Seven-Dimensional Super-Yang–Mills Theory in $\mathcal{N} = 1$ Superfields

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Motivation

- Superspace convenient for $\mathcal{N} = 1$ model building
- Higher-dimensional models have more supersymmetry
- For higher SUSY, superspace not so useful anymore
- Superfield formulations desirable
- Idea: Single out one SUSY to be manifest in superfields
- Even in higher-dimensional models, want $\mathcal{N} = 1$ in 4D, e.g. by coupling to lower-dimensional subspaces which preserve some, but not all SUSY
- This talk: Seven-dimensional case (D6 branes in IIA, ADE singularities in $M$ theory on $G_2$ manifolds)
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Superfields for Higher-Dimensional SUSY

- Choosing embedding into $\mathcal{N} = 1$ superfields singles out one manifest supersymmetry
- Can turn this around: $\mathcal{N} = 1$ SUSY & higher-dimensional Lorentz symmetry implies higher SUSY
- Ten-dimensional SYM: [Marcus, Sagnotti, Siegel '83]
- Renewed interest: [Arkani-Hamed et al. '01]; [Marti, Pomarol '01] in 5D, including radion
- However, gauge symmetry not manifest
- Supergravity: [Linch, Luty, Phillips '02] for linearised 5D SUGRA; [Schmidt, Paccetti Correia, Tavartkiladze '04-06] in 5D, including compactification
- Gauge-covariant formulation in 5D: [Hebecker '01]
- Here: Extend this to seven dimensions
General Dimensions

- Higher-dimensional SUSY is either $\mathcal{N} = 2$ or $\mathcal{N} = 4$ from 4D perspective
- Degree-of-freedom-wise, SYM multiplet corresponds to one vector and one/three chiral multiplets of $\mathcal{N} = 1$
- However, theory has more structure than just 4D $\mathcal{N} = 4$ SUSY: higher-dimensional Lorentz symmetry, different for different dimensions
- In general, action cannot be written in terms of field strengths and covariant derivatives only
- Action contains $V$ explicitly (i.e. not just $e^V$ and $W_\alpha$)
  $\Rightarrow$ extra term to maintain gauge invariance (vanishes in WZ gauge)
- Reason: non-Abelian $V$ transforms nonpolynomially
5D Case: Covariant Description

- 5d gauge multiplet: Vector $A_M$, scalar $B$, symplectic Majorana spinor pair $\psi_I$
- Off-shell component description known – includes $SU(2)$ triplet of auxiliary fields
- Identify vector and chiral superfields $V$, $\Phi = A_5 + iB + \cdots$
- Crucial point: Define covariant derivative $\nabla_5 = \partial_5 + \Phi$
- Define covariantly transforming “field strength” $Z = e^{-2V}\nabla_5 e^{2V}$
- Simplest possible action, symbolically

$$\mathcal{L} \sim \text{tr } W^\alpha W_\alpha + \text{tr } Z^2,$$

reproduces component action (already off-shell)
Symmetry Argument: 5 and 7 OK

Approach does not work in any dimension, for a symmetry reason:
In 4D language, we have $\mathcal{N} = 2$ or $\mathcal{N} = 4$ with extra structure, so naïve $R$ symmetry is broken to the geometrical symmetry $SO(d)$.

- In five or six dimensions, choosing one supersymmetry manifest breaks $SU(2)_R \rightarrow$ nothing. Geometrical symmetry is nothing for 5D, $SO(2)$ for 6D
- In seven to ten dimensions, naïve manifest $R$ symmetry is $SU(3)$. Only $SO(3)$ is subgroup, hence only 7D works.
- Form technical point of view, want scalar components of chiral multiplets to be $A_i + iB_i$, so need equal number of extra gauge field components and non-gauge scalars.
Seven-dimensional SYM can be obtained from 10D by dimensional reduction. 10D Super-Yang–Mills theory has the “minimal” field content: Vector $A_{\hat{M}}$ and Majorana–Weyl gaugino $\Lambda$. (Only on-shell description!)

**Action:**

$$\mathcal{L}_{10} = -\frac{1}{4} F_{\hat{M}\hat{N}} F^{\hat{M}\hat{N}} + \frac{1}{2} \bar{\Lambda} \Gamma_{\hat{M}} D_{\hat{M}} \Lambda$$

**SUSY transformations:**

$$\delta A_{\hat{M}} = i \frac{\Gamma_{\hat{M}} \Lambda}{2 \bar{\epsilon}} , \quad \delta \Lambda = -\frac{1}{4} F_{\hat{M}\hat{N}} \Gamma^{\hat{M}\hat{N}} \epsilon .$$
When reducing to seven dimensions, the fermions are a bit subtle due to different possible spinor types.

- **Symmetries of the seven-dimensional theory**
  - Lorentz symmetry $SO(1, 6)$
  - Supersymmetry
  - $R$ symmetry $SU(2)$ – this is the remnant Lorentz $SO(3)$ which reappears as automorphism of the superalgebra, even for minimal 7D supersymmetry

- **Fields:**
  - Vector $A_M \sim (7; 1)$
  - $SU(2)$ triplet of adjoint scalars $B_i \sim (1; 3)$
  - $SU(2)$ doublet of spinors $\Psi_I \sim (8; 2)$ with symplectic Majorana condition
    \[ \Psi_I = \varepsilon_{IJ} C \Psi_J^T \]
The action is

\[ \mathcal{L} = -\frac{1}{4} \text{tr} F_{MN} F^{MN} - \frac{1}{2} \text{tr} D_M B_i D^M B^i + \frac{1}{4} g^2 \text{tr} [B_i, B_j] [B^i, B^j] + \frac{i}{2} \text{tr} \bar{\Psi}_I \Gamma^M D_M \Psi_I + \frac{i}{2} g \text{tr} \bar{\Psi}_I [B_i \sigma^i_{IJ}, \Psi_J] \]

SUSY transformations:

\[ \delta A_M = -\frac{i}{2} \bar{\epsilon}_I \Gamma_M \Psi_I, \quad \delta B_i = -\frac{1}{2} \sigma^i_{IJ} \bar{\epsilon}_I \Psi_J, \]

\[ \delta \Psi_I = -\frac{1}{4} F_{MN} \Gamma^{MN} \epsilon_I + \frac{i}{2} \Gamma^M D_M (B_i \sigma_i)_{IJ} \epsilon_J + \frac{1}{4} g \epsilon_{ijk} [B_i, B_j] (\sigma_k)_{IJ} \epsilon_J \]
4D Degrees of Freedom

4D description reduces manifest symmetry:
\[ SO(1, 6) \times SU(2) \rightarrow SO(1, 3) \times SO(3) \times SU(2) \]

In 4D language, the fields are:

- gauge vector \( A_\mu \sim (4; 1, 1) \)
- three “gauge scalars” \( A_i \sim (1; 3, 1) \)
- three adjoint scalars \( B_i \sim (1; 1, 3) \)
- four (complex) Weyl spinors \( \lambda_r \sim (2; 2, 2) \)

In full dimensional reduction, \( R \) symmetry and extra-dimensional Lorentz symmetry would form \( SU(2) \times SU(2) = SO(4) \), enhanced to \( SU(4) \) – the scalars in a \( (3, 1) \oplus (1, 3) = 6 \), spinors in a \( 4 \). Singling out one supersymmetry breaks \( SU(4) \rightarrow SU(3) \) – scalars now in \( 3 \), spinors in \( 1 \oplus 3 \).

Here, however, the \( A_i \) are different from the \( B_i \) – keep the diagonal \( SO(3) \).
Superfield Embedding

Scalars and three spinors form triplet of chiral multiplets,
\[ \Phi_i = A_i + iB_i + 2\theta \psi_i + \theta^2 F_i \]

\[ \delta (A_i + iB_i) \sim \epsilon \psi_i, \quad \delta \psi_i \sim F_{\mu i}\sigma^\mu \bar{\epsilon} - F_i \epsilon. \]

The other spinor and the vector form vector multiplet
\[ V = -\theta \sigma^\mu \bar{\theta} A_\mu + i\theta^2 \bar{\theta} \chi - i\bar{\theta}^2 \theta \chi + \frac{1}{2} \theta^4 D \]

\[ \delta A_\mu \sim \epsilon \sigma_\mu \bar{\chi} + \text{H.c.}, \quad \delta \chi \sim F_{\mu \nu} \sigma^{\mu \nu} \epsilon - D \epsilon. \]

Note: Still on-shell description – auxiliary fields fixed to be
\[ F_i = \varepsilon_{ijk} \left( \partial_j \Phi_k + \frac{1}{2} i [\Phi_j, \Phi_k] \right) = \frac{1}{2} \varepsilon_{ijk} \left( F_{jk} + 2iD_j B_k - i [B_j, B_k] \right), \]
\[ D = -D_i B_i. \]
As usual, closure of SUSY transformations on the vector multiplet requires additional field-dependent gauge transformation:

\[ [\delta_\varepsilon, \delta_\eta] A_\mu = -2i \left( \varepsilon \sigma^\nu \eta - \eta \sigma^\nu \varepsilon \right) \partial_\nu A_\mu + \delta_{\text{gauge}} \]

where \( \delta_{\text{gauge}} \) is a gauge transformation with parameter

\[ \Lambda = 2i \left( \varepsilon \sigma^\mu \eta - \eta \sigma^\mu \varepsilon \right) A_\mu \]

Here, the same happens for the chiral multiplets,

\[ [\delta_\varepsilon, \delta_\eta] \phi_i = -2i \left( \varepsilon \sigma^\mu \eta - \eta \sigma^\mu \varepsilon \right) \partial_\mu \phi_i + \delta_{\text{gauge}} , \]

with the same \( \Lambda \) – makes right-hand side gauge covariant.
The superfields transform as

\[ e^{2V} \longrightarrow e^{-i\bar{\Lambda}} e^{2V} e^{i\Lambda}, \quad \Phi_i \longrightarrow e^{-i\Lambda} (\Phi_i - i\partial_i) e^{i\Lambda}. \]

We can define a covariant derivative in the extra dimensions as

\[ \nabla_i = \partial_i + i\Phi_i, \quad \nabla_i \rightarrow e^{-i\Lambda} \nabla_i e^{i\Lambda} \]

This allows to define two field strength superfields:

\[ W_\alpha = -\frac{1}{4} \tilde{D}^2 e^{-2V} D_\alpha e^{2V}, \]
\[ Z_i = e^{-2V} \nabla_i e^{2V}, \]

which both transform as

\[ Z_i \rightarrow e^{-i\Lambda} Z_i e^{i\Lambda}, \quad W_\alpha \rightarrow e^{-i\Lambda} W_\alpha e^{i\Lambda}. \]
Action — $W^2 + Z^2$

The action contains three pieces:

• The usual four-dimensional gauge kinetic term,

$$\frac{1}{16} \int d^2 \theta \, tr \, W^\alpha W_\alpha + \text{H.c.} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - i \chi \sigma^\mu D_\mu \bar{\chi} + \frac{1}{2} D^2$$

• The kinetic term of the chiral multiplets,

$$\frac{1}{4} \int d^4 \theta \, tr \, Z_i Z_i = -\frac{1}{2} F_{\mu i} F^{\mu i} - \frac{1}{2} D_\mu B_i D^\mu B_i + DD_i B_i + 2F_i \bar{F}_i - i \psi_i \sigma^\mu D_\mu \bar{\psi}_i - (\chi D_i \psi_i - \chi [B_i, \psi_i] + \text{H.c.})$$

Note: $tr \, Z_i Z_i$ is Hermitean although $Z_i$ itself is not!
The purely extra-dimensional piece of the action is provided by a superpotential-like term

\[ \frac{1}{4} \int d^2 \theta \, \text{tr} \, \varepsilon_{ijk} \Phi_i \left( \partial_j \Phi_k + \frac{i}{3} [\Phi_j, \Phi_k] \right) + \text{H.c.} = \]

\[ = \frac{1}{4} \varepsilon_{ijk} F_i \left( F_{jk} + 2i D_j B_k - i [B_j, B_k] \right) \]

\[ - \frac{1}{2} \varepsilon_{ijk} \psi_i D_j \psi_k + \frac{1}{2} \varepsilon_{ijk} \psi_i [B_j, \psi_k] + \text{H.c.} \]

This is a superfield Chern–Simons term \( \sim A \wedge dA + \frac{2}{3} A^3 \) – so it is gauge invariant up to a “winding number” due to \( \pi_3(G) = \mathbb{Z} \):

\[ \delta \mathcal{L}_{CS} \sim \int d^2 \theta \, \text{tr} \, \varepsilon_{ijk} (g^{-1} \partial_i g) (g^{-1} \partial_j g) (g^{-1} \partial_k g) \]

For gauge transformations that preserve WZ gauge, \( \mathcal{L}_{CS} \) is invariant.
Complete Action

The full action

\[ \mathcal{L} = \mathcal{L}_{W^2} + \mathcal{L}_{Z^2} + \mathcal{L}_{CS}, \]

leads to the auxiliary fields

\[ F_i = \frac{1}{2} \varepsilon_{ijk} (F_{jk} + 2iD_j B_k - i [B_j, B_k]), \quad D = -D_i B_i, \]

consistent with 7D transformations.

After eliminating, one gets back the original action

\[ \mathcal{L}_{SF} = -\frac{1}{4} F_{MN} F^{MN} - \frac{1}{2} D_M B_i i^M B_i + \frac{1}{4} [B_i, B_j] [B_i, B_j] \]

\[ - i \chi \sigma^\mu D_\mu \bar{\chi} - i \psi_i \sigma^\mu D_\mu \bar{\psi}_i \]

\[ - \left[ \chi (D_i \psi_i - [B_i, \psi_i]) + \frac{1}{2} \varepsilon_{ijk} \psi_i (D_j \psi_k - [B_j, \psi_k]) + \text{H.c.} \right]. \]

Note: \( \mathcal{N} = 1 \) SUSY and 7D Lorentz symmetry imply \( \mathcal{N} = 4 \) – fixes coefficients.
Higher-Dimensional Operators in 7D

(still work in progress)

• Application of the formalism: Study of supersymmetric higher-dimensional operators
• Once three dimensions are compactified, full Lorentz symmetry is not required anymore
• For $U(1)$ gauge groups, $W_\alpha$ and $Z_i$ are already gauge invariant
• Example: Gaugino masses from internal flux via

$$\int d^4 \theta Z_i Z_i W^{\alpha} W_\alpha \sim F_{ij} F^{ij} \chi^{\alpha} \chi_{\alpha}$$

• For string theory compactification, coupling to supergravity required – should at least include relevant moduli
D6 Intersections

Another application: coupling to matter on subspaces, e.g. at brane intersections

Consider two 6-brane (stacks), intersecting on four-dimensional subspace. $\Sigma$ carries chiral multiplet in bifundamental.
Σ preserves $\mathcal{N} = 1$ SUSY, possibly spontaneously broken – restrictions on couplings from SUSY and gauge symmetry:

- Clearly, kinetic term is
  \[ \int d^4 \theta \rho^\dagger e^2 (\nu + \bar{\nu}) \rho \]

- No direct coupling to the $\Phi_i$ – come with derivative
- Nontrivial gauge group cross-coupling necessarily involves $\rho$
- Exception: $U(1)$ gauge theory – $W_\alpha$ and $Z_i$ are gauge invariant, geometry still constrains, e.g. $g^{i\hat{i}} Z_i \hat{Z}_\hat{i} \sim \cos(\alpha) Z \hat{Z}$, where $\alpha$ is the intersection angle
- As example, take transverse brane to carry $U(1)$ and a nonzero $F$ term. Then
  \[ \int d^4 \theta \left| \hat{Z}_\hat{i} \right|^2 \rho^\dagger e^2 (\nu + \bar{\nu}) \rho \sim \left| \hat{F}_\hat{i} \right|^2 \bar{\rho} \rho \]
Summary

- Gauge-covariant $\mathcal{N} = 1$ superspace description of seven-dimensional SYM theory
- Possible in 5D and 7D
- Construction of Lagrangean straightforward – coefficients determined by higher-dimensional Lorentz symmetry
- Facilitates systematic study of higher-dimensional operators
- Application: Coupling to lower-dimensional subspaces, as e.g. in brane intersections – $\mathcal{N} = 1$ SUSY is manifest
- still work in progress...