Anomaly Cancellation in 6D Sugra from the Heterotic String

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[W. Buchmüller, CL, J. Schmidt, in progress]

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Contents

Overview

- We consider a 6D supergravity theory obtained from the $E_8 \times E_8$ heterotic string compactified on an anisotropic $T^6/\mathbb{Z}_6$ orbifold [Buchmüller et al. 06]
- 4D limit is known, gives the MSSM spectrum
- Compactification proceeds in two steps:
  - A First, compactify four dimensions on $T^4/\mathbb{Z}_3$ and go to the 6D limit: Only zero modes and localised fields remain, all effectively bulk fields
  - B Compactify the remaining two dimensions on $T^2/\mathbb{Z}_2$: Heterotic string determines the localised fields (first, third and fifth twisted sector)
- Here: Study anomaly cancellation in the 6D theory (bulk (A) and branes (B))

And now for...

Anomalies

- Anomalies: Classical symmetry not preserved in quantised theory
- OK for global symmetries, anomalies lead to distinct predictions (e.g. for $\pi \to \gamma\gamma$)
- Anomalies are fatal for gauge symmetries: Effective action Γ actually is not gauge invariant,

$$\mathcal{A}(\Lambda) = \delta_\Lambda \Gamma \neq 0$$

- Here gauge symmetries include local Lorentz transformations (equivalent to coordinate transformations) $\rightsquigarrow$ gravitational anomalies.
- Group structure of gauge transformations induces the Wess–Zumino consistency condition

$$[\delta_{\Lambda_1}, \delta_{\Lambda_2}]\,\Gamma = \delta_{[\Lambda_1,\Lambda_2]}\Gamma \quad \Longrightarrow \quad \delta_{\Lambda_1}\mathcal{A}(\Lambda_2) - \delta_{\Lambda_2}\mathcal{A}(\Lambda_1) = \mathcal{A}([\Lambda_1, \Lambda_2])$$

Anomaly Polynomial and Descent Equations

- Wess–Zumino condition is solved by descent equations:
  - Start from a closed and gauge invariant $(D+2)$-form I_{D+2} which is a polynomial in the gauge and gravitational field strengths $R = \mathrm{d}\Omega + \Omega^2$ and $F = \mathrm{d}A + A^2$ (where Ω and A are spin and gauge connection one-forms)
  - Since I_{D+2} is closed, it (locally) defines a Chern–Simons form via $I_{D+2} = \mathrm{d}I^{(0)}_{D+1}$
  - The gauge variation of I_{D+1} is again closed, hence $\delta I^{(0)}_{D+1} = \mathrm{d}I^{(1)}_{D}$
  - The anomaly defined as

$$\mathcal{A} = \int I^{(1)}_D$$

    automatically satisfies the WZ consistency condition
- For anomaly cancellation, it is most convienient to analyse the anomaly polynomial I_{D+2} instead of $\mathcal{A}$ itself

Contributions

- Only chiral fields contribute, so anomalies only occur in even dimensions
- Pure gravitational anomalies are only possible in $D = 4n + 2$, because in $D = 4n$ CPT ensures equal numbers of right- and left-handed fields
- Fields of different chirality contribute with opposite signs
- In 6D, the anomaly polynomial is an 8-form and receives various contributions: [Erler 93]
  - Gravitino (left-handed) and dilatino (right-handed) are gauge singlets and thus only contribute to gravitational anomalies $\sim \operatorname{tr} R^4$, $\sim \left(\operatorname{tr} R^2\right)^2$
  - Gauginos (left-handed) and hypermultiplet fermions (right-handed) also contribute to gauge anomalies $\sim \operatorname{tr} F^4$, $\sim \operatorname{tr} F^2 \operatorname{tr} F'^2$, and mixed gauge–gravity anomalies $\sim \operatorname{tr} R^2 \operatorname{tr} F^2$
  - Antisymmetric tensors with (anti)selfduality conditions also induce anomalies. However, the model contains one self-dual and one anti-self-dual tensor, so their effects cancel.

Green–Schwarz Mechanism

- Certain anomalies can be cancelled by the Green–Schwarz mechanism: Anomaly polynomial must be reducible,

$$I_{D+2} = X_4 Y_{D-2}\,,$$

  where X_4 and Y_{D-2} are both closed and gauge invariant
- If the anomaly is reducible, one can exploit the peculiar transformation properties of antisymmetric tensor fields to introduce new terms in the Lagrangean whose gauge variation cancels the anomaly.
- In the heterotic theory, we only have the NS two-form field B_2, so the X_4 in the factorisation must be of a special form related to the transformation of B_2

Green–Schwarz Mechanism: B-Field

B_2 transforms under local Lorentz and gauge transformations as (I labels gauge group factors, $G = \prod_I G_I$)

$$\delta B = \underbrace{\mathrm{tr}\left(\Theta \mathrm{d}\Omega\right)}_{\omega^{(1)}_{2\,R}} - \sum_I \alpha_I \underbrace{\mathrm{tr}\left(\Lambda_I \mathrm{d} A_I\right)}_{\omega^{(1)}_{2\,I}}$$

Here Θ and Λ_I are the transformation parameters. The α_I are determined by the gauge groups: $\alpha_{SU(N)} = 2$, $\alpha_{SO(N)} = 1$.

Crucial point: The 2-forms $\omega^{(1)}_{2\,R}$ and $\omega^{(1)}_{2\,I}$ can be obtained from $\mathrm{tr}\, R^2$ and $\mathrm{tr}\, F_I^2$ via the descent equations, so δB_2 can also be written as a descent of a closed and gauge invariant 4-form,

$$\delta B_2 = \text{descent of } \left(\mathrm{tr}\, R^2 - \sum_I \alpha_I\, \mathrm{tr}\, F_I^2\right) \stackrel{!}{=} X_4 \,.$$

This can be used to cancel anomalies if X_4 appears in the factorisation of the anomaly polynomial

$$I_{D+2} = X_4\, Y_{D-2} \,.$$

And now for...

Model A: Field content

Gauge Group: $SU(6) \times U(1)^3 \times \left[SU(3) \times SO(8) \times U(1)^2\right]$

Sector	Multiplet	Representation	#
Gravity		Graviton & Tensor	
	Hyper		2
Untwisted	Vector	$(\mathbf{35}; 1, 1)$	35
		$(1; \mathbf{8}, 1)$	8
		$(1; 1, \mathbf{28})$	28
		$5 \times (1; 1, 1)$	5
Untwisted	Hyper	$(\mathbf{20}; 1, 1)$	20
		$(1; 1, \mathbf{8}) + (1; 1, \mathbf{8}_s) + (1; 1, \mathbf{8}_c)$	24
		$4 \times (1; 1, 1)$	4
Twisted	Hyper	$9 \times (\mathbf{6}; 1, 1) + 9 \times (\bar{\mathbf{6}}; 1, 1)$	108
		$9 \times (1; \mathbf{3}, 1) + 9 \times (1; \bar{\mathbf{3}}, 1)$	54
		$3 \times (1; 1, \mathbf{8}) + 3 \times (1; 1, \mathbf{8}_s) + 3 \times (1; 1, \mathbf{8}_c)$	72
		$36 \times (1; 1, 1)$	36
		# (Vector Multiplets)	76
		# (Hypermultiplets)	320

Irreducible Terms

GS mechanism can only cancel reducible anomalies. Some terms cannot be reducible, so their coefficients have to vanish:

- $\sim \operatorname{tr} R^4$: This term imposes the condition

$$\#\,(\text{hypermultiplets}) - \#\,(\text{vector multiplets}) = 244$$

- $\sim \operatorname{tr} F_I^4$: Relevant for non-Abelian groups with fourth-order Casimir, gives constraints on number and representations of matter multiplets: The hypermultiplet fermions have to cancel the gaugino contribution
- $\sim \operatorname{tr} F_I^3\, F_{U(1)}$: Involves also $U(1)$ charges of hypermultiplets

Indeed, all irreducible terms cancel.

Reducible Terms, Factorisation

The remaining anomaly polynomial is reducible:
(A: non-Abelian factors; $u, v, \ldots$: $U(1)$'s)

$$I_8 \propto \left(\operatorname{tr} R^2\right)^2 + \frac{1}{6}\left(\operatorname{tr} R^2\right)\left(\sum_A m_A \operatorname{tr} F_A^2 - \sum_{u,v} m_{uv} F_u F_v\right) + 4 \sum_{A,u,v} d_{A\,uv} \left(\operatorname{tr} F_A^2\right) F_u F_v + \sum_{u,v,w,x} h_{uvwx} F_u F_v F_w F_x$$

$$= \left[\operatorname{tr} R^2 - 2 \operatorname{tr} F_{SU(6)}^2 - 2 \operatorname{tr} F_{SU(3)}^2 - \operatorname{tr} F_{SO(8)}^2 - 2 \sum_u F_u^2\right] \times \left[\operatorname{tr} R^2 - \sum_{u,v} \beta_{uv} F_u F_v\right]$$

$$= X_4 Y_4$$

So all bulk anomalies can be cancelled!

And now for...

Brane Anomalies

- Up to now, we considered anomalies in the 6D effective theory of a compactification on $T^4/\mathbb{Z}_2$ (step A)
- Now step B: we further compactify on a $T^2/\mathbb{Z}_2$: Four fixed points($\equiv$ branes), one Wilson line
  $\rightsquigarrow$ two pairs of equivalent fixed points, labelled by $n_2 = 0, 1$
- Bulk fields may survive the orbifold projection at $n_2 = 0$, $n_2 = 1$ or both; only those surviving at both have zero modes
- States at the fixed points are determined by string theory (i.e., the twisted sectors)

$\Rightarrow$ In the orbifold theory, there can be anomalies in the bulk and localised at the fixed points.

- In the 4D limit, anomalies reduce to those of zero modes and all localised fields.

Brane Anomalies

- Vanishing of (integrated) 4D anomaly is not sufficient, bulk and brane anomalies have to vanish separately (at each fixed point)
- Bulk anomalies vanish by Green–Schwarz mechanism (inherited from before)
- For brane anomalies, there are two contributions at a given fixed point:
  - Brane-localised fields
  - Bulk fields that survive the projection at that fixed point (i.e. not just zero modes!) — these contribute with a factor $\frac{1}{4}$ (four fixed points)

    [Lee et al. 03, Groot Nibbelink et al. 03]
- Localised anomaly polynomial again has to factorise,

$$I_6^{(\mathrm{loc})} = X_4 \, Y_2^{(\mathrm{loc})}$$

  with the same X_4 as before (but resticted to appropriate subgroups at the fixed point).
- If $I_6^{(\mathrm{loc})}$ factorises, the anomaly is cancelled by a Green–Schwarz mechanism, localised on the fixed point

Model B:Localised Fields

$$SU(5) \times U(1)^4 \times \left[SU(3) \times SO(8) \times U(1)^2\right]$$

	surviving bulk hypermultiplets	brane hypermultiplets
$n_2 = 0$:	$(\mathbf{10}; 1, 1), (\overline{\mathbf{10}}; 1, 1)$ $10 \times (\mathbf{5}; 1, 1),\ 10 \times (\overline{\mathbf{5}}; 1, 1)$ $9 \times (1; \mathbf{3}, 1),\ 9 \times (1; \overline{\mathbf{3}}, 1)$ $12 \times (1; 1, \hat{\mathbf{8}}),\ 58 \times (1; 1, 1)$	$(\mathbf{10}; 1, 1),\ (\overline{\mathbf{5}}; 1, 1)$ $2 \times (1; \mathbf{3}, 1),\ 2 \times (1; \overline{\mathbf{3}}, 1)$ $(1; 1, \mathbf{8}_c)$ $8 \times (1; 1, 1)$

$$SU(2) \times SU(4) \times U(1)^4 \times \left[SU(2) \times SU(4) \times U(1)^4\right]$$

	surviving bulk hypermultiplets	brane hypermultiplets
$n_2 = 1$:	$(\mathbf{2}, \mathbf{4}; 1, 1),\ (\mathbf{2}, \overline{\mathbf{4}}; 1, 1),\ (\mathbf{2}, \mathbf{6}; 1, 1)$ $10 \times (1, \mathbf{4}; 1, 1),\ 10 \times (1, \overline{\mathbf{4}}; 1, 1)$ $18 \times (\mathbf{2}, 1; 1, 1),\ 20 \times (1, 1; \mathbf{2}', 1)$ $8 \times (1, 1; 1, \mathbf{4}'),\ 8 \times (1, 1; 1, \overline{\mathbf{4}}')$ $5 \times (1, 1; 1, \mathbf{6}'),\ 66 \times (1, 1; 1, 1)$	$4 \times (\mathbf{2}, 1; 1, 1)$ $16 \times (1, 1; 1, 1)$

Anomaly Cancellation and Anomalous $U(1)$'s

- No pure gravitational anomaly in 4D, so no restriction on absolute number of multiplets
- The only irreducible terms are cubic non-Abelian, $\sim \mathrm{tr}\, F_A^3$, which cancel at both fixed points
- Remaining anomaly polynomials factorise in the required way

$\Rightarrow$ Localised Green–Schwarz terms can be added to cancel these

- Since factorisation involves a 2-form, $I_6^{(\mathrm{loc})} = X_4 Y_2^{(\mathrm{loc})}$, only $U(1)$ anomalies are possible
- At each fixed point, we can find one linear combination of generators to be the "anomalous $U(1)$", such that all other $U(1)$'s are anomaly free
- However, anomalus $U(1)$'s at different fixed points differ, so no "global" anomalous $U(1)$ [Gmeiner et al. 02]

Anomalous $U(1)$ and 4D Limit

- Anomalous $U(1)$ corresponds to non-zero trace of corresponding generator
- Generates localised FI terms with $\xi_{\mathrm{FI}} = gM_{\mathrm{P}}^2 / \left(192\pi^2\right) \operatorname{tr} T_{\mathrm{an}}$
- 4D FI term is sum of localised ones, 4D anomalous $U(1)$ is linear combination of local ones, weighted with $\operatorname{tr} T_{\mathrm{an}}$ at each fixed point $\rightsquigarrow \operatorname{tr} T_{\mathrm{an}}|_{\mathrm{4D}} = 88$
- In the 4D limit, anomalous $U(1)$, and possibly other gauge groups, are spontaneously broken, some fields acquire large VEVs

And now for...

Conclusion and Outlook

- String-derived 6D supergravity model, with MSSM 4D limit
- Anomaly freedom checked explicitly
- Starting point for phenomenological investigations
- Potentially interesting features:
    - FI term from anomalous $U(1)$ is $\mathcal{O}(\text{GUT scale}) \rightsquigarrow$ determine scalar potential
    - Depending on VEVs of non-Abelian singlets, a truly hidden sector can be achieved
    - Blow-up of orbifold singularities, generalisation to K3?
    - Seesaw mechanism can be realised [Buchmüller et al. 07]
- $\mathcal{O}(100)$ similar models exist [Lebedev et al. 06]
- To Do: Investigation of vacuum structure, D-terms, $U(1)$ breaking,...