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2-Loop Supersymmetric Renormalization Group Equations Including R-Parity Violation and Aspects of Unification

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Summary :

1. Introduction
2. Gauge coupling Unification
3. 2-loop bounds from perturbative Unification
4. b-τ Yukawa Unification
5. Infrared Fixed point structure
6. Conclusions

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Introduction

• The conservation of the baryon and lepton number is an automatic consequence of the gauge invariance and renormalizibility in the SM

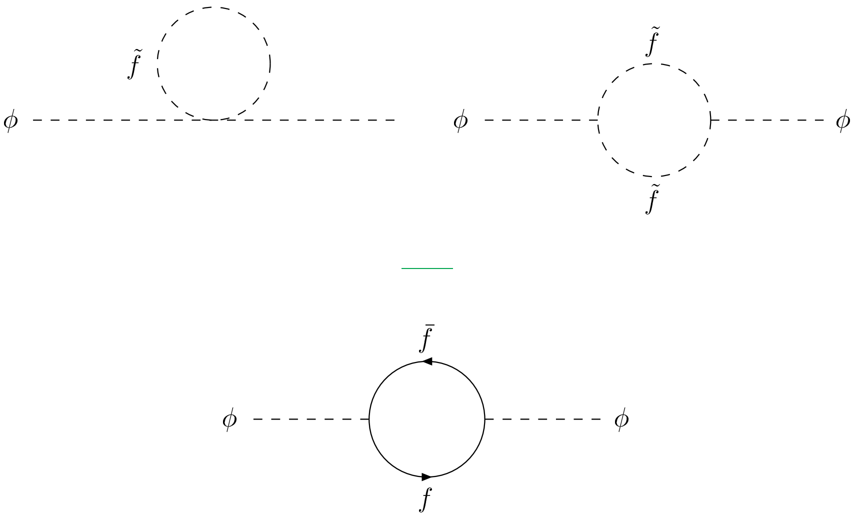
• B and L non-conservation is generally considered in the context of GUT Models. In this case their effects are suppressed by powers of the $M_{GUT} \sim 2 \times 10^{16}$ GeV.

• In Supersymmetric extensions of the SM, gauge invariance, renormalizability and analyticity of the superpotential no longer assure B and L number violation.

Supersymmetry

- Supersymmetry is a symmetry which relates particles of different spins. It has a virtue of allowing non trivial interactions between fermions and bosons.

- It is possible for the quadratic divergences to cancel between the bosonic and fermionic loops. To insure that this cancellation persists to all orders we need Supersymmetry.



- Supersymmetry is certainly not a symmetry in nature: for example, there is certainly no bosonic particle with the mass and charge of the electron. Supersymmetry must be broken at a scale of the electroweak symmetry breaking in order to keep in touch with the solution of the hierarchy problem.

Minimal SuperSymmetric Standard Model

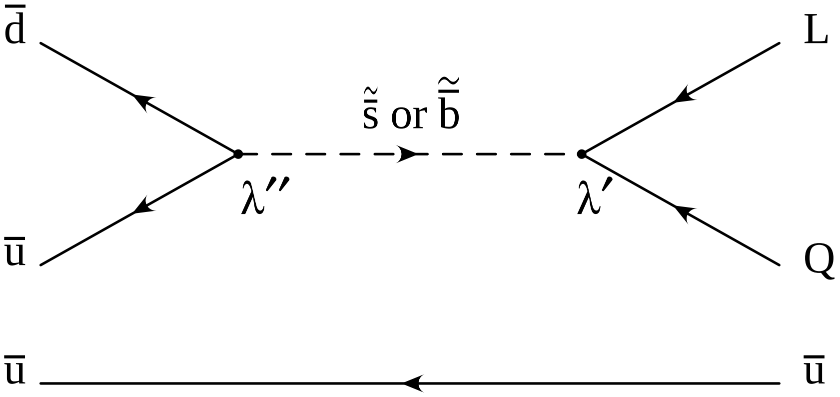
Superfields	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Components
$Q_r^{i,\alpha}$	$(\mathbf{3},\mathbf{2},\frac{1}{6})$	$\begin{pmatrix}\tilde{u}\\ \tilde{d}\end{pmatrix}_L \,,\, \begin{pmatrix}u\\ d\end{pmatrix}_L$
$D_{r,\alpha}^c$	$(\bar{\mathbf{3}},\mathbf{1},\frac{1}{3})$	$\tilde{d}^c \,,\, d^c$
$U_{r,\alpha}^c$	$(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})$	$\tilde{u}^c \,,\, u^c$
L_r^i	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$\begin{pmatrix}\tilde{\nu}_e\\ \tilde{e}\end{pmatrix}_L \,,\, \begin{pmatrix}\nu_e\\ e\end{pmatrix}_L$
E_r^c	$(\mathbf{1},\mathbf{1},1)$	$\tilde{e}^c \,,\, e^c$
H_1^i	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$\begin{pmatrix}\tilde{H}_1^0\\ \tilde{H}_1^-\end{pmatrix} \,,\, \begin{pmatrix}H_1^0\\ H_1^-\end{pmatrix}$
H_2^i	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$\begin{pmatrix}\tilde{H}_2^+\\ \tilde{H}_2^0\end{pmatrix} \,,\, \begin{pmatrix}H_2^+\\ H_2^0\end{pmatrix}$

Superfields	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Components
$V_3^{(R)}$	$(\mathbf{8},\mathbf{1},0)$	$\tilde{G}^{(R)} \,,\, G_\mu^{(R)}$
$V_2^{(\Gamma)}$	$(\mathbf{1},\mathbf{3},0)$	$\tilde{W}^{(\Gamma)} \,,\, W_\mu^{(\Gamma)}$
V_1	$(\mathbf{1},\mathbf{1},0)$	$\tilde{B} \,,\, B_\mu$

Superpotential

$$\begin{aligned} W \;=\; & \epsilon_{ab}\left[(\mathbf{Y}_E)_{ij}L_i^a H_1^b \bar{E}_j + (\mathbf{Y}_D)_{ij}Q_i^{ax}H_1^b\bar{D}_{jx} + (\mathbf{Y}_U)_{ij}Q_i^{ax}H_2^b\bar{U}_{jx}\right] \\ & + \epsilon_{ab}\,\mu H_1^a H_2^b \\ & + \epsilon_{ab}\left[\frac{1}{2}\lambda_{ijk}L_i^a L_j^b \bar{E}_k + \lambda'_{ijk}L_i^a Q_j^{xb}\bar{D}_{kx}\right] \qquad \rightarrow \not{L} \text{ violation} \\ & + \frac{1}{2}\epsilon_{xyz}\lambda''_{ijk}\bar{U}_i^x\bar{D}_j^y\bar{D}_k^z \qquad \rightarrow \not{B} \text{ violation} \\ & + \epsilon_{ab}\left[\kappa^i L_i^a H_2^b\right] \qquad \rightarrow \not{L} \text{ violation.} \end{aligned}$$

$$\Downarrow$$



Proton Decay!

- B number is violated by 9 λ''_{ijk} couplings
- L bumber is violated by 9 λ_{ijk} + 27 λ'_{ijk} + 3 κ_i couplings

R-Parity Symmetry

• In 1978 Fayet and Farrar proposed a discrete multiplicative symmetry which is called R-Parity

$$R = (-1)^{3B+L+2S}$$

B : Baryon number, L : Lepton number, S : Spin

• R-Parity automatically guarantees the B and L conservation

• It acts like

R(SM like particles)=+1 , R(SUSY particles)=-1

• The ad hoc imposition of the R-parity is not theoretically well motivated (R-parity is imposed by hand!), i.e., not sufficient for suppressing all the dangerous B and L violating terms.

⇒Example : If we consider the MSSM as an effective field theory, then,

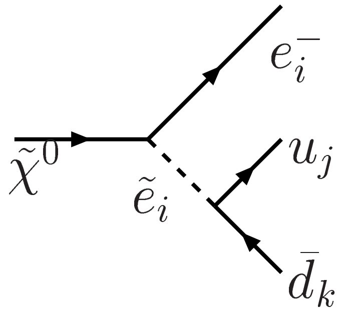
$$\mathcal{W} = \ldots + \left(\frac{k_{ijkl}}{\Lambda}\right)(Q_i Q_j)(Q_k L_l) + \ldots.$$

which respects R-parity but not B and L number!!

•The presence of the R-Parity violating terms produces several changes in the phenomenology of the MSSM

1. Lepton or Baryon number is violated

2. The LSP is not stable and can decay in the detector. It is no longer a dark matter candidate



$$\Gamma_{\tilde{\chi}^0} = \frac{3\alpha_{em}}{128\pi^2}\lambda'^2_{121}\frac{M^5_{\tilde{\chi}^0}}{m_{\tilde{e}^4}} \Rightarrow \lambda'_{121} > 1.4\times 10^{-6}$$

$$\text{for} \quad m_{\tilde{e}} = 200 \text{ GeV} \quad \text{and} \quad m_{\tilde{\chi}^0} = 100 \text{ GeV}$$

Direct bound from ALEPH+OPAL exp.

If there are few individual couplings, strong limits exist i.e., $\lambda'_{ijk}\times\lambda''_{lmn} \leq 10^{-10}$ $\Rightarrow$all the experiments assume one $R\!\!\!/_p$ coupling at a time.

3. The neutralino is not necessarily the LSP

$$\text{LSP} \in \left\{\tilde{\chi}^0_1, \tilde{\chi}^\pm_1, \tilde{g}, \tilde{t}, \tilde{l}, \tilde{\nu}\right\}$$

In each case the collider phenomenology can be distinct

4. The single production of supersymmetric particles is possible (ALEPH)

There are resonant single production mechanisms i.e.,

$$e^+ + e^- \to \tilde{\nu}_j \text{ through } L_1 L_j E_1^c$$

$$e^- + u_j \to \tilde{d}_{Rk}^c \text{ through } L_1 Q_j D_k^c$$

Other bounds

- Neutrino Mass

$$\delta m_{\nu_e} \simeq \frac{\lambda_{133}^2}{8\pi^2}\frac{m_\tau^2}{m_{\tilde{\tau}}} \qquad \overset{\delta m_{\nu_e} \lesssim 5\text{eV}}{\Longrightarrow} \quad \lambda_{133} < 0.003$$

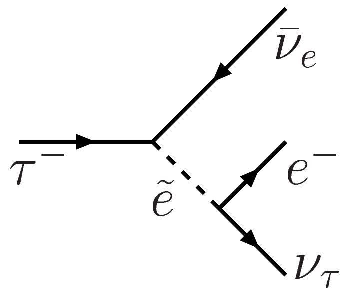
- Neutrinoless double beta decay, $\nu_\mu - e$ scattering , atomic parity violation, Quark mixings, LEP precision measurements....

Indirect Bounds

The $\not{R}_p$ interactions can contribute to various processes through the virtual exchange of SUSY particles.

Example :

The operator $L_1 L_3 E_k^c$ can contribute to the decay $\tau \to e\nu\bar{\nu}$ via the diagram



For large slepton masses, $m_{\tilde{e}_R}$ this interaction is described by the effective 4-fermion Lagrangian

$$\mathcal{L}_{eff} \;=\; \frac{|\lambda_{13k}|^2}{2m_{\tilde{e}}^2}\left(\bar{e}_L\gamma^\mu\nu_{e_L}\right)\left(\bar{\nu}_\tau\gamma_\mu\tau_L\right)$$

$\to$ contributions to the Fermi constant, G_F.

$$R_\tau \equiv \frac{\Gamma(\tau\to e\nu\bar{\nu})}{\Gamma(\tau\to\mu\nu\bar{\nu})}$$

The contribution from $\not{R}_p$relative to the SM contributions is

$$R_\tau = R_\tau(SM)[1 + 2\frac{M_W^2}{g_W^2}\frac{|\lambda_{13k}|^2}{m_{\tilde{e}_{Rk}}^2}]$$

By making use of the experimental value

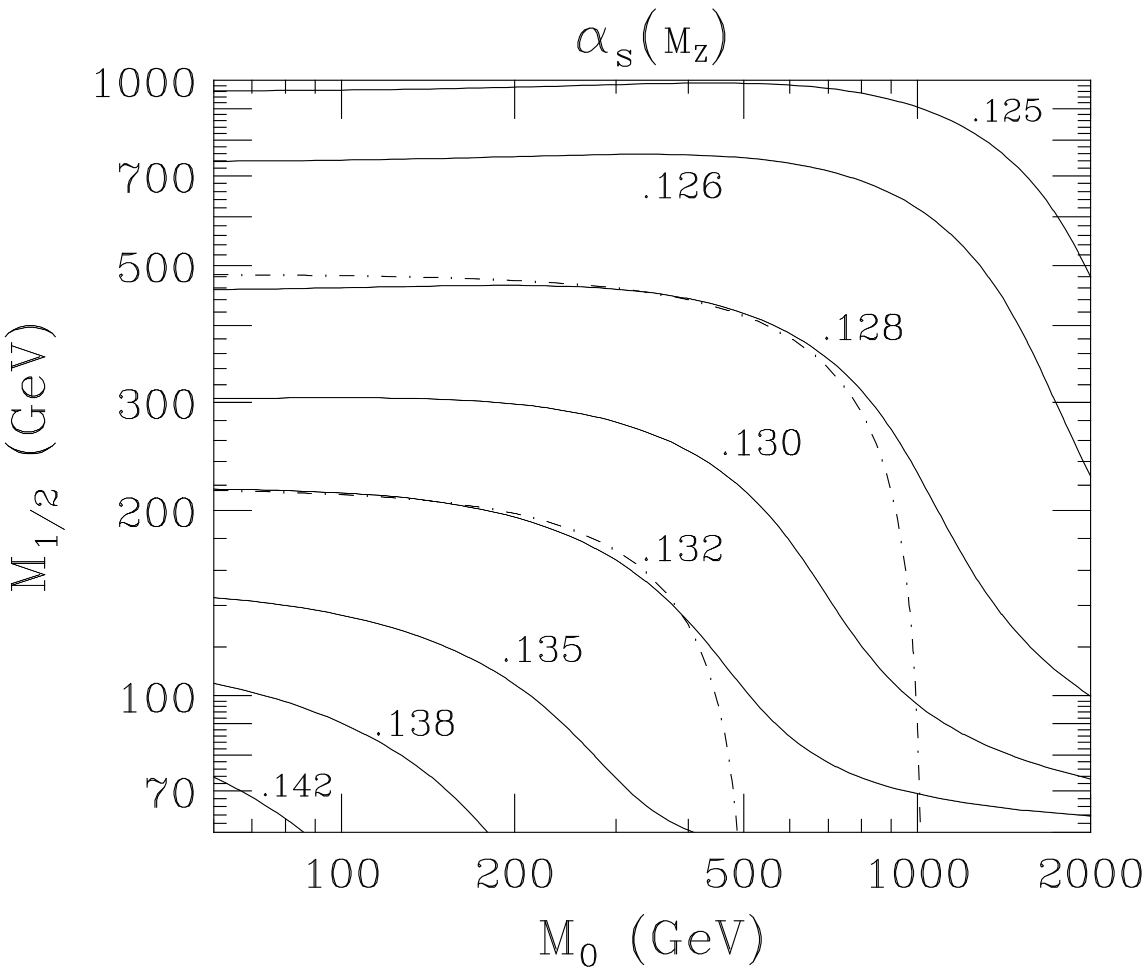
$$\frac{R_\tau}{R_\tau(SM)} = 1.0006 \pm 0.0103 \Rightarrow |\lambda_{13k}| < 0.06 \left(\frac{m_{\tilde{e}_{Rk}}}{100\ \mathrm{GeV}}\right)$$

Similarly, for $m_{slepton} = 100$ GeV

$$\lambda_{323} < 0.06$$
$$\lambda'_{333} < 0.26\ , \left(R_l = \frac{\Gamma_{had}(Z^0)}{\Gamma_l(Z^0)}\right)$$
$$\lambda''_{323} < 0.43$$

Gauge coupling Unification in the MSSM

• 2 loop + thresholds from SUSY particles



$\Rightarrow$ but $\alpha_s^{exp} = 0.119 \pm 0.002 \Rightarrow$ We need a GUT model !!

• Minimal SU(5) model : Large $\alpha_s > 0.13$ • Missing Doublet model based on SU(5) gauge group : Small $\alpha_s < 0.12$ • Flipped SU(5) and SO(10) correct values of α_s

2-loop RGE's including R-parity violation ..

• We have found the complete 2-loop RGE's of the superpotential parameters for the MSSM including the full set of R-parity violating superpotential couplings. We used these equations to do a study of

a) Gauge coupling Unification

b) 2-loop bounds on the $\not{R}_p$couplings from perurbative unification

c) $b - \tau$ Yukawa Unification

d) (Quasi) Fixed Point structure of the Yukawa couplings

RGE's

Example: Y_E The RGE for the 'electron' Yukawa couplings (including full family dependence) is given by

$$\frac{d}{dt}(\mathbf{Y}_E)_{ij} = (\mathbf{Y}_E)_{ik}\Gamma^{E_j}_{E_k} + (\mathbf{Y}_E)_{ij}\Gamma^{H_1}_{H_1} - \lambda_{kij}\Gamma^{H_1}_{L_k} + (\mathbf{Y}_E)_{kj}\Gamma^{L_i}_{L_k}$$

Γ^j_i are the anomalous 1- and 2-loop dimensions.

Bilinear terms

$$\begin{aligned}
\frac{d}{dt}\mu &= \mu\left\{\Gamma^{H_1}_{H_1} + \Gamma^{H_2}_{H_2}\right\} + \kappa^i \Gamma^{H_1}_{L_i}, \\
\frac{d}{dt}\kappa^i &= \kappa^i \Gamma^{H_2}_{H_2} + \kappa^p \Gamma^{L_i}_{L_p} + \mu \Gamma^{L_i}_{H_1}.
\end{aligned}$$

Assumptions

In order to follow the experimental bounds, we consider one coupling at a time:

$$\mathbf{LL\bar{E}} : \ \lambda_{323}, \quad \mathbf{LQ\bar{D}} : \ \lambda'_{333}, \quad \mathbf{\bar{U}\bar{D}\bar{D}} : \ \lambda''_{323},$$

$\Rightarrow$ less constraint bounds from experiment

Gauge coupling Unification in the MSSM+$\not{R}_p$

In the case of the MSSM without R-parity the effect of the new couplings comes at the 2-loop level :

$$\begin{aligned}
\frac{d}{dt} g_a &= \frac{g_a^3}{16\pi^2} B_a^{(1)} + \frac{g_a^3}{(16\pi^2)^2} \left[\sum_{b=1}^{3} B_{ab}^{(2)} g_b^2 \right. \\
&- \left. \sum_{x=u,d,e} \left(C_a^x \mathrm{Tr}(\mathbf{Y}_x^\dagger \mathbf{Y}_x) + A_a^x \sum_{i=1}^{3} \mathrm{Tr}(\mathbf{\Lambda}_{x_i}^\dagger \mathbf{\Lambda}_{x_i}) \right) \right] .
\end{aligned}$$

$$\Delta \left(\frac{d}{dt} g_a \right) \equiv \frac{d}{dt} g_a^{\not{R}_p - MSSM} - \frac{d}{dt} g_a^{R_p - MSSM}$$

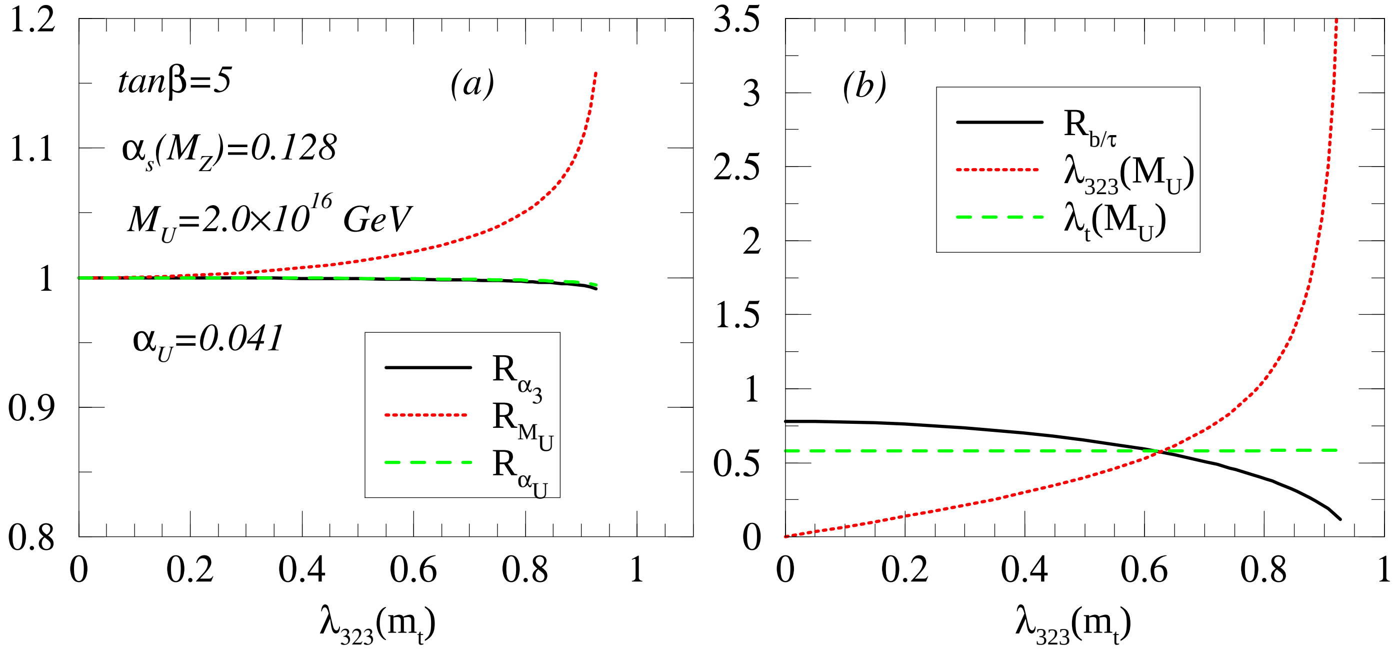
where

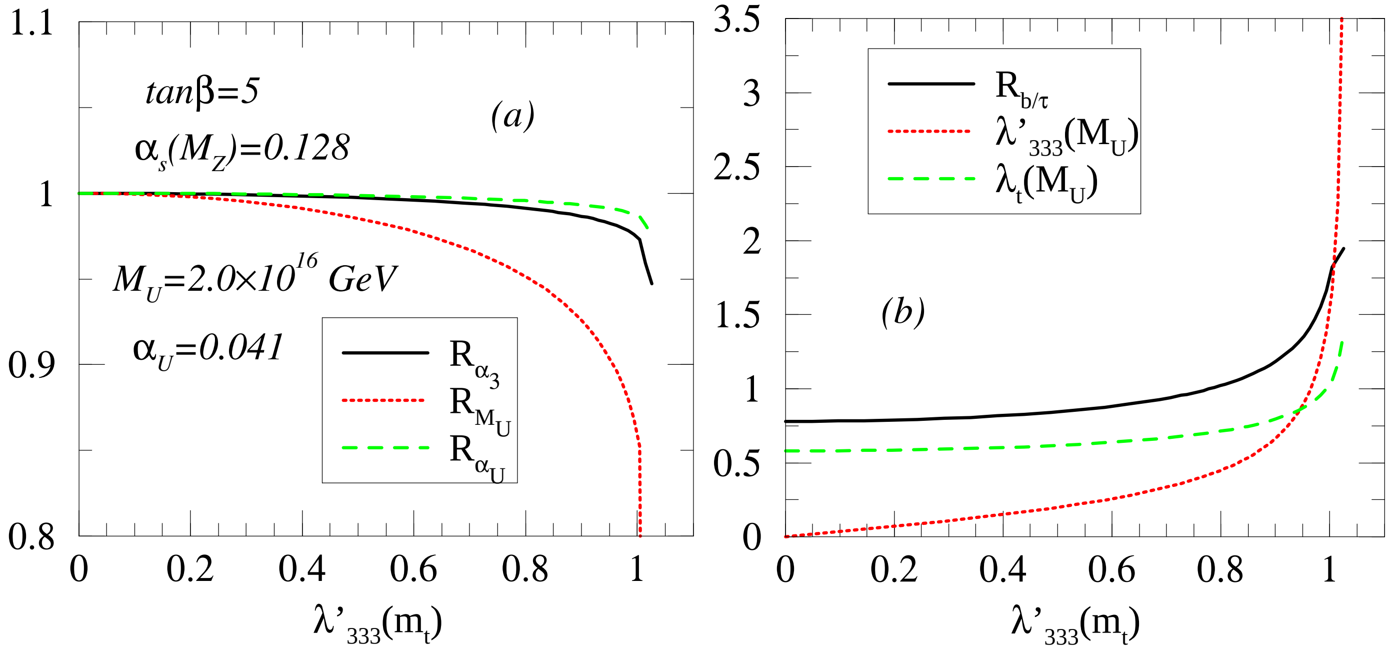
$$\begin{aligned}
\Delta \left(\frac{d}{dt} g_3 \right) &= -\frac{g_3^3}{(16\pi^2)^2} \left[6\lambda''^2_{323} + 4\lambda'^2_{333} \right] \\
\Delta \left(\frac{d}{dt} g_2 \right) &= -\frac{g_2^3}{(16\pi^2)^2} \left[6\lambda'^2_{333} + 2\lambda^2_{323} \right] \\
\Delta \left(\frac{d}{dt} g_1 \right) &= -\frac{g_1^3}{(16\pi^2)^2} \left[\frac{24}{5}\lambda''^2_{323} + \frac{14}{5}\lambda'^2_{333} + \frac{18}{5}\lambda^2_{323} \right] .
\end{aligned}$$

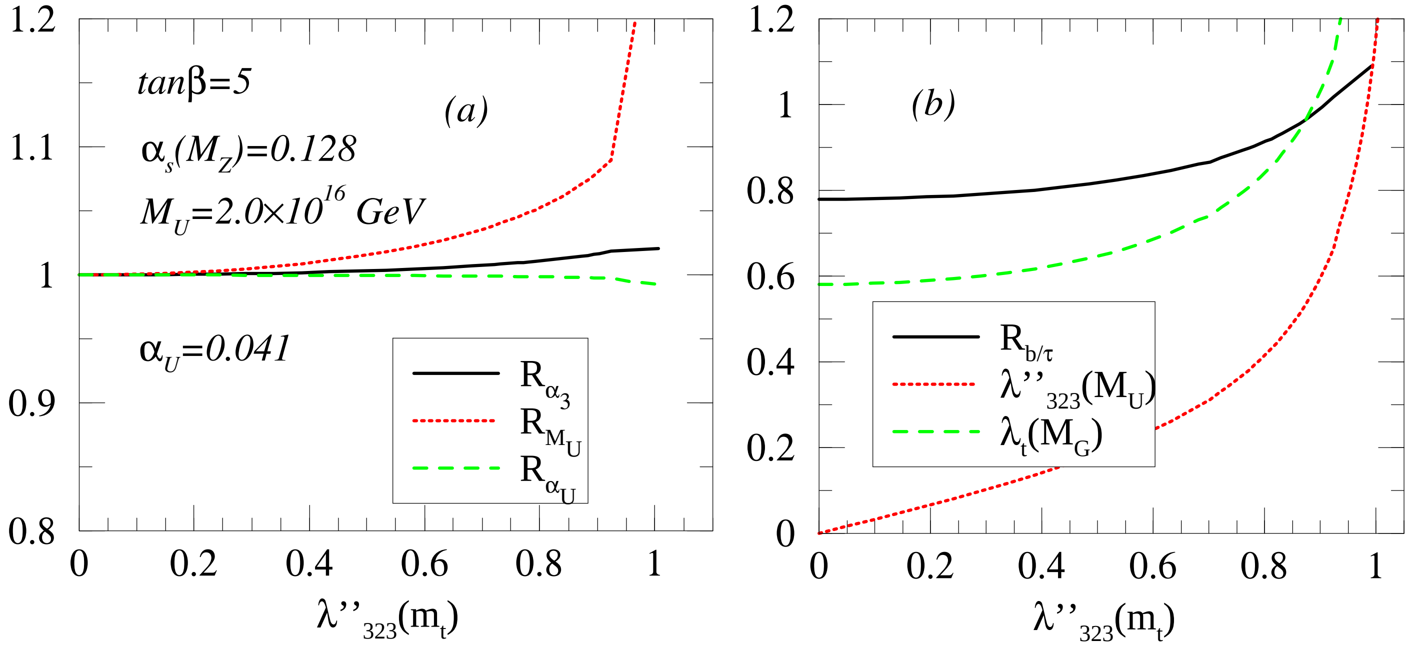
Numerical Results

- Definitions

$$
\begin{aligned}
R_{\alpha_3}(\lambda_{\not R_p}) &= \frac{\alpha_3(M_Z, \lambda_{\not R_p})}{\alpha_3(M_Z, 0)}, \\
R_{M_U}(\lambda_{\not R_p}) &= \frac{M_U(\lambda_{\not R_p})}{M_U(0)}, \\
R_{\alpha_U}(\lambda_{\not R_p}) &= \frac{\alpha_U(\lambda_{\not R_p})}{\alpha_U(0)}.
\end{aligned}
$$




tanβ=5
α_s(M_Z)=0.128
M_U=2.0×10^16 GeV
α_U=0.041
(a)
R_α3
R_MU
R_αU
λ'_333(m_t)
(b)
R_b/τ
λ'_333(M_U)
λ_t(M_U)
λ'_333(m_t)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R_MU
R_αU
λ''_323(m_t)
(b)
R_b/τ
λ''_323(M_U)
λ_t(M_G)
λ''_323(m_t)

$\boxed{\alpha_s \,:\, \text{decreases by } 5\% \text{ from } \lambda'_{333}}$

$$\alpha_s^{MSSM}{=}0.128 \Rightarrow \alpha_s^{MSSM+\not{R}_p}{=}0.122$$

$$\downarrow$$

1.5σ from the experimental value!

$\boxed{\alpha_U \,:\, \text{remains unchanged from } \not{R}_p}$

$\boxed{M_U \,:\, 20\ \% \text{ up by } \lambda''_{323} \text{ but not string scale unification!}}$

Bounds from perturbative Unification

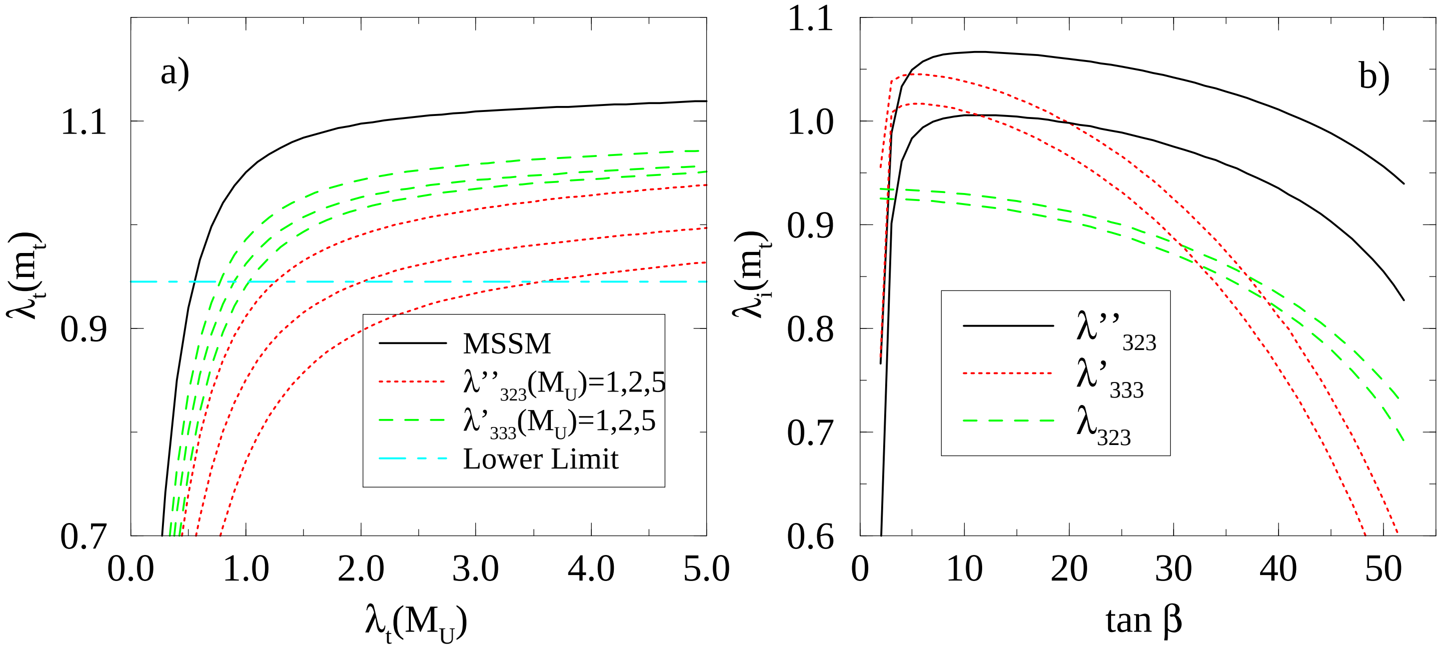
- Perturbative limit : $\frac{\lambda^2}{4\pi} \lesssim 1|_{M_U} \Rightarrow \lambda \lesssim 3.5$ at M_U

$$\boxed{\begin{array}{c} \lambda_{323}(m_t) \lesssim 0.93 \\ \lambda'_{333}(m_t) \lesssim 1.06 \\ \lambda''_{323}(m_t) \lesssim 1.07 \\ \text{at } \tan\beta = 5. \end{array}}$$

It is worth pointing out that this is the same as the "laboratory" bound for slepton masses at 1.5 TeV!

For heavy spectrum the "laboratory" bounds are no stricter than the perturbative limits

Do the perturbativity bounds depend on the $\tan\beta$?



• Two loop effects on the bounds :

$$\lambda_{323} \to 5\%$$
$$\lambda'_{333} \to 12\%$$
$$\lambda''_{323} \to 10\%$$

b-τ Unification

•Does the model feel the existence of an SU(5) model ?

$$R_{b/\tau}(M_U) = \frac{\lambda_b(M_U, \lambda_{\not{R}_p})}{\lambda_\tau(M_U, \lambda_{\not{R}_p})}.$$

For $\lambda_{\not{R}_p} = 0$, $\tan\beta = 5$ we have

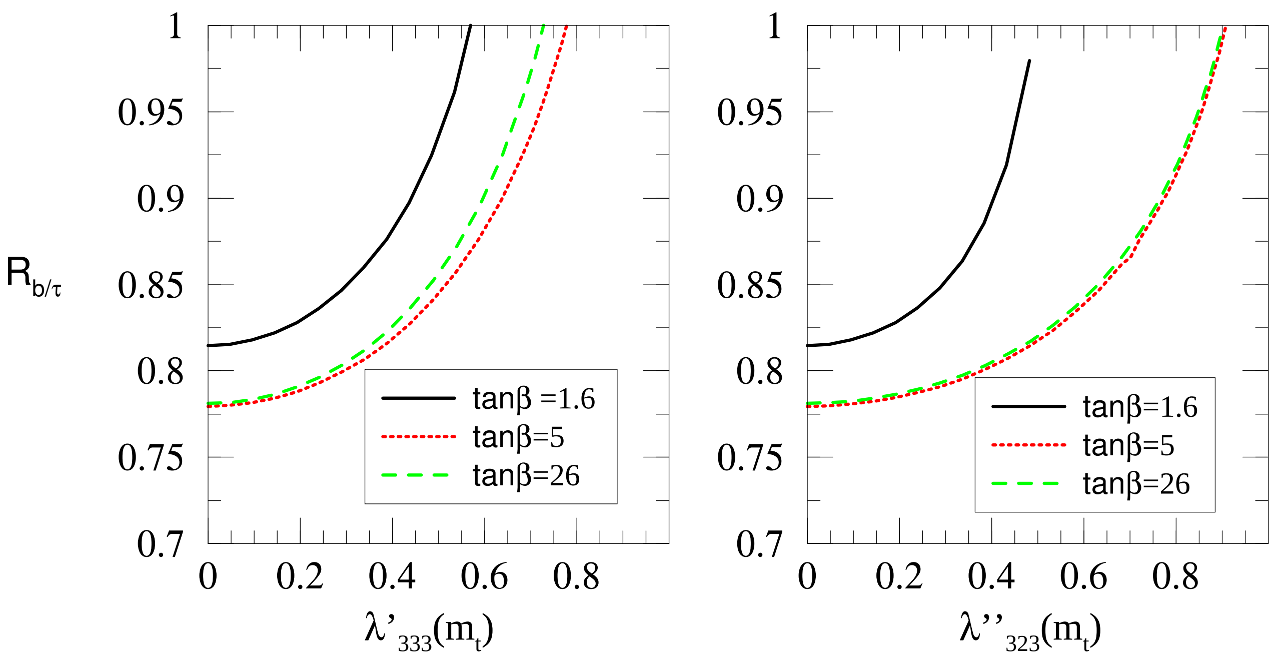
$$R_{b/\tau}(M_U) = 0.78.$$

$\rightarrow$ we have to go to very high values of $\tan\beta \sim 40$!

$$\begin{aligned} 16\pi^2 \frac{dR_{b/\tau}(t)}{dt} &= R_{b/\tau}(t) \Big[\lambda_t^2 + 3\lambda_b^2 - 3\lambda_\tau^2 - 4\lambda_{323}^2 + 3\lambda'^2_{323} + 2\lambda''^2_{323} \\ &+ \frac{4}{3} g_1^2 - \frac{16}{3} g_3^2 \Big]. \end{aligned}$$

• λ_{323} strongly reduces $R_{b/\tau}$ and thus can not lead to $b - \tau$ unification

Only the case with $\lambda' \neq 0$ or $\lambda'' \neq 0$ works!



$\boxed{b-\tau \text{ unification for } \lambda'_{333} = 0.8 \text{ and for every value of } \tan\beta}$

$\boxed{b-\tau \text{ unification for } \lambda''_{323} = 0.9 \text{ and for every value of } \tan\beta}$

- Unification of all Yukawa couplings ?

$$\boxed{\lambda''_{323} \sim 0.9 \Rightarrow \lambda_t = \lambda_b = \lambda_\tau \text{ at the unification scale } M_U}$$

The fixed point structure of the theory

MSSM infrared stable fixed point

$$X_t \equiv \frac{\lambda_t^2}{g_3^2} = \frac{7}{18}$$

is too low to accommodate $m_t = 175 \pm 5$ GeV !

(QFP) $\Rightarrow$ the top Yukawa coupling at the EW scale is independent of the choice of its input value at the GUT scale

If $\lambda_t(m_t)$ is nearer to its Quasi-Fixed point $\lambda_t \approx 1.1$, then, for $m_t(m_t) = 165 \pm 5$ GeV $\Rightarrow \sin\beta = \frac{\sqrt{2}\ m_t(m_t)}{v\ \lambda_t(m_t)}$

$$\Downarrow$$

$$\tan\beta = 1.7 \pm 0.2$$

MSSM+$\not{R}_p$

1) Infrared Stable Fixed points

We analyse the fixed points in the RGE's including the couplings λ'_{333}, λ''_{323} one at a time. We neglect also the effect of λ_b, λ_τ.

$$X_t = \frac{1}{3} \quad \text{if we include} \quad \lambda'_{333}$$

$$X_t = \frac{1}{8} \text{ if we include } \lambda''_{323}$$

excluded by the top quark mass!
No infrared stable fixed points in the MSSM+$\not{R}_p$

2) Quasi Fixed Points

Large $\lambda_t(M_U) > 1$ and small $\not{R}_p$ couplings

The behaviour of the R_p conserving superpotential parameters will be similar to their behaviour in the R_p conserving MSSM. In this case, we can solve the one-loop RGEs for the $\not{R}_p$ Yukawa couplings analytically:

$$\begin{aligned}
\lambda_{323}(\mu) &= \lambda_{323}(M_U)\delta_2^{-3}\delta_1^{-3/11}, \\
\lambda'_{333}(\mu) &= \lambda'_{333}(M_U)\delta_3^{40/27}\delta_2^{-5/2}\delta_1^{-29/594}\delta_t^{1/6}, \\
\lambda''_{323}(\mu) &= \lambda''_{323}(M_U)\delta_3^{56/27}\delta_2\delta_1^{-95/297}\delta_t^{1/3},
\end{aligned}$$

where $\delta_{1,2,3} \equiv g_{1,2,3}(\mu)/g(M_U)$ and $\delta_t \equiv \lambda_t(\mu)/\lambda_t(M_U)$.

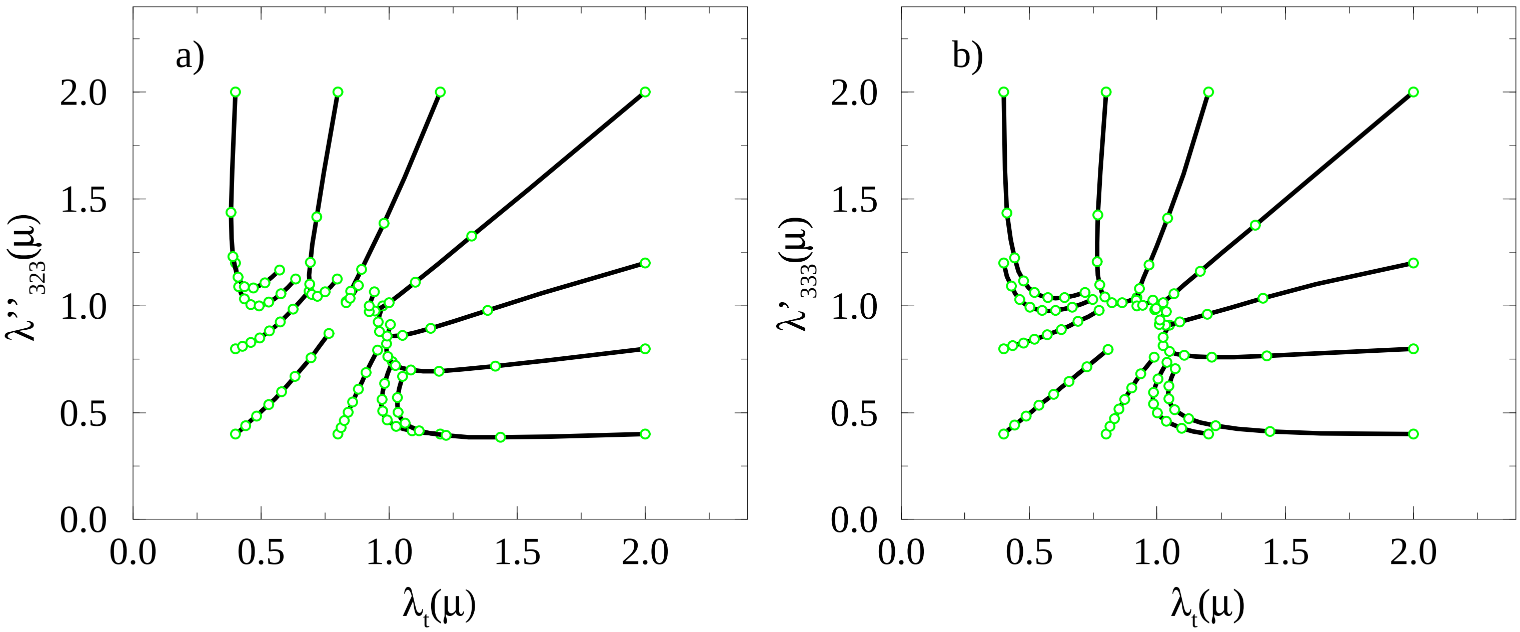
This corresponds to a straight line for the $\lambda_{\not{R}_p}(M_U)$ curves for $\tan\beta < 15$ and $\lambda_{\not{R}_p}(m_t) \lesssim 0.3$!!

3) Two large Yukawa couplings

Numerical solutions to the two-loop equations, in the cases of large λ_{323}, or λ''_{323} with large $\lambda_t(m_t)$.

When λ'_{333} is switched on, then $\tan\beta = 2.1\pm^{0.9}_{0.1}$

When λ''_{323} is switched on then $\tan\beta = 3.0\pm^{1.5}_{0.7}$.



Both couplings are running toward 1 in the infrared regime!

Conclusions

$\surd$ Complete 2-loop RGE's for the superpotential couplings and masses

$\surd$ We consider three models of $R\!\!\!/_p$: $L_3 L_2 \bar{E}_3$, $L_3 Q_3 \bar{D}_3$, $\bar{U}_3 \bar{D}_2 \bar{D}_3$

We found :

$\alpha_s(M_Z)$ can be changed at most by 5%

The unification scale is shifted by up to 20%

2-loop limit from perurbative unification. They are 5-12% weaker than the 1-loop

For λ'_{333}, $\lambda''_{323} \neq 0$ we found new points for $b - \tau$ Yukawa unification for almost all values of $\tan\beta$

The QFP structure is changed resulting in higher values of $\tan\beta$ and thus in higher masses for the lightest CP-even Higgs