(Non-)Universal Anomalies and Discrete Symmetries from the Heterotic String

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XXIV Workshop Beyond the Standard Model
Bad Honnef 2012

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[arXiv:1203.xxxx]

Motivation

- MSSM superpotential contains (potentially) bad terms:

$$W_{\text{bad}} \supset \mu H_u H_d + QLd^c + u^c d^c d^c + LLe^c + QQQL + u^c u^c d^c e^c + \cdots$$

- Forbid/constrain these operators by (discrete) symmetries, (matter parity, proton hexality, $\mathbb{Z}_4^R$,...) [many papers, many people at this workshop]
- In string constructions, discrete symmetries can arise as remnants of gauge and internal Lorentz symmetries

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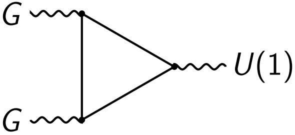
- In string constructions, discrete symmetries can arise as remnants of gauge and internal Lorentz symmetries
- Discuss anomalies in such models and the appearance of discrete symmetries in the $E_8 \times E_8$ heterotic string
- In particular: Consider $\mathbb{Z}_3$ orbifold and its blowup

Contents

Anomalies in MSSM Extensions

In four dimensions, anomalies come from triangle diagrams, e.g.



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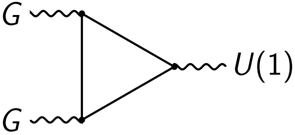

$$\propto \quad A_{G^2-U(1)} = \sum_f q_f \ell(\boldsymbol{r}_f)$$

Bottom-up: Focus on $G^2 - U(1)_A$, because

- $U(1)_A^2 - U(1)_Y$ anomaly should vanish
- $U(1)_A^3$ anomaly depends on extra SM singlets
- consider $\mathbb{Z}_N$ symmetries which arise as remnants of $U(1)_A$

[Araki et al. '08]

Top-down: We (in principle) know the spectrum, and so the anomaly coefficients

Anomaly Coefficients for MSSM with extra $U(1)$

Take MSSM with additional $U(1)_X$, charges $q_Q, \ldots, q_{H_d}$
$\rightsquigarrow$ Anomaly coefficients generically not universal

Impose e.g.

- allowed Yukawa couplings,
- $U(1)_X$ is flavour-blind and commutes with $SU(5)$ (but assume doublet-triplet splitting, i.e. no Higgs triplets),
- may or may not be an R symmetry (i.e. $R = 0$ or $R = 1$)

$$\begin{aligned}
A_{SU(3)^2-U(1)_X} &= 3\,(3q_{10} + q_{\bar{5}})\,-6\,,\\
A_{SU(2)^2-U(1)_X} &= 2\,(3q_{10} + q_{\bar{5}})\,-6\,,\\
A_{U(1)_Y^2-U(1)_X} &= 2\,(3q_{10} + q_{\bar{5}})\,-9\,.
\end{aligned}$$

For $\mathbb{Z}_N$ symmetries, coefficients might be universal $\mod N$ or $\mod \frac{N}{2}$

Anomaly Polynomial

[Wess, Zumino '71; Stora '84; Alvarez-Gaumé, Ginsparg '84]

Variation of path integral measure under gauge transformation:

$$\int \mathcal{D}\psi e^{\mathrm{i}S} \longrightarrow \int \mathcal{D}\psi e^{\mathrm{i}\mathcal{A}} e^{\mathrm{i}S}\,, \quad \mathcal{A} = \int I_d^{(1)}$$

Wess–Zumino consistency conditions $\rightsquigarrow$ descent equations

$$\mathrm{d}I_d^{(1)} = \delta I_{d+1}^{(0)}\,, \qquad \mathrm{d}I_{d+1}^{(0)} = I_{d+2}\,.$$

- Anomaly form $I_d^{(1)}$: linear in transformation parameter, polynomial in gauge connections and field strengths
- Chern–Simons form $I_{d+1}^{(0)}$: polynomial in gauge connections and field strengths
- Anomaly polynomial I_{d+2}: closed gauge invariant polynomial in field strengths – contains all relevant information about the anomaly

Treat $I_{d+1}^{(0)}$ and I_{d+2} as formal objects, but can be made rigorous

Green–Schwarz Mechanism

[Green, Schwarz '84]

Green–Schwarz mechanism: Cancel transformation of measure with variation of action – requires

a) factorisation of anomaly polynomial, $I_{d+2} = Y_{d+2-k} X_k$, and
b) $(k-2)$-form field B_{k-2} with gauge transformation

$$\delta B_{k-2} = -X^{(1)}_{k-2} \,,$$

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$\curvearrowright$ anomaly cancelled by variation of GS action

$$S_{\text{GS}} = \int \frac{1}{2} \left| \mathrm{d}B_{k-2} + X^{(0)}_{k-1} \right|^2 + B_{k-2} Y_{d+2-k} + \cdots$$

- Generalisation: sum of factorised anomalies $I_{d+2} = \sum_a Y^i X^i$ is cancelled by set of form fields B^i with appropriate transformation
- Exchanging $Y_{d+2-k} \leftrightarrow X_k$ corresponds to dualising $B_{k-2} \to \widetilde{B}_{d-k}$.

GS Mechanism in 10D

Two possible multiplets:

- supergravity multiplet $\left(e_A^M, \Psi_M, B_2, \chi, \phi\right)$
- vector multiplet $(\mathfrak{A}_M, \Lambda)$

Anomalies arise from gravitino, dilatino and gauginos, cancellation by Kalb–Ramond two-form B_2

$\rightsquigarrow$ gauge group must be $SO(32)$ or $E_8 \times E_8$, with $I_{12} = X_4^{\text{uni}} Y_8$ and

$$X_4^{\text{uni}} = \operatorname{tr} \mathfrak{R}^2 - \operatorname{tr} \mathfrak{F}'^2 - \operatorname{tr} \mathfrak{F}''^2$$
$$\delta B_2 = \operatorname{tr} \Theta \mathrm{d}\Omega - \operatorname{tr} \lambda' \mathrm{d}\mathfrak{A}' - \operatorname{tr} \lambda'' \mathrm{d}\mathfrak{A}''$$

[Bergshoeff et al. '82]

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[Bergshoeff et al. '82]

Field strength of B_2 has nontrivial Bianchi identity,

$$H_3 = \mathrm{d}B_2 + X_3^{\text{uni},(0)} \quad \Longrightarrow \quad \mathrm{d}H_3 = X_4^{\text{uni}}$$

$$\Longrightarrow \int_C X_4^{\text{uni}} = 0 \quad \text{for all four-cycles } C$$

GS Mechanism in 4D

$I_6 = X_4 Y_2 \rightsquigarrow Y_2 = \mathrm{d}A_A$ is field strength of the "anomalous $U(1)$"
Cancellation by two-form b_2 or scalar a ("axion") – by dualising, can restrict to scalars with transformation $a \to a - \lambda$

$$\Longrightarrow S_{\mathrm{GS}} = \int \frac{1}{2} \left|\mathrm{d}a + A_A\right|^2 + aX_4$$

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- Axion kinetic term gives Stueckelberg mass term for $U(1)_A \Rightarrow$ anomalous $U(1)$s get broken, but remain as (perturbative) selection rules
- X_4 contains field strengths of other gauge group factors G_i, weighted with the anomaly coefficients:

$$X_4 = A_{\mathrm{grav}-U(1)} \operatorname{tr} R^2 + \sum_i A_{G_i^2-U(1)_A} \operatorname{tr} F_i^2$$

- In particular: If anomaly is cancelled by Kalb–Ramond b_2 $\Rightarrow X_4$ is reduction of X_4^{uni}, and universal axion a_0 couples universally to all gauge groups

Overview of Heterotic GS Mechanisms

Distinguish Orbifolds and smooth Calabi–Yaus with vector bundles:

Orbifolds	Calabi–Yau X with Gauge Bundle
$B_2 = b_2$: single two-form in 4D $\rightsquigarrow$ universal axion a_0	$B_2 = b_2 + \beta_r E_r$, $E_r \in H^2(X)$ $\rightsquigarrow$ additional axions β_r
Universality: a_0 couples to reduction of X_4^{uni}	Couplings $\beta_r X_4^r$ depend on gauge background and curvature – some remnants of universality
(at most) one anomalous $U(1)$	Number of anomalous $U(1)$s given by rank of bundle

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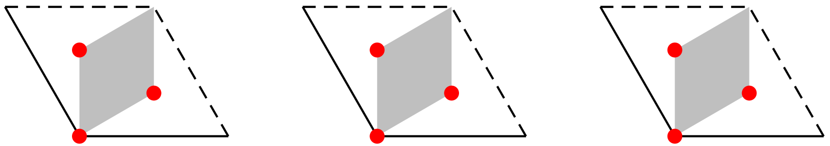
$T^6/\mathbb{Z}_3$ Orbifold

For illustration, consider simple $T^6/\mathbb{Z}_3$ orbifold model



$V = \frac{1}{3}\left(1, 1, -2, 0^5\right)\left(0^8\right)$, no Wilson lines
$\Rightarrow$ standard embedding, 27 equivalent fixed points

Gauge group	$E_6 \times SU(3)\,[\times E_8]$
spectrum	$3\left(\mathbf{27}, \overline{\mathbf{3}}\right) + 27\left[\left(\mathbf{27}, \mathbf{1}\right) + 3\left(\mathbf{1}, \mathbf{3}\right)\right]$

In particular, no anomalous $U(1)$, hence universal axion does not shift under gauge transformations, and no FI term has to be cancelled

Blowup

[Groot Nibbelink et al. 07-09]

Idea: VEVs for twisted states (blow-up modes) corresponds to smoothing out singularities – connection to smooth Calabi–Yau with bundles

In particular: VEVs for twisted non-oscillator states $(\mathbf{27}, \mathbf{1})$

$\leftrightarrow$ line bundles (i.e. Abelian fluxes)

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$\leftrightarrow$ line bundles (i.e. Abelian fluxes)

Procedure:

1. Replace fixed points by exceptional divisors E_r ($\mathbb{P}^2$s)
2. Turn on Abelian gauge flux along the exceptional divisors,

$$\mathcal{F} = V_r^I H_I E_r \,, \quad H_I\text{: Cartan generators of } E_8$$

   Bundle vector V_r given by shifted momentum of blow-up mode

Note: Line bundles don't reduce the rank, axion shifts do:

$$B_2 = b_2 - \beta_r E_r \,, \quad \delta B_2 = -\operatorname{tr} \lambda \mathcal{F} \quad \Rightarrow \quad \delta \beta_r = \operatorname{tr} \lambda V_r$$

Donaldson–Uhlenbeck–Yau equation

Bundle has to satisfy (analogue of D-term eqaution)

$$0 = \frac{1}{2} \int_X J \wedge J \wedge \mathcal{F} = \sum_r \mathrm{vol}(E_r) V_r \,, \quad \text{with all } \mathrm{vol}(E_r) > 0$$

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Simple choice:

1. Take three bundle vectors $V_{(1,2,3)}$ which sum to zero, $V_{(1)} + V_{(2)} + V_{(3)} = 0$
2. Assign $V_{(1)}$ to first k exceptional divisors, $V_{(2)}$ to the next p and $V_{(3)}$ to remaining $27 - k - p = q$

$\Rightarrow$ DUY Equation becomes

$$\sum_{r=1}^{k} \mathrm{vol}\, E_r = \sum_{r=k+1}^{k+p} \mathrm{vol}\, E_r = \sum_{r=k+p+1}^{27} \mathrm{vol}\, E_r$$

Bundle Vectors

Choose bundle vectors from p_{sh} of twisted **27**,
$\rightsquigarrow$ flux quantisation and Bianchi Identity fulfilled automatically, break $E_6 \to SO(10) \times U(1)$ at each E_r

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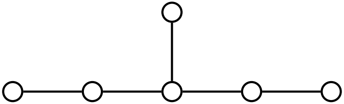
Specifically, choose

$$V_{(1)} = \frac{1}{3}\left(2, 2, 2, 0^5\right)$$
$$V_{(2)} = \frac{1}{3}\left(-1, -1, -1, 3, 0^4\right)$$



$V_{(3)}$ is linearly dependent, does not break further
$\Rightarrow$ gauge group $SO(8) \times U(1)_A \times U(1)_B \times SU(3)$

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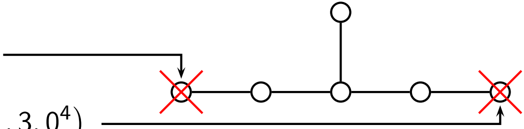

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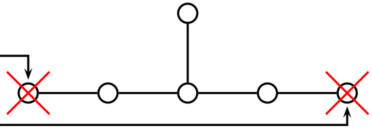

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Decomposition

$$\mathbf{27} \longrightarrow \mathbf{8}_{\mathrm{S}(1,-1)} \oplus \mathbf{8}_{\mathrm{C}(1,1)} \oplus \mathbf{8}_{\mathrm{V}(-2,0)} \oplus \mathbf{1}_{(-2,-2)} \oplus \mathbf{1}_{(-2,2)} \oplus \mathbf{1}_{(4,0)}$$

Massless spectrum depends on (k, p, q)

Blowup Anomalies

Orbifold: No anomaly
Blowup: two $U(1)$'s, different spectrum
$\Rightarrow$ anomaly polynomial $I_6 = \int_X I_{12}$ with backgrounds inserted

$$\Rightarrow I_6 \sim {F_A}^3 \cdot \left(\frac{k-6}{12}\right) + F_A F_B^2 \cdot \left(\frac{k-18}{4}\right)$$
$$+ F_A \left[\operatorname{tr} F_{SU(3)}^2 + \operatorname{tr} F_{SO(8)}^2 + \frac{7}{48} \operatorname{tr} R^2\right] \cdot \left(\frac{k-9}{2}\right)$$
$$+ F_B \left[\frac{1}{8} F_B^2 + \frac{1}{48} F_A^2 + \operatorname{tr} F_{SU(3)}^2 + \operatorname{tr} F_{SO(8)}^2 + \frac{7}{48} \operatorname{tr} R^2\right] \cdot \left(\frac{p-q}{2}\right)$$

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- For $p = q$, $U(1)_B$ is omalous, while $U(1)_A$ is always anomalous
- Remnant universality: Coefficients of non-Abelian groups from one E_8 are equal, and proportional to gravitational anomaly (only true if one E_8 unbroken)

Axion Shifts and massive $U(1)_B$

Axions β_r shift under $U(1)_{A,B}$ – universal axion does not!
$\Rightarrow$ $U(1)_A$ and $U(1)_B$ always massive, even if one of them is omalous:

$$\int_X H_3 \wedge *H_3 = A^I_\mu A^{\mu\, J} M^2_{IJ} + \cdots \,, \qquad M^2_{IJ} = V^I_r V^J_s \cdot \int_X E_r \wedge *_6\, E_s$$

Mass matrix is positive definite and always rank-two (and depends on the Kähler parameters)

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$\rightarrow$ Stueckelberg mass possible without anomaly (but not *vice versa*)

Note: Still a coupling of the universal axion to X_4, as required by supersymmetry

Contents

Gauge Symmetry

Remnant non-R symmetries: discrete subgroup of $U(1)_A \times U(1)_B$ which leaves VEVs invariant
Blow-up modes:

$$\mathbf{1}_{4,0}\,, \quad \mathbf{1}_{-2,-2}\,, \quad \mathbf{1}_{-2,2}$$

$\Rightarrow$ discrete remnant $\mathbb{Z}_4 \times \mathbb{Z}_4$, generated by

$$T_\pm : \phi_{(q_A,q_B)} \longrightarrow \exp\left\{\frac{2\pi \mathrm{i}}{4}\,(q_A \pm q_B)\right\} \phi_{(q_A,q_B)}$$

However: Charges of all massless fields are even under both $\mathbb{Z}_4$s
$\rightsquigarrow$ only $\mathbb{Z}_2 \times \mathbb{Z}_2$ realised on massless spectrum

Both $\mathbb{Z}_2$ factors are omalous

R Symmetries

- R symmetries do not commute with SUSY $\leftrightarrow \theta$ transforms, and different components of SUSY multiplets have different charges
- Only defined up to mixing with non-R symmetries
- For $\mathcal{N} = 1$ SUSY, only one $\theta \rightsquigarrow$ only one $U(1)$ or $\mathbb{Z}_N$ R symmetry – otherwise can redefine generators such that only one acts on θ
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- In compactifications, internal Lorentz transformations treat spinors and scalars differently $\rightsquigarrow$ can lead to R symmetries in 4D
- Orbifolds are special points in moduli space, so expect more symmetries – in particular, for general smooth spaces, expect no R symmetry in general

R Symmetries from Orbifolds

R transformations from sublattice rotations act as

$$\mathcal{R} : \Phi \longrightarrow e^{2\pi \mathrm{i} v R} \Phi$$

where (for Z_3 orbifolds) $v = \left(\underline{\tfrac{1}{3}, 0, 0}\right)$, $R = q_{\mathrm{sh}} - \Delta N$

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where (for Z_3 orbifolds) $v = \left(\underline{\frac{1}{3}, 0, 0}\right)$, $R = q_{\text{sh}} - \Delta N$
Symmetry conventions somewhat tricky:

- For bosons, both v and R quantised in units of $\frac{1}{3}$, so $\mathbb{Z}_9$ symmetry (i.e. $\mathcal{R}^9 = \mathbb{1}$)
- For fermions, $R^{\text{f}} = R^{\text{b}} - \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$, so θ has charge $\frac{1}{6}$ (i.e. $\mathbb{Z}_6$ "R symmetry")
- Hence: $\mathbb{Z}_{18}$ symmetry with charges for

$$(\text{bosons}, \text{fermions}, \theta) = \frac{1}{18} (2n, 2n - 3, 3)$$

- Can redefine charges such that θ has charge 1 and superpotential has charge 2 mod 6, but then fields have non-integer charges

Model: VEV picture

Our blow-up modes have

$$R = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

Seek unbroken combinations of the three sublattice rotations and $U(1)_{A,B}$:

$$\begin{aligned} \mathbf{1}_{4,0} &\longrightarrow \mathcal{R}_1^p \mathcal{R}_2^q \mathcal{R}_3^r \, T_A \, T_B \mathbf{1}_{4,0} = \mathbf{1}_{4,0} \,, \\ \mathbf{1}_{-2,-2} &\longrightarrow \mathcal{R}_1^p \mathcal{R}_2^q \mathcal{R}_3^r \, T_A \, T_B \mathbf{1}_{-2,-2} = \mathbf{1}_{-2,-2} \,, \\ \mathbf{1}_{-2,2} &\longrightarrow \mathcal{R}_1^p \mathcal{R}_2^q \mathcal{R}_3^r \, T_A \, T_B \mathbf{1}_{-2,2} = \mathbf{1}_{-2,2} \end{aligned}$$

However, this implies $p + q + r = 3 \Rightarrow$ only a $\mathbb{Z}_2$ R symmetry survives in blow-up

Model: GLSM Description

[Witten '93;Groot Nibbelink '10; Blaszczyk et al. '11]
(cf. talk by Fabian Ruehle this afternoon)

Algebraically, describe the orbifold by ($\mathbb{P}^2[3]$ is a T^2)

$$\frac{\mathbb{P}^2[3] \times \mathbb{P}^2[3] \times \mathbb{P}^2[3]}{\mathbb{Z}_3}$$

Blowup (crepant resolution) in $(0,2)$ GLSM description:

- Introduce extra coordinates (exceptional divisors) and $U(1)$s
- Geometry given by F and D term equations, GLSM FI terms become CY Kähler parameters
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- Geometry given by F and D term equations, GLSM FI terms become CY Kähler parameters
- Bundle given by "chiral-Fermi" superfields Λ_I with charges determined by the bundle vectors

$\exists$ discrete automorphisms of the coordinates which leave F and D terms invariant $\rightsquigarrow$ discrete symmetries – these are R symmetries if holomorphic three-form ω transforms (ω transforms like W, so $\mathbb{Z}_2$ is invisible)

GLSM for $(k, p, q) = (9, 9, 9)$ Model

Two types of R symmetries:

- $\mathbb{P}^2$ coordinates $z_{i\alpha}$ only appear as $z_{i\alpha}^3$ or $|z_{i\alpha}|^2 \Rightarrow \mathbb{Z}_3$ rotations
  Presumably broken by deformations of Kähler potential terms (schematically, ϕ_{4d} massless modes in 4d)

$$\int \mathrm{d}^2\theta^+ \phi_{\text{4d}}(x^\mu) N(z, x_i)\, \Lambda\bar{\Lambda}$$

  Fits with orbifold: Bundle corresponds to blowup

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  $$\int \mathrm{d}^2\theta^+ \phi_{4\mathrm{d}}(x^\mu) N(z, x_i) \Lambda\bar{\Lambda}$$

  Fits with orbifold: Bundle corresponds to blowup
- For certain values of Kähler parameters, permutation symmetries: Group-wise exchanges of exceptional divisors – in orbifold, similar interpretation for equal VEVs

$\Rightarrow$ Generically, no R symmetry (except at most $\mathbb{Z}_2$), but enhanced at certain loci of parameter space

Contents

Conclusions

- Discussed anomalies in 4D low-energy theories from the heterotic string
- Anomalies are generically not universal – the orbifold anomalous $U(1)$ is the exception because it is cancelled by the universal axion
- For blowups of heterotic orbifolds, many axions possible

Conclusions

- Discussed anomalies in 4D low-energy theories from the heterotic string
- Anomalies are generically not universal – the orbifold anomalous $U(1)$ is the exception because it is cancelled by the universal axion
- For blowups of heterotic orbifolds, many axions possible
- Some rest of universality: Non-Abelian subgroups of one E_8 have universal coefficients (but for $SU(5)$, this is before doublet-triplet splitting)
- Line bundles do reduce the rank via the axion shift – also omalous $U(1)$s can become massive
- Blow-ups can leave gauged discrete subgroups unbroken – important for phenomenology
- On orbifold, R symmetries exist but are broken by the blow-up

Outlook

- Found nice agreement between orbifold and blow-up picture, up to some subtleties
- "Geometry part" of GLSM generically has many "unbreakable" R-like symmetries – seem to be broken by the bundle, but better understanding of their breaking required
- Linked to determination of charged massless spectrum
- Different non-generic type of R symmetries: Exchange symmetries, appearing for certain loci in Kähler moduli space, e.g. exchange of exceptional divisors if their volumes are equal
- Study these symmetries for more realistic models, including Wilson lines etc.