The Potential Fate of Local Model Building

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CL, Hans Peter Nilles, Claudia Christine Stephan
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Motivation

- F-Theory: Type IIB vacua with general branes, exceptional symmetries available for GUT model building
- Usually, consider local models: Focus on brane stack or points within the stack and decouple bulk of the compactification manifold
- Advantage: Simple, physics basically fixed by symmetry
- Obvious question: Existence of global completion
- Proton stability generic problem of GUTs
- Dimension-four proton decay operators forbidden by matter parity or variants
- To keep spirit (and predictivity) of local models, matter parity should also be defined locally
- For heterotic analogue, see e.g. talk by Patrick Vaudrevange

Approach

- First, consider local $SU(5)$ GUT models where all matter curves meet at one point – symmetry enhanced to E_8
- Define a matter parity to forbid dimension-four proton decay
- Dimension-five proton decay avoided by assignment of zero modes to matter curves
- Heavy top quark by tree-level Yukawa coupling, other masses involve singlet VEVs – we require masses for all quarks and leptons
- Three generations can com from three, two or one matter curve
- In semilocal embedding, try to justify zero mode assignment by explicit fluxes: This will turn out to be impossible

Contents

Global, Semilocal, Local

[Beasley, Heckman, Vafa; Donagi, Wijnholt; Marsano, Saulina, Schäfer-Nameki; Hayashi, Kawano, Tatar, Watari; Dudas, Palti; Choi, . . .]

For F-Theory GUTs, different degrees of locality:

- *Global* model: Specify full compactification space (CY fourfold): Includes all branes, fluxes, obeys consistency conditions, can stabilise moduli etc.

  [Blumenhagen, Grimm, Jurke, Weigand; Braun, Hebecker, CL, Valandro,. . .]
- *Semilocal* model: Focus on the GUT surface (brane stack) S and matter curves within S: Decouples bulk of compactification space, certain consistency conditions included
- *Local* model: Consider only points within S where matter curves intersect and interactions are localised: Simple, and hope for predictivity because any good global model must contain good local model and bulk physics decoupled. Certain question cannot be answered, and actual existence of global completion is not guaranteed.

GUT Surface, Matter Curves

[Conventions of Dudas, Palti]

- $SU(5)$ GUT on two-complex-dimensional surface S in the base of elliptically fibred CY fourfold X, locally given by $w = 0$
- Elliptic fibration over S described by Tate model [Bershadsky et al.]

$$y^2 = x^3 + b_5 xy + b_4 x^2 w + b_3 y w^2 + b_2 x w^3 + b_0 w^5$$

  b_k: functions on S (actually sections in certain line bundles)
- Localised matter on matter curves (brane intersections), representation can be derived from decomposition of adjoint of enhanced symmetry group G_Σ
- Explicitly, $SU(5)$ is enhanced

$$\begin{aligned} \text{to } SU(6): \quad & b_3^2 b_4 - b_2 b_3 b_5 + b_0 b_5^2 = 0 \quad \Rightarrow \mathbf{5} \\ \text{to } SO(10): \quad & b_5 = 0 \quad \Rightarrow \mathbf{10} \end{aligned}$$

Yukawa Couplings

- 6D Matter in hypermultiplets – 4D zero modes determined by flux (via index theorem)
- Hypercharge flux can split multiplets, e.g. doublet-triplet splitting [BHV, DW]
- Triple intersections of matter curves: Pointlike enhancement to even higher group, triple adjoint interaction determines Yukawa couplings
- For $SU(5)$ GUT require $SO(12)$ enhancement for down-type Yukawas,

$$(\mathbf{66})^3 \supset \bar{\mathbf{5}}_{H_d} \bar{\mathbf{5}}_M \, \mathbf{10}_M$$

and E_6 enhancement for up-type,

$$(\mathbf{78})^3 \supset \mathbf{5}_{H_u} \mathbf{10}_M \, \mathbf{10}_M$$

Point of E_8

[Heckman, Tavanfar, Vafa]

- Need E_6 and $SO(12)$ enhancements for up- and down-type Yukawas
- CKM matrix: Favourably, these points coincide (no geometric suppression of quark mixing) $\rightsquigarrow E_7$
- For PMNS matrix: Further enhancement to E_8 (but we do not consider neutrinos in the following)
- Hence: One single Yukawa "point of E_8", all interactions localised here
- Simple: All (superpotential) interactions determined by group theory, geometric data can be largely ignored

Gauge Theory Description

- Consider SYM theory on worldvolume of S: E_8 GUT, broken to $SU(5)$ by adjoint Higgs
- Commutant of $SU(5) \subset E_8 = SU(5)_\perp$
- Actually, rank-preserving breaking

$$E_8 \longrightarrow (SU(5) \times SU(5)_\perp) \longrightarrow SU(5) \times U(1)^4$$

- Extra $U(1)$'s generically massive in F-Theory by geometric Stueckelberg effect, but this cannot be analysed in local model – $U(1)$'s remain as global selection rules [Grimm, Weigand]
- Higgs field varies over S – matter curves now visible as vanishing loci of Higgs eigenvalues

E_8 Higgs

$$E_8 \longrightarrow SU(5) \times SU(5)_\perp$$
$$\mathbf{248} \longrightarrow (\mathbf{24},\mathbf{1}) \oplus (\mathbf{1},\mathbf{24}) \oplus \left[(\mathbf{10},\mathbf{5}) \oplus \left(\mathbf{5},\overline{\mathbf{10}}\right) \oplus \text{c.c.}\right]$$

$$\text{Higgs} \quad \Phi \sim \begin{pmatrix} t_1 & & & & \\ & t_2 & & & \\ & & t_3 & & \\ & & & t_4 & \\ & & & & t_5 \end{pmatrix} \in (\mathbf{1},\mathbf{24}) \ , \sum_i t_i = 0$$

Connection to Tate model: $b_k \sim$ symmetric polynomials in the t_i of order k, matter curves now given by

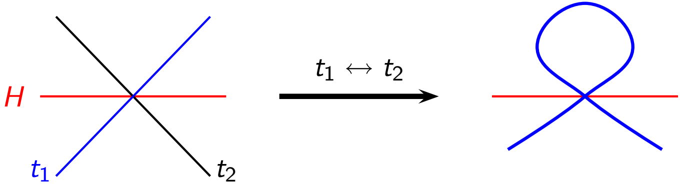
$$\mathbf{10}: t_i = 0\,, \qquad \mathbf{5}: -(t_i + t_j) = 0\,,\ i \neq j$$

t_i double as charges: gauge-invariant terms must have zero $\sum t_i$

Monodromy

[Bershadsky et al.]

- The b_k in the Tate model are symmetric polynomials in the t_i
  $\Rightarrow$ Invariant under permutations of the t_i
- Interpretation: Self-intersection, locally distinct-looking branes are the same



- Heavy top requires coupling $\mathbf{5}_{H_u}\mathbf{10}_{\text{top}}\mathbf{10}_{\text{top}}$
  $\rightsquigarrow$ (at least) $\mathbb{Z}_2$ monodromy $t_1 \leftrightarrow t_2$
- Fixes top and up-type Higgs curve: $\mathbf{10}_{\text{top}} \sim \{t_1, t_2\}$, $\mathbf{5}_{H_u} \sim -t_1 - t_2$
- Reduces $SU(5)_\perp$ to lower rank

Yukawa Couplings

Good couplings: Quark and lepton masses, weak-scale μ term

$$W_{\text{good}} = \mu \mathbf{5}_{H_u} \overline{\mathbf{5}}_{H_d} + Y_u \mathbf{5}_{H_u} \mathbf{10}_M \mathbf{10}_M + Y_d \overline{\mathbf{5}}_{H_d} \overline{\mathbf{5}}_M \mathbf{10}_M$$

Yukawa Couplings

Good couplings: Quark and lepton masses, weak-scale μ term

$$W_{\text{good}} = \mu \mathbf{5}_{H_u} \bar{\mathbf{5}}_{H_d} + Y_u \mathbf{5}_{H_u} \mathbf{10}_M \mathbf{10}_M + Y_d \bar{\mathbf{5}}_{H_d} \bar{\mathbf{5}}_M \mathbf{10}_M$$

Bad couplings: Baryon and lepton number violating operators

$$\left.\begin{aligned} W_{\text{bad}} &= \beta \mathbf{5}_{H_u} \bar{\mathbf{5}}_M + \lambda \bar{\mathbf{5}}_M \bar{\mathbf{5}}_M \mathbf{10}_M \\ &\quad + W^1 \mathbf{10}_M \mathbf{10}_M \mathbf{10}_M \bar{\mathbf{5}}_M + W^2 \mathbf{10}_M \mathbf{10}_M \mathbf{10}_M \bar{\mathbf{5}}_{H_d} \\ &\quad + W^3 \bar{\mathbf{5}}_M \bar{\mathbf{5}}_M \mathbf{5}_{H_u} \mathbf{5}_{H_u} + W^4 \bar{\mathbf{5}}_M \bar{\mathbf{5}}_{H_d} \mathbf{5}_{H_u} \mathbf{5}_{H_u} \\ K_{\text{bad}} &= K^1 \mathbf{10}_M \mathbf{10}_M \mathbf{5}_M + K^2 \bar{\mathbf{5}}_{H_u} \bar{\mathbf{5}}_{H_u} \mathbf{10}_M \end{aligned}\right\} \text{dim-5}$$

dim-3/4

Coefficients can contain singlet VEVs, suppressed by M_{GUT} [Conlon, Palti]

Yukawa Couplings

Good couplings: Quark and lepton masses, weak-scale μ term

$$W_{\text{good}} = \mu \mathbf{5}_{H_u} \overline{\mathbf{5}}_{H_d} + Y_u \mathbf{5}_{H_u} \mathbf{10}_M \mathbf{10}_M + Y_d \overline{\mathbf{5}}_{H_d} \overline{\mathbf{5}}_M \mathbf{10}_M$$

Bad couplings: Baryon and lepton number violating operators

$$\begin{aligned} W_{\text{bad}} &= \beta \mathbf{5}_{H_u} \overline{\mathbf{5}}_M + \lambda \overline{\mathbf{5}}_M \overline{\mathbf{5}}_M \mathbf{10}_M \\ &\quad + W^1 \mathbf{10}_M \mathbf{10}_M \mathbf{10}_M \overline{\mathbf{5}}_M + W^2 \mathbf{10}_M \mathbf{10}_M \mathbf{10}_M \overline{\mathbf{5}}_{H_d} \\ &\quad + W^3 \overline{\mathbf{5}}_M \overline{\mathbf{5}}_M \mathbf{5}_{H_u} \mathbf{5}_{H_u} + W^4 \overline{\mathbf{5}}_M \overline{\mathbf{5}}_{H_d} \mathbf{5}_{H_u} \mathbf{5}_{H_u} \\ K_{\text{bad}} &= K^1 \mathbf{10}_M \mathbf{10}_M \mathbf{5}_M + K^2 \overline{\mathbf{5}}_{H_u} \overline{\mathbf{5}}_{H_u} \mathbf{10}_M \end{aligned}$$

dim-3/4

dim-5

Coefficients can contain singlet VEVs, suppressed by M_{GUT} [Conlon, Palti]

Some terms related by interchange $\overline{\mathbf{5}}_{H_d} \leftrightarrow \overline{\mathbf{5}}_M$

Matter Parity

[Dimopoulos, Raby, Wilczek; Ibanez, Ross; Dreiner, Luhn, Thormeier]

Various discrete symmetries help for proton stability. Compatibility with $SU(5)$ favours $\mathbb{Z}_2$ which matter parity distinguishes Higgs and matter:

	$\mathbf{5}_{H_u}, \bar{\mathbf{5}}_{H_d}$	$\mathbf{10}_M, \bar{\mathbf{5}}_M$
P_M	$+1$	-1

Forbids all baryon and lepton number violating operators except

$$W^1 \mathbf{10}_M \mathbf{10}_M \mathbf{10}_M \bar{\mathbf{5}}_M \quad \text{and} \quad W^3 \bar{\mathbf{5}}_M \bar{\mathbf{5}}_M \mathbf{5}_{H_u} \mathbf{5}_{H_u}$$

W^3 generates neutrino masses (Weinberg operator), can be tolerated if suppression scale high enough (but will not be generated, so ignore from now on)
W^1 forbidden at any order ($W^1 \supset QQQL$, $\bar{u}\bar{u}\bar{d}\bar{e}$, ...): Main constraint in finding models

Contents

Model Requirements

For the local model we require

- P_M defined at the point of E_8
- heavy top quark (i.e. rank-one up-type Yukawa matrix at tree level)
- No dim-5 proton decay (the W^1 operator forbidden at all orders)
- Masses for all quarks and leptons after switching on VEVs (down-type Yukawa matrix can be rank-zero or one, but not rank-two)

Model Requirements

For the local model we require

- P_M defined at the point of E_8
- heavy top quark (i.e. rank-one up-type Yukawa matrix at tree level)
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Local model building freedom: Freely choose

- Monodromy (at least $\mathbb{Z}_2$)
- Assignment of matter and Higgs zero modes to curves
- Singlet VEVs (for matter parity even singlets)

Matter Parity

Define $\mathbb{Z}_2$ matter parity in terms of the t_i (i.e. as subgroup of $SU(5)_\perp$):

$$P_M = (-1)^{c_i t_i}\ , \quad c_i = 0, 1 \qquad (\text{defined} \mod 2)$$

- Monodromy $t_1 \leftrightarrow t_2$ requires $c_1 = c_2 = 1$ so $\mathbf{10}_{\text{top}}$ is odd
- Up-type masses always allowed once gauge invariant
- Down-type masses give constraint:

	$\bar{\mathbf{5}}_{H_d}$	$\bar{\mathbf{5}}_M$	$\mathbf{10}_M$
charge	$t_i + t_j$	$t_k + t_l$	t_m

  Gauge invariant iff all t_i distinct – can only be matter parity even if even number of $c_i = 1$ (singlets have charge $t_i - t_j$, so don't change the argument)

- Note: W^1 operator has same charge structure

Two Possibilities

Hence, two possible definitions of matter parity:

Case I: $P_M = (-1)^{t_1+t_2+t_3+t_4}$

Case II: $P_M = (-1)^{t_1+t_2}$

Now analyse matter, Higgs and VEV assignment for both cases: $\mathbf{10}_{\text{top}}$ and $\mathbf{5}_{H_u}$ already fixed, need to distribute remaining matter and $\overline{\mathbf{5}}_{H_d}$ according to their matter parity

Main restriction: Forbid W^1, but allow down-type Yukawas

VEVs allowed only for even matter parity singlets

Case I: Matter and VEV Assignment

Matter **10** Curves			
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	
$\mathbf{10}_3$	t_4	$-$	
Matter **5** Curves			
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	
Even Singlet Curves			
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	
$\mathbf{1}_7$	$t_1 - t_2$	$+$	

Case I: Matter and VEV Assignment

Matter **10** Curves			
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	
$\mathbf{10}_3$	t_4	$-$	
Matter 5 Curves			
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	
Even Singlet Curves			
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	
$\mathbf{1}_7$	$t_1 - t_2$	$+$	

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

Case I: Matter and VEV Assignment

Matter **10** Curves

$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	no matter
$\mathbf{10}_3$	t_4	$-$	matter
Matter 5 Curves			
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	matter
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	no matter
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	matter
Even Singlet Curves			
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	
$\mathbf{1}_7$	$t_1 - t_2$	$+$	

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

$\rightsquigarrow$ no matter on $\mathbf{10}_2$, $\mathbf{5}_5$

Case I: Matter and VEV Assignment

Matter **10** Curves			
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	no matter
$\mathbf{10}_3$	t_4	$-$	matter
Matter **5** Curves			
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	matter
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	no matter
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	matter
Even Singlet Curves			
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	
$\mathbf{1}_7$	$t_1 - t_2$	$+$	

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

$\rightsquigarrow$ no matter on $\mathbf{10}_2$, $\mathbf{5}_5$

- W^1 with singlets:

e.g. $\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_6\mathbf{1}_4\,,$

$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_3\mathbf{1}_1$

Case I: Matter and VEV Assignment

	Matter **10** Curves		
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	no matter
$\mathbf{10}_3$	t_4	$-$	matter
	Matter **5** Curves		
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	matter
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	no matter
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	matter
	Even Singlet Curves		
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	no VEV
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	VEV
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	no VEV
$\mathbf{1}_7$	$t_1 - t_2$	$+$	VEV

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

$\rightsquigarrow$ no matter on $\mathbf{10}_2$, $\mathbf{5}_5$

- W^1 with singlets:

e.g. $\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_6\mathbf{1}_4\,,$

$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_3\mathbf{1}_1$

$\rightsquigarrow$ no VEVs for $\mathbf{1}_1$, $\mathbf{1}_4$ (because of t_3)

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	
$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	
$\bar{\mathbf{5}}_2$	$-t_{1,2} - t_4$	
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	
$\bar{\mathbf{5}}_2$	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	either rank-two Yukawa matrix, or no up-type masses with singlets
$\bar{\mathbf{5}}_2$	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two

Case I: Down-Type Higgs

Higgs-like $\mathbf{5}$ Curves		Down-type Yukawas
$\overline{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	No masses at tree level or with singlets μ term
$\overline{\mathbf{5}}_1$	$-t_{1,2} - t_3$	either rank-two Yukawa matrix, or no up-type masses with singlets
$\overline{\mathbf{5}}_2$	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\overline{\mathbf{5}}_4$	$-t_3 - t_4$	

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two
- String-scale μ term for both Higgses on one curve

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$\bar{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	No masses at tree level or with singlets μ term
$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	either rank-two Yukawa matrix, or no up-type masses with singlets
$\bar{\mathbf{5}}_2$	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	Rank-one Yukawa matrix, bottom quark heavy

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two
- String-scale μ term for both Higgses on one curve
- $\bar{\mathbf{5}}_4 = \bar{\mathbf{5}}_{H_d}$ is unique choice, tree-level coupling $\bar{\mathbf{5}}_{H_d}\mathbf{10}_{\text{top}}\bar{\mathbf{5}}_3$

Case I: Yukawas and CKM

- Third generation: $\mathbf{10}_1$ and $\bar{\mathbf{5}}_3$,light generations: $\mathbf{10}_3$ and $\bar{\mathbf{5}}_6$
- Higgses: $\bar{\mathbf{5}}_{H_u}$ and $\bar{\mathbf{5}}_4$, only $\langle \mathbf{1}_2 \rangle \sim \epsilon$ required at first order
- Ignore $\mathbf{1}_7$ and $\mathcal{O}(1)$ coefficients
- Yukawa matrices (schematically):

$$Y^u \sim Y^d \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}$$

- CKM matrix:

$$V_{\mathrm{CKM}} \sim \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}$$

- Masses and mixings possible though not a great fit
- Degeneracy because three generations come from two curves

Case II

$$P_M = (-1)^{t_1+t_2}$$

$\rightsquigarrow$ split t's into $t_{\text{odd}} = \{t_1, t_2\}$ and $t_{\text{even}} = \{t_3, t_4, t_5\}$

- Symmetric setup, possible monodromy acting on t_{even}
- Only one matter $\mathbf{10}$ curve, $\mathbf{10}_{\text{top}}$
- Down-type Higgs unique
- Three possible matter $\bar{\mathbf{5}}$ curves (charges $t_{\text{odd}} - t_{\text{even}}$), model building choice
- Matter-parity even singlets do not mix t_{odd} and t_{even}
- W^1 operator cannot be generated: Charge $4t_{odd} + t_{even}$ cannot be compensated by matter-parity even singlets
- Masses and mixings possible, choice of singlet VEVs not unique

Local Model Summary

- Two possible definitions of matter parity at the point of E_8
- In both cases, assignments of matter and Higgses is unique or strongly constrained, some freedom in choice of VEVs
- Restrictions mainly from forbidding W^1 while allowing for down-type masses
- W^3 operator (neutrino masses) is not generated in any case
- Masses for all matter and mixing possible
- Involves choices of zero modes and VEVs by hand – these cannot be calculated in the local framework

Contents

Semilocal Approach

[Friedman, Morgan, Witten;Donagi, Wijnholt]

Now *semilocal* picture: Consider GUT surface S, using spectral cover approach

Main aim: Find homology classes of matter curves which allow to find the flux restrictions and thus the zero mode spectrum.

Two types of fluxes:

- $U(1) \subset SU(5)_\perp$ fluxes on matter curves (from the transverse branes): Determines (chiral) 4D zero modes for full GUT multiplets (by index theorem). These are still free parameters up to anomaly cancellation requirements.
- Hypercharge flux on S (globally trivial so hypercharge stays unbroken): Restrictions to matter curves splits $SU(5)$ multiplets; homological relations between matter curves lead to relations between the splittings.

Spectral Cover

Consider threefold

$$\mathbb{P}(K_S \oplus \mathcal{O}_S)$$

with homogeneous coordinates U, V. Because of $\mathbb{Z}_2$ monodromy, spectral equation must factorise:

$$\begin{aligned} 0 &= b_0 U^5 + b_2 U^3 V^2 + b_3 U^2 V^3 + b_4 U V^4 + b_5 V^5 \\ &= \left(a_1 V^2 + a_2 UV + a_3 U^2\right)\left(a_4 V + a_7 U\right)\left(a_5 V + a_8 U\right)\left(a_6 V + a_9 U\right) \end{aligned}$$

b_k are sections in $\eta - kc_1 = (6-k)\, c_1(S) + c_1\left(N_{S/X}\right)$. This determines the bundles for the a_m, and in turn for the matter curves, in terms of three unspecified line bundles $\chi_{7,8,9}$.

Fluxes and Zero Modes

$U(1)$ fluxes: Given by integers $M_{\mathbf{5}}$, $M_{\mathbf{10}}$. Free up to consistency conditions [Dudas, Palti; Marsano]

$$\sum M_{\mathbf{10}} + \sum M_{\mathbf{5}} = 0\,, \qquad M_{\mathbf{10}_1} = -\left(M_{\mathbf{5}_1} + M_{\mathbf{5}_2} + M_{\mathbf{5}_3}\right)$$

Hypercharge flux must be globally trivial:

$$0 = F_Y \cdot c_1 = F_Y \cdot \eta \quad \rightsquigarrow \quad \sum_{\mathbf{5}} F_Y = \sum_{\mathbf{10}} F_Y = 0$$

Restrictions to matter curves gives by integer $N_Y = F_Y \cdot (\text{homology class})$. For curve with flux numbers M and N_Y, chiral zero modes given by

$$\begin{aligned} \mathbf{5}: &\qquad n_{(\mathbf{3},\mathbf{1})} = M_{\mathbf{5}}\,, \quad n_{(\mathbf{1},\mathbf{2})} = M_{\mathbf{5}} + N_Y\,, \\ \mathbf{10}: &\quad n_{(\mathbf{3},\mathbf{2})} = M_{\mathbf{10}}\,, \quad n_{(\bar{\mathbf{3}},\mathbf{1})} = M_{\mathbf{10}} - N_Y\,, \quad n_{(\mathbf{1},\mathbf{1})} = M_{\mathbf{10}} + N_Y \end{aligned}$$

Upshot

10 Curves

	M	N_Y
$\mathbf{10}_1$	$-(M_{\mathbf{5}_1} + M_{\mathbf{5}_2} + M_{\mathbf{5}_3})$	$-\widetilde{N}$
$\mathbf{10}_2$	$M_{\mathbf{10}_2}$	N_7
$\mathbf{10}_3$	$M_{\mathbf{10}_3}$	N_8
$\mathbf{10}_4$	$M_{\mathbf{10}_4}$	N_9

5 Curves

	M	N_Y
$\mathbf{5}_{H_u}$	$M_{\mathbf{5}_{H_u}}$	$\widetilde{N}$
$\mathbf{5}_1$	$M_{\mathbf{5}_1}$	$-\widetilde{N}$
$\mathbf{5}_2$	$M_{\mathbf{5}_2}$	$-\widetilde{N}$
$\mathbf{5}_3$	$M_{\mathbf{5}_3}$	$-\widetilde{N}$
$\mathbf{5}_4$	$M_{\mathbf{5}_4}$	$N_7 + N_8$
$\mathbf{5}_5$	$M_{\mathbf{5}_5}$	$N_7 + N_9$
$\mathbf{5}_6$	$M_{\mathbf{5}_6}$	$N_8 + N_9$

- Three free parameters $N_{7,8,9}$ for the hypercharge flux, corresponding to three unspecified line bundles
- $\widetilde{N} = N_7 + N_8 + N_9$
- Up-type Higgs and top **10** curve split with the same parameter

Case I: Matter Sector is Fine

- Doublet-triplet splitting for Higgses ($\widetilde{N} \neq 0$) inevitably splits $\mathbf{10}_{\text{top}}$ and at least one more **10** curve
- However, splitting of matter multiplets is OK as long as there are three generations of zero modes in the end, i.e. other **10** curve must have "opposite" split
- Matter on $\mathbf{10}_1$, $\mathbf{10}_3$, $\mathbf{5}_3$ and $\mathbf{5}_6$, so to have full net generations, we require

$$N_7 = N_9 = 0 \quad \Rightarrow \quad \text{only } N_8 \text{ left free}$$

- No exotics from **10**'s and remaining matter-like **5** curve can be satisfied by choosing appropriate M's
- Satisfactory matter sector can be engineered easily

Case I: Higgs Sector is not Fine

- Higgs sector:

	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$
$\mathbf{5}_{H_u}$	$M_{\mathbf{5}_{H_u}}$	$M_{\mathbf{5}_{H_u}} + N_8$
$\mathbf{5}_1$	$M_{\mathbf{5}_1}$	$M_{\mathbf{5}_1} - N_8$
$\mathbf{5}_2$	$M_{\mathbf{5}_2}$	$M_{\mathbf{5}_2} - N_8$
$\mathbf{5}_4$	$M_{\mathbf{5}_4}$	$M_{\mathbf{5}_4} + N_8$

- We can pairwise decouple triplets from $\mathbf{5}_{H_u}$ and $\mathbf{5}_2$, and from $\mathbf{5}_1$ and $\mathbf{5}_4$
- However: Requiring a light up-type Higgs doublet without light triplets impossible
- Separately, down-type Higgs on $\mathbf{5}_4$ cannot be realised
- Cannot be cured by allowing exotics from the matter sector
- Either way: Light triplets, or no light doublets, i.e. doublet-triplet splitting problem not solved

Case II: Again not Fine

- Only one matter **10** curve, split with same parameter as up-type Higgs
- Higgs triplets cannot be decoupled by even matter parity singlets
- No exotic matter $\Rightarrow$ no doublet-triplet splitting
- Again: even allowing for exotics, no doublet-triplet splitting

Case II: Again not Fine

- Only one matter **10** curve, split with same parameter as up-type Higgs
- Higgs triplets cannot be decoupled by even matter parity singlets
- No exotic matter $\Rightarrow$ no doublet-triplet splitting
- Again: even allowing for exotics, no doublet-triplet splitting

Upshot: In both cases, doublet-triplet splitting in Higgs sector does not work, even when allowing for exotics from the matter sector!

Contents

Conclusions

- Analysed F-Theory GUT at "point of E_8" and in semilocal approach
- Goal: Find a locally defined matter parity to ensure proton stability
- Local model is very constrained: Two cases only
- Neither case can be embedded in semilocal framework (using spectral cover)
- Problem is doublet-triplet splitting in the Higgs sector
- Predictivity of local point in question – Crucial model features required to have nonlocal origin?
- Possible loophole: Spectral cover not the most general framework – T-Branes might help to get rid of exotics