; hence its contribution to $\delta a_\mu^{\rm NP}$ can be neglected in (3). Thus, to a good approximation,

$$\delta a_\mu^{\rm Higgs} \simeq \delta a_\mu^h \simeq \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}}\left(\frac{m_\mu^2}{m_h^2}\right)\tan^2\beta\left[\ln\left(\frac{m_h^2}{m_\mu^2}\right)-\frac{7}{6}\right]. \qquad (4)$$

One can check that a light Higgs boson with a mass of around 10 GeV and with $\tan\beta=35$ gives $\delta a_\mu^{\rm Higgs} \simeq 280\times 10^{-11}$, which is within the 90% CL allowed range for $\delta a_\mu^{\rm NP}$ quoted in (1,2). Contour lines corresponding to a full numerical evaluation of the Higgs sector one-loop contribution to $\delta a_\mu^{\rm Higgs}$ [in units of 10^{-11}] are exhibited in fig. 1a, for $\sin(\beta-\alpha)=0$ and $m_H=m_A=m_{H^\pm}=200$ GeV.[d] The

[b] For the purposes of this presentation we refer to the Two Higgs Doublet Model (II) [2HDM(II)]. For the Model I see the discussion in Ref [9] and for Model III see Ref [11].

[c] Part of the dominant two-loop corrections have been discussed in Ref. [17].

[d] The results are insensitive to the values of the heavy Higgs masses above 100 GeV.

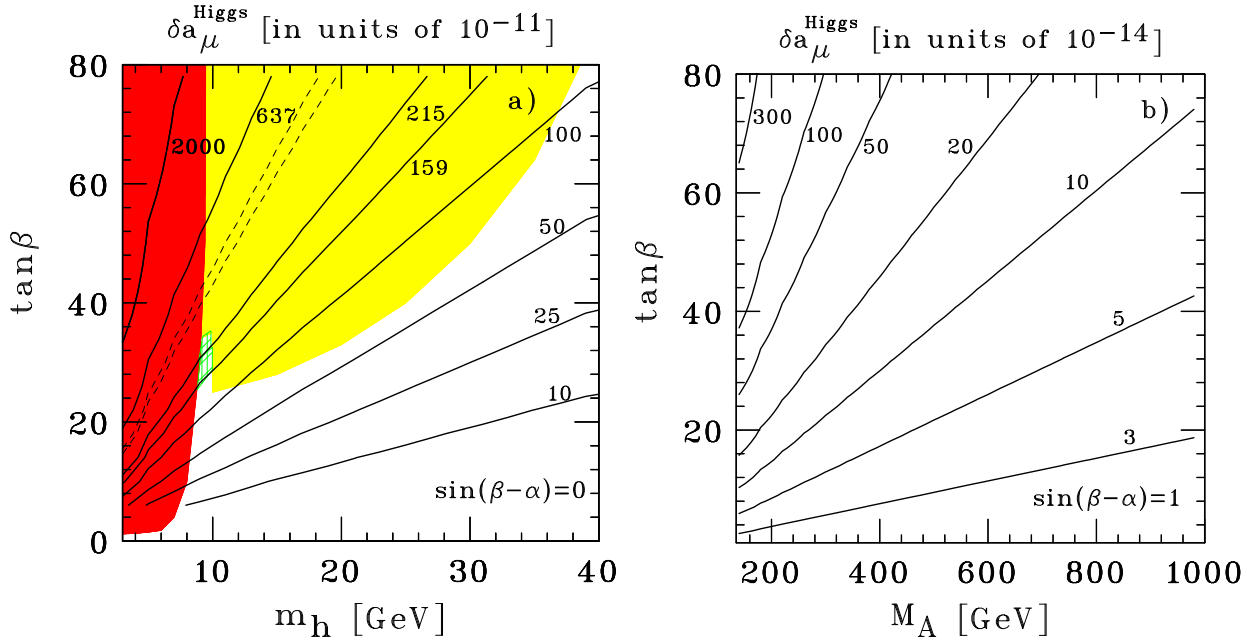

Figure 1: **a)** Contours of the predicted one-loop Higgs sector contribution to the muon anomalous magnetic moment, $\delta\alpha_\mu^{\rm Higgs}$ (in units of 10^{-11}) in the 2HDM, assuming that $\sin(\beta-\alpha)=0$, and $m_H = m_A = m_{H^\pm} = 200$ GeV (there is little sensitivity to the heavier Higgs masses). The dashed line contours correspond to the central value of $\delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM}$, as reported by Brown *et.al* and by Narison. The contour lines marked 215 and 637 correspond to 90% CL limits for the contribution of new physics to a_μ (1). The contour 159 corresponds to the lower bound on $\delta a_\mu^{\rm NP}$ given in (2). The dark-shaded (red) region is excluded by the CUSB Collaboration search for $\Upsilon \to h\gamma$ at CESR. The light-shaded (yellow) region is excluded at 95% CL by the ALEPH and DELPHI searches for $e^+e^- \to hf\bar{f}$ ($f = b$ or τ) at LEP. In the small hatched region (green) nestled between the two experimentally excluded shaded regions, above the 159 contour line and centered around $m_h \simeq 10$ GeV, the Higgs sector contribution to $\delta a_\mu^{\rm NP}$ lies within the 90% CL allowed range (1,2). **b)** Contour plot of the Higgs 1-loop contributions to the muon $g-2$ anomalous magnetic moment $\delta a_\mu^{\rm Higgs}$ (in units of 10^{-14}), on a plane $M_A - \tan\beta$, in the decoupling limit, *i.e.*, $\sin(\beta-\alpha)=1$. The light Higgs boson mass has been fixed to be just above the current experimental bound, $M_h = 115$ GeV. The heavy CP-even and the charged Higgses are taken to be equal to M_A. Note the residual enhancement with the $\tan\beta$ even in the ("decoupled") region of large M_A.

relevant experimental bounds are also displayed in fig. 1; [see discussion below]. A careful inspection of the excluded region in the m_h *vs.* $\tan\beta$ parameter space shows that a light Higgs boson of around 10 GeV mass and $25 \lesssim \tan\beta \lesssim 35$ is permitted depending on the lower bound of Eq.(1,2). In this parameter regime, we obtain a value for $\delta a_\mu^{\rm NP}$ within the 90% CL allowed range of (1). However, the central values of $\delta a_\mu^{\rm NP}$ given in Refs [3,7] lie within the excluded regions of fig. 1a.

2.2 Decoupling limit: $\sin(\beta-\alpha)=1$

In the decoupling limit, where $\sin(\beta-\alpha) \simeq 1$ and $m_A \gg m_Z$, the couplings of the light Higgs boson, h, are (nearly) identical to those of the SM Higgs boson. As a result, the LEP SM Higgs mass bound of $m_h \gtrsim 113.5$ GeV applies. For $\sin(\beta-\alpha)=1$, the H couplings to down-type fermion pairs are enhanced by $\tan\beta$. Thus, the Higgs sector contribution to $\delta a_\mu^{\rm NP}$ is given by (3), with m_h replaced by m_H. In the decoupling limit, $m_H \simeq m_A \simeq m_{H^\pm}$ [the mass differences are of $\mathcal{O}(m_Z^2/m_A)$]. Setting $\sin(\beta-\alpha)=1$ and $m_H = m_A = m_{H^\pm}$, we find

$$\delta a_\mu^{\rm Higgs} \simeq \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}} \left(\frac{m_\mu^2}{m_A^2}\right) \tan^2\beta \left[\frac{1}{2} - \left(\frac{2m_\mu^2}{m_A^2}\right) \ln\left(\frac{m_A^2}{m_\mu^2}\right)\right] . \qquad (5)$$

The contribution of h is not $\tan\beta$–enhanced and is thus negligible. It is interesting to note that for values of $m_A \lesssim m_h \tan\beta$, the heavier ("decoupled") Higgs bosons actually dominate in the Higgs sector contribution to $\delta a_\mu^{\rm NP}$.[e] However, for 100 GeV $< m_A <$ 1000 GeV, and $30 < \tan\beta < 100$, the Higgs sector contribution to a_μ ranges from about 3×10^{-12} to 3×10^{-14}, which is three to five orders of magnitude below what is needed to explain the BNL measurement of a_μ [see fig.1b].

2.3 Experimental constraints from CESR and LEP

Let us consider the 2HDM in which $\sin(\beta-\alpha) = 0$, $\tan\beta \gg 1$ and $m_h \sim \mathcal{O}(10$ GeV), which are necessary conditions if the Higgs sector is to be the source for $\delta a_\mu^{\rm NP}$ in the range given by Eq.(1,2). The hAZ coupling is maximal , so we must assume that m_A is large enough so that $e^+e^- \to hA$ is not observed at LEP. The tree-level hZZ coupling is absent, which implies that the LEP SM Higgs search based on $e^+e^- \to Z \to Zh$ does not impose any significant constraints on m_h.[f] However, there are a number of constraints on light Higgs masses that do not rely on the hZZ coupling. For Higgs bosons with $m_h \lesssim 5$ GeV, the SM Higgs boson was ruled out by a variety of arguments that were summarized in [8]. For 5 GeV $\lesssim m_h \lesssim 10$ GeV, the relevant Higgs boson constraint can be derived from the absence of Higgs production in $\Upsilon \to h\gamma$ [see red region in Fig.1a]. A second bound on m_h can be derived from the non-observation of Higgs bosons at LEP via the process $e^+e^- \to hf\bar{f}$ ($f = b$, τ). However, only preliminary results have appeared so far by DELPHI [20] and ALEPH [21] [see yellow region in Fig.1a]. One noteworthy consequence of $m_h \sim 10$ GeV is the possibility of mixing between the h and the 0^{++} $b\bar{b}$ bound states $\chi_{b0}(1P)$ and $\chi_{b0}(2P)$, as discussed in [16,22]. As a result, the decay $\chi_{b0} \to \tau^+\tau^-$ should be prominent. For more details see Ref.[9].

3 Results

If we combine the experimental bounds on the Higgs mass discussed in section 2, we conclude that a light Higgs boson can be responsible for the observed 2.6σ deviation of the BNL measurement of the muon anomalous magnetic moment at the 90% CL in the framework of a two-Higgs-doublet model with Model II Higgs-fermion Yukawa couplings only if the model parameters satisfy the following requirements:

$$\begin{aligned} & m_\Upsilon \lesssim m_h \lesssim 2m_B \ , \\ & \sin(\beta-\alpha) \simeq 0 \ , \\ & 30 \lesssim \tan\beta \lesssim 35 \quad [\text{Davier} - \text{Hocker}(98)] \ , \\ & 25 \lesssim \tan\beta \lesssim 35 \quad [\text{Narison}(01)] \ . \end{aligned} \tag{6}$$

In addition, the other Higgs bosons should be heavy enough to satisfy the LEP bounds. Although, the parameter space is highly constraint we think that LEP (and maybe CLEO) ought to search for such a possibility.

To justify the word "Crisis" in the title, is enough to indicate the enormous activity [23,24,25,26,27] and excitement on a possible Physics Beyond the SM from the BNL muon anomalous magnetic moment measurements.

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[e] If we formally take $m_A \to \infty$, we recover the Standard Model Higgs contribution to a_μ.

[f] Presumably, radiative corrections would lead to a small effective value for $\sin(\beta-\alpha)$. The LEP Higgs search yields an excluded region in the $\sin(\beta-\alpha)$ *vs.* m_h plane, and implies that for $m_h \sim 10$ GeV, $|\sin(\beta-\alpha)| \lesssim 0.06$ is not excluded at 95% CL [18,19].

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