The Rotation of the R-parity violating bilinear term in N=1 Supersymmetry with a minimal content of fields.

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We prove that once the billinear R-parity violating term is set to zero at some scale, it will never reappeared at any other one in perturbation theory. Consequently, we show how to rotate away the bilinear R-parity violating term in N=1 Supersymmetry with Standard Model (super)fields.

Consider the most general renormalizable, gauge invariant and holomorphic superpotential involving the indistinguisable lepton L_i, and Higgs H_1 superfields in an N=1 minimal supersymmetric model,

$$\begin{aligned} \mathcal{W} &= (Y_E)_{ij} L_i H_1 \bar{E}_j + (Y_D)_{ij} Q_i H_1 \bar{D}_j \\ &+ \frac{1}{2}(\Lambda_{E^k})_{ij} L_i L_j \bar{E}_k + (\Lambda_{D^k})_{ij} L_i Q_j \bar{D}_k \\ &+ \mu H_1 H_2 + \kappa_i L_i H_2 \ . \end{aligned} \tag{1}$$

The latin indices $i, j = 1, ..3$ are the flavour indices while the SU(2) and SU(3) indices are supressed. It is well known [1] that even if we set the billinear κ-term equal to zero this will be generated from loop corrections due to the appearence of the non-zero anomalous dimensions $\gamma_{H_1}^{L_i}$. The same also will happens for vanishing (at tree level) Yukawa couplings-this time due to the non-vanishing anomalous dimension, $\gamma_{L_i}^{L_j}, i \neq j$ in addition to $\gamma_{H_1}^{L_i}$. Thus it seems an "apparent" violation of the celebrated nonrenormalization theorem [2]. Of course this is not happend because in this framework the kinetic terms of the superfields are *not* canonically normalized. Thus, the most general Kahler potential involving the renormalized lepton superfileds L_i and the "Higgs" doublet H_1 can be written in the form,

$$K(L, L^*) = \sum_{\alpha,\beta=1}^{4} g_{\alpha\beta} L^{*\alpha} L^{\beta} \ , \tag{2}$$

with $\alpha = \{i, 4\}$ i.e., $L_4 \equiv H_1$. The matrix $g_{\alpha\beta}$ must be hermitian (because the action must be real) with a positive definite norm[4]. So that, the general form of $g_{\alpha\beta}$ to all orders in perturbation theory is

$$g_{\alpha\beta} = \begin{pmatrix} g_{ij} & g_{i4} \\ g_{i4}^* & g_{44} \end{pmatrix} , \tag{3}$$

with $g_{ij} = g_{ji}^*$ and $g_{44} \in R_+$. More explicitly, we can write this matrix in the form,

$$g_{\alpha\beta} = \begin{pmatrix} \delta_{ij} + \gamma_{ij} & \gamma_{i4} \\ \gamma_{i4}^* & 1 + \gamma_{44} \end{pmatrix} , \tag{4}$$

where $\gamma_{ij} = \gamma_{L_i}^{L_j}$, $\gamma_{i4} = \gamma_{L_i}^{H_1} = \gamma_{H_1}^{L_i *}$ and $\gamma_{44} = \gamma_{H_1}^{H_1}$ are the wave function renormalization matrices. Since $g_{\alpha\beta}$ is a hermitian matrix it can always be diagonilized by a matrix N,

$$(N^\dagger g N)_{\alpha\beta} = \delta_{\alpha\beta} \ . \tag{5}$$

This can be done by decomposing N into a unitary matrix $U_{\alpha\beta}$ which diagonilizes the hermitian $g_{\alpha\beta}$ times a diagonal matrix whose elements are the inverse square roots of the eigenavalues ω_α of $g_{\alpha\beta}$, i.e, $N_{\alpha\beta} = \frac{U_{\alpha\beta}}{\sqrt{\omega_\alpha}}$, (no sum over α). The diagonilization of the metric means also a transformation of the superfields,

$$L_i = \sum_\alpha N_{i\alpha} \mathcal{L}_\alpha \ , \tag{6}$$

$$L_4 = \sum_\alpha N_{4\alpha} \mathcal{L}_\alpha \ . \tag{7}$$

under which the Kahler potential is diagonal, i.e. $K(L, L^*) = \sum_\alpha \mathcal{L}_\alpha^* \mathcal{L}_\alpha$. The renormalized superpotential (1) also changes into

$$\begin{aligned} \mathcal{W} = \frac{1}{2}\Big[(Y_E)_{ik} N_{i\alpha} N_{4\beta} - (Y_E)_{ik} N_{i\beta} N_{4\alpha} + (\Lambda_{E^k})_{ij} N_{i\alpha} N_{j\beta}\Big] \mathcal{L}_\alpha \mathcal{L}_\beta \bar{E}_k + \Big[-(Y_D)_{jk} N_{4\alpha} + (\Lambda_{D^k})_{ij} N_{i\alpha}\Big] \mathcal{L}_\alpha Q_j \bar{D}_k \\ + \Big[\mu N_{4\alpha} + \kappa_i N_{i\alpha}\Big] \mathcal{L}_\alpha H_2 \ , \end{aligned} \tag{8}$$

Now from (8) and at 1-loop order the anomalous dimension for the superfields $\mathcal{L}_\alpha$ and $\mathcal{L}_\beta$ with $\alpha \neq \beta$ is given by (we use here the relation $\sum_\gamma N^*_{\gamma\alpha}N_{\gamma\beta} = \frac{\delta_{\alpha\beta}}{Tr(\omega)}$ which is followed from the fact that U is unitary in the decomposition of the matrix N)

$$
\begin{aligned}
16\pi^2\gamma_{\mathcal{L}_\alpha}^{\mathcal{L}_\beta(1-loop)} &= 3\Big[Tr(Y_DY_D^\dagger)N_{4\alpha}N^*_{4\beta} - (Y_D)_{jk}(\Lambda^*_{D^k})_{lj}N_{4\alpha}N^*_{l\beta} - (Y^*_D)_{jk}(\Lambda_{D^k})_{ij}N_{i\alpha}N^*_{4\beta} + (\Lambda_{D^k}\Lambda^\dagger_{D^k})_{il}N_{i\alpha}N^*_{l\beta}\Big] \\
&+ \frac{1}{Tr(\omega)}\Big[(Y_EY_E^\dagger)_{il}N_{i\alpha}N^*_{l\beta} + Tr(Y_EY_E^\dagger)N_{4\alpha}N^*_{4\beta} + (\Lambda_{E^k}\Lambda^\dagger_{E^k})_{il}N_{i\alpha}N^*_{l\beta} \\
&- (\Lambda^*_{E^k}Y_E)_{lk}N_{4\alpha}N^*_{l\beta} - (\Lambda_{E^k}Y^*_E)_{lk}N_{l\alpha}N^*_{4\beta}\Big] \; . \qquad (9)
\end{aligned}
$$

But the similarity transformation (5) implies that for $(\alpha \neq \beta)$

$$
\sum_{ij} N^*_{i\alpha}\gamma_{ij}N_{j\beta} + \sum_i N^*_{i\alpha}\gamma_{i4}N_{4\beta} + \sum_i N^*_{4\alpha}\gamma^*_{i4}N_{i\beta} + \gamma_{44}N^*_{4\alpha}N_{4\beta} = 0 \qquad (10)
$$

Substituting the explicit formulae for the anomalous dimensions of the superfields appeared in the superpotential (1) at 1-loop from Ref. [1] and for $(i \neq j)$

$$
16\pi^2\gamma_{ij} = (Y_EY_E^\dagger)_{ji} + (\Lambda_{E^q}\Lambda^\dagger_{E^q})_{ji} + 3(\Lambda_{D^q}\Lambda^\dagger_{D^q})_{ji} \; , \qquad (11)
$$

$$
16\pi^2\gamma_{i4} = 16\pi^2\gamma^*_{4i} = -3(\Lambda^*_{D^q}Y_D)_{iq} - (\Lambda^*_{E^q}Y_E)_{iq} \; , \qquad (12)
$$

into (10) we get

$$
\gamma_{\mathcal{L}_\alpha}^{\mathcal{L}_\beta(1-loop)} = 0 \quad (\alpha \neq \beta) \; . \qquad (13)
$$

Since this is valid at 1-loop it valids also to *all* orders in perturbation theory because of the non-renormalization theorem. One could also see that neither of μ or κ_i could be generated in the superpotential. Their RGE's are now decoupled

$$
\frac{d\mu}{dt} = \mu\left[\gamma^{\mathcal{H}_1}_{\mathcal{H}_1} + \gamma^{H_2}_{H_2}\right] \; , \qquad (14)
$$

$$
\frac{d\kappa_i}{dt} = \kappa_i\left[\gamma^{\mathcal{L}_i}_{\mathcal{L}_i} + \gamma^{H_2}_{H_2}\right] \; , \qquad (15)
$$

where $\mathcal{H}_1 \equiv \mathcal{L}_4$ is the Higgs doublet responsible in giving masses to down quarks and leptons. Thus the billinear κ-term can be set to zero at some scale and it will never reappeared at any other one. Note that, soft supersymmetry breaking *does not* affect the proof since the non-renormalization theorem is still valid and thus the superfield wave functions do not depend on the soft breaking parameters.

One should notice that the superpotential (8) has the same form like in (1) subject to the constraint (13) and $L \to \mathcal{L}$ and $\Lambda \to \Lambda', \kappa \to \kappa', \mu \to \mu'$. We show that it is possible to rotate away the bilinear interactions $\kappa'_i\mathcal{L}_iH_2$ by a unitary transformation thus preserving the canonical form of the Kähler potential. In order to do this we introduce new fields L'_1, L'_2, L'_3, and H'_1. The old fields L_1, L_2, L_3, H_1 are given in terms of the new fields by the rotation

$$
\mathcal{L}_\alpha = \mathcal{O}_{\alpha\beta}\mathcal{L}'_\beta \; , \qquad (16)
$$

where

$$
\begin{pmatrix} \mathcal{L}_1 \\ \mathcal{L}_2 \\ \mathcal{L}_3 \\ \mathcal{L}_4 \end{pmatrix} = \begin{pmatrix} c_2c_3 & -s_2 & 0 & c_2s_3 \\ c_1s_2c_3 & c_1c_2 & -s_1 & c_1s_2s_3 \\ s_1s_2c_3 & s_1c_2 & c_1 & s_1s_2s_3 \\ -s_3 & 0 & 0 & c_3 \end{pmatrix} \begin{pmatrix} \mathcal{L}'_1 \\ \mathcal{L}'_2 \\ \mathcal{L}'_3 \\ \mathcal{L}'_4 \end{pmatrix} \qquad (17)
$$

where $c_i = \cos\theta_i$ and $s_i = \theta_i$ are rotation angles. They are given explicitly by

$$
\begin{aligned}
c_1 &= \frac{\kappa'_2}{\sqrt{\kappa'^2_2 + \kappa'^2_3}}, \quad & s_1 &= \frac{\kappa'_3}{\sqrt{\kappa'^2_2 + \kappa'^2_3}} \\
c_2 &= \frac{\kappa'_1}{\sqrt{\vec{\kappa'}^2}}, \quad & s_2 &= \frac{\sqrt{\kappa'^2_2 + \kappa'^2_3}}{\sqrt{\vec{\kappa'}^2}} \qquad (18) \\
c_3 &= \frac{\mu}{\sqrt{\mu^2 + \vec{\kappa'}^2}}, \quad & s_3 &= \frac{\sqrt{\vec{\kappa'}^2}}{\sqrt{\mu^2 + \vec{\kappa'}^2}}
\end{aligned}
$$

where we have introduced the notation $\vec{\kappa'}^2 = \kappa'^2_1 + \kappa'^2_2 + \kappa'^2_3$. After the above field redefinition the only remaining superfield bi-linear term is

$$
\mu'' H'_1 H_2 \qquad (19)
$$

with $\mu'' = \sqrt{\mu'^2 + \vec{\kappa'}^2}$ and $H'_1 \equiv \mathcal{L}'_4$. Notice that the orthogonal transformation (17) is not the Hall-Suzuki [3] transformation. It is easy to show that the constraint (13) is not violated by the transformation (17). From [1] and from (13) we obtain at 1-loop :

$$
\begin{aligned}
\gamma^{\mathcal{L}'_\beta}_{\mathcal{L}'_\alpha} &\propto Tr(\Lambda''\Lambda''^\dagger) = Tr(\Lambda'\mathcal{O}\mathcal{O}^\dagger\Lambda'^\dagger) \\
&= Tr(\Lambda'\Lambda^{\dagger'}) = \gamma^{\mathcal{L}_\beta}_{\mathcal{L}_\alpha} = 0 \qquad (20)
\end{aligned}
$$

where $\Lambda^{''} \equiv (\Lambda^{''}_{\mathcal{L}^\alpha})_{Q_j \bar{D}_k}$ or $\Lambda^{''} \equiv (\Lambda^{''}_{\mathcal{L}^\alpha})_{\mathcal{L}_\beta \bar{E}_k}$ are the new superpotential couplings after the transformation (17). Thus one can rotate away the billiear R-parity violating term to all orders in prturbation theory and that completes the proof. In what follows we prove that it is possible to rotate at the same time away the R-parity violating bilinear superpotential term toghether with the R-parity violating soft SUSY breaking bilinear term in the Lagrangian. Thus, the bilinear terms (we remove the primes now) are written in the form

$$\mathcal{L}^{bil} = \int d^2\theta \Big\{ \mu \mathcal{L}_4 H_2 + \kappa_i \mathcal{L}_i H_2 \Big\} + \text{H.c} + \int d^2\theta \eta \Big\{ \widetilde{B} \mathcal{L}_4 H_2 + \widetilde{D} \mathcal{L}_i H_2 \Big\} + \text{H.c} \ , \tag{21}$$

where $\eta \equiv \theta^2$ is the spurion superfield and $\theta_\alpha, a = 1, 2$ (as well as $\bar{\theta}$) are the spinorial Gransmann components of the superspace. The transformation which rotates away the terms proportional to κ_i and $\widetilde{D}$ is the orthogonal matrix (17) with the replacement in (19)

$$\begin{aligned} \mu &\to \mu - \eta \widetilde{B} \\ \kappa_i &\to \kappa_i - \eta \widetilde{D}_i \ . \end{aligned} \tag{22}$$

Therefore, we end up with the Lagrangian of the form

$$\mathcal{L}^{'bil} = \int d^2\theta \Big\{ \mu' \mathcal{L}'_4 H_2 - \eta \widetilde{B}' \mathcal{L}'_4 H_2 \Big\} + \text{H.c} \ , \tag{23}$$

where[5]

$$\begin{aligned} \mu' &= \sqrt{\mu^2 + \vec{\kappa}^2} \ , \\ \widetilde{B}' &= \frac{\mu \widetilde{B} + \vec{\kappa} \cdot \widetilde{D}}{\sqrt{\mu^2 + \vec{\kappa}^2}} \ . \end{aligned} \tag{24}$$

[1] B. C. Allanach, A. Dedes and H. K. Dreiner, Phys. Rev. **D60**, 056002 (1999) [hep-ph/9902251].

[2] M. T. Grisaru, W. Siegel and M. Rocek, Nucl. Phys. **B159** (1979) 429; N. Seiberg, Phys. Lett. **B318** (1993) 469 [hep-ph/9309335].

[3] L. J. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984). The transformation presented in this paper is $U = \begin{pmatrix} 1 & -\delta_i \\ \delta_i & I \end{pmatrix}$ where it is assumed that $\delta_i \equiv \frac{\kappa_i}{\mu}$ is less than a unity. The transformation is unitary only to the first order in δ_i, and thus does not preserve the Kähler metric in general.

[4] This is required to make the canonical commutation relations consistent and it is not an assumption.

[5] We have made use of the relation : $\sqrt{a + b\eta} = \sqrt{a} + \eta \frac{b}{2\sqrt{a}}$.