The Radiative Electroweak Symmetry Breaking in the MSSM at 2-loops

A. Dedes
Physikalisches Institut der Universitat Bonn,
Nussallee 12, D-53115 Bonn, Germany

R. Espinosa
Madrid, Madrid, Autonoma U., IFT, Spain

R. J. Zhang
Princeton, Inst. Advanced Study , USA

(Dated: February 1, 2001)

Abstract

We find the 2-loop tadpole corrections to the minimization conditions of the effective potential in the MSSM. We show that with the two-loop improvement, we almost exactly cancel the renormalization group scale dependence of the vevs (or of the parameters μ and B). Next, we delineate the parameter space where EWSB occurs. We find...

I. THE GENERAL SET-UP

First of all we follow the notation of Ref.[1]. We ignore contributions from the electroweak gauge couplings and keep only terms in the Lagrangian which depend on the SU(3) gauge coupling g_3 and the top Yukawa coupling Y_t. **(For large $\tan\beta$ values we should also keep contributions from sbottoms but at this point I do not know if we can straiforwardly include this correction into your two loop effective potential. But I expect that we can do it.)**. First we write the Higgs potential in terms of the field h_1 and h_2 dependent masses,

$$V(h_1,h_2) = V_{vac} + V_0(h_1,h_2) + V_1(h_1,h_2) + V_2(h_1,h_2) \ , \tag{1}$$

where V_{vac} is the field independent vacuum self energy (which is non-zero when SUSY is broken), V_0 is the tree level potential

$$V_0(h_1,h_2) = \frac{1}{2}(m_{H_1}^2 + \mu^2)h_1^2 + \frac{1}{2}(m_{H_2}^2 + \mu^2)h_2^2 + B_\mu h_1 h_2 \ , \tag{2}$$

where all the parameters have been taken to be real, V_1 is the 1-loop effective potential in the Landau gauge,

$$16\pi^2 V_1(h_1,h_2) = 3\Big[H(m_{\tilde t_1}^2) + H(m_{\tilde t_2}^2) - 2H(m_t^2)\Big] + ... \tag{3}$$

where "..." means other corrections proportional to the electroweak gauge couplings,

$$\begin{aligned} &m_{\tilde t_1}^2 = m_{\tilde t}^2 + m_t X_t \ , \quad m_{\tilde t_2}^2 = m_{\tilde t}^2 - m_t X_t \ , \\ &m_{\tilde t}^2 \simeq M_S^2 + m_t^2 \ , \quad m_t X_t = \frac{h_t}{\sqrt{2}}(\mu h_1 + A_t h_2) \ , \quad m_t^2 = \frac{1}{2}h_t^2 h_2^2 \ , \\ &M_S = M_{\tilde Q} = M_{\tilde U} \ , \end{aligned} \tag{4}$$

and the function H is

$$H(m^2) = \frac{m^4}{2}\Big(\ln\frac{m^2}{Q^2} - \frac{3}{2}\Big) \ . \tag{5}$$

In the above we have assumed that all supersymmetric particles have roughly the same mass $M_S >> M_Z$. V_2 represents the 2-loop contributions to the effective potential and this is given in the Appendix D of [1].

Inspection of the extrema of the two variables function $V(h_1, h_2)$ leads to the following cases:

i) Radiative Electroweak Symmetry Breaking : The $SU(2) \times U(1)$ symmetry is broken at some scale $Q < Q_R$ where Q_R is called REWSB-scale. The potential (1) should satisfy the condition

$$\left[V_{h_1 h_1}(Q_R) V_{h_2 h_2}(Q_R) - V^2_{h_1 h_2}(Q_R) \right]_{h_1=h_2=0} = 0 \; , \tag{6}$$

where $V_{h_i h_j} = \frac{\partial^2 V}{\partial h_i \partial h_j}$.

ii) Stability of the potential : There must be a scale $Q = Q_I$ such that the potential $V(h_1, h_2)$ is bounded from below,

$$V(h_1 = h_2 \to \infty; Q_I) > 0 \; . \tag{7}$$

iii) Global EW minimum : Neccesary and sufficient conditions for a global minimum arround the EW vacuum expectation values v_1, v_2 are,

$$\left[V_{h_1} = V_{h_2} \right]_{h_1=v_1, h_2=v_2} = 0 \; , \tag{8}$$

$$\left[V_{h_1 h_1} \quad , \quad V_{h_2 h_2} \right]_{h_1=v_1, h_2=v_2} > 0 \; , \tag{9}$$

$$\left[V_{h_1 h_1} V_{h_2 h_2} - V^2_{h_1 h_2} \right]_{h_1=v_1, h_2=v_2} > 0 \; , \tag{10}$$

$$\left[V(h_1, h_2) \right]_{h_1=v_1, h_2=v_2} < \left[V(h_1, h_2, \Phi_i) \right]_{h_1=v_1, h_2=v_2, \Phi_i=x_i} \; , \tag{11}$$

where $V_{h_i} = \frac{\partial V}{\partial h_i}$ and Φ_i are other scalar fields of the MSSM with v.e.v's x_i. In this letter we *assume* the validity of (11). Eqs (9,10) imply $\det(\mathcal{M}^2_h) > 0$.

Ones eqs.(8-11) are satisfied then also eq.(6) should be satisfied. Then the only thing that is left is to calculate the tadpole corrections *i.e., eqs.((8,9)* at 2 loop level. After checking the scale invariance of the VEVs we constraint the running parameters of the MSSM under the requirement that the 2-minimization conditions, (8,9) are satisfied or not.

II. 1-LOOP CORRECTIONS

At one loop it is rather trivial to prove the scale invariance of the potential. Thus, with $\Omega_\Lambda = -V(h_1 = h_2 = 0) = -\frac{3}{16\pi^2} M_S^4 \ln \frac{M_S^2}{Q^2}$ and the 1-loop RGE's

$$\frac{\partial m_1^2}{\partial t} = \frac{3h_t^2}{16\pi^2}\mu^2 \tag{12}$$

$$\frac{\partial m_2^2}{\partial t} = \frac{3h_t^2}{16\pi^2}\Big[m_2^2 + 2M_S^2 + A_t^2\Big] \tag{13}$$

$$\frac{\partial B_\mu}{\partial t} = \frac{3h_t^2}{16\pi^2}\Big[\mu A_t + \frac{1}{2}B_\mu\Big] \tag{14}$$

$$\frac{\partial h_2^2}{\partial t} = -\frac{3h_t^2}{16\pi^2}h_2^2\ , \tag{15}$$

with $m_{1,2}^2 = m_{H_{1,2}}^2 + \mu^2$ and $t = \ln Q^2$. Then up to two loop corrections,

$$\frac{dV}{dt} = \frac{d\Omega_\Lambda}{dt} + \frac{dV_0}{dt} + \frac{dV_1}{dt} = 0\ . \tag{16}$$

The field dependent first and second derivatives of the potential are

$$V_{h_1} = m_1^2 h_1 + B_\mu h_2 + \frac{3}{16\pi^2}\frac{h_t\mu}{\sqrt{2}}\Big[\mathcal{G}(m_{\tilde{t}_1}^2) - \mathcal{G}(m_{\tilde{t}_2}^2)\Big] \tag{17}$$

$$\begin{aligned} V_{h_2} = {} & m_2^2 h_2 + B_\mu h_1 + \frac{3}{16\pi^2}\Bigg[h_t^2 h_2\Big(\mathcal{G}(m_{\tilde{t}_1}^2) + \mathcal{G}(m_{\tilde{t}_2}^2) - 2\mathcal{G}(m_t^2)\Big) \\ & + \frac{h_t A_t}{\sqrt{2}}\Big(\mathcal{G}(m_{\tilde{t}_1}^2) - \mathcal{G}(m_{\tilde{t}_2}^2)\Big)\Bigg] \end{aligned} \tag{18}$$

$$V_{h_1 h_1} = m_1^2 + \frac{3}{16\pi^2}\frac{h_t^2\mu^2}{2}\left[\ln\frac{m_{\tilde{t}_1}^2}{Q^2} + \ln\frac{m_{\tilde{t}_2}^2}{Q^2}\right] \tag{19}$$

$$\begin{aligned} V_{h_2 h_2} = {} & m_2^2 + \frac{3}{16\pi^2}\Bigg\{\left[h_t^2 h_2 + \frac{h_t A_t}{\sqrt{2}}\right]^2 \ln\frac{m_{\tilde{t}_1}^2}{Q^2} + \left[h_t^2 h_2 - \frac{h_t A_t}{\sqrt{2}}\right]^2 \ln\frac{m_{\tilde{t}_2}^2}{Q^2} \\ & + h_t^2\Big(\mathcal{G}(m_{\tilde{t}_1}^2) + \mathcal{G}(m_{\tilde{t}_2}^2) - 6E(m_t^2)\Big)\Bigg\} \end{aligned} \tag{20}$$

$$V_{h_1 h_2} = B_\mu + \frac{3}{16\pi^2}\left[\frac{(h_t^2\mu)(h_t h_2)}{\sqrt{2}}\left(\ln\frac{m_{\tilde{t}_1}^2}{Q^2} - \ln\frac{m_{\tilde{t}_2}^2}{Q^2}\right) + \frac{h_t^2\mu A_t}{2}\left(\ln\frac{m_{\tilde{t}_1}^2}{Q^2} + \ln\frac{m_{\tilde{t}_2}^2}{Q^2}\right)\right]\ , \tag{21}$$

where $\mathcal{G}(m^2) = m^2\Big(\ln\frac{m^2}{Q^2} - 1\Big)$ and $E(m^2) = m^2\Big(\ln\frac{m^2}{Q^2} - \frac{1}{3}\Big)$.

III. TWO LOOP CORRECTIONS

IV. NUMERICAL ANALYSIS

V. CONCLUSIONS

REFERENCES

[1] J. R. Espinosa and R. Zhang, Nucl. Phys. **B586** (2000) 3 [hep-ph/0003246].