Anomaly Cancellation and FI Terms in Heterotic Models

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[W. Buchmüller, CL, J. Schmidt, JHEP09(2007)113]

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Contents

1. Overview
2. Anomalies
3. FI Terms and Vacuum Configurations

Overview

- In orbifold theories, anomalies can arise in bulk and on fixed point
- Cancellation of anomalies is guaranteed by string theory, so it provides a non-trivial check of any calculated spectrum
- On four dimensional fixed points and in the four-dimensional limit, anomalous $U(1)$'s occur, together with corresponding large FI terms
- For unbroken SUSY, these FI terms need to be cancelled by VEVs of charged fields
- In four-dimensional limit, the set of VEVs determines phenomelology
- From higher-dimensional perspective, local cancellation of FI terms is much more involved

Anomalies on the Orbifold

- Anomalies in bulk and on brane are chracterised by anomaly polynomial: $(d+2)$–form I_{d+2}
- For $T^2/\mathbb{Z}_2$ limit, polynomial is an 8-form::

$$I_8 = \frac{1}{2} I_{T^2} + \sum_f I_f \delta^2(y - y_f)\, \mathrm{d}y^5 \mathrm{d}y^6$$

- I_{T^2}: Anomaly polynomial of bulk fields on T^2 (hence factor $\frac{1}{2}$)
- I_f: Localised 6-form at fixed point f,

$$I_f = I_f(\text{brane fields}) + \frac{1}{4} I_f(\text{surviving bulk fields})\,.$$

Bulk fields surviving the projection at fixed point f contribute with a factor of $\frac{1}{4}$ because there are four fixed points.

Green–Schwarz Mechanism

- Green–Schwarz mechanism can cancel reducible anomalies, i.e. those for which

$$I_{T^2} = X_4 Y_4 \qquad\qquad I_f = X_4|_f \, Y_{2,f}$$

- Here X_4 is fixed by the variation of the B_2 field via descent equations to be

$$X_4 = \mathrm{tr}\, R^2 - \sum_{A \text{ non-Abelian}} \alpha_A \, \mathrm{tr}\, F_A^2 - \sum_u \alpha_u F_u^2 \,,$$

- A labels non-Abelian gauge group factors, u labels $U(1)$'s.
- $X_4|_f$ is the projection of the fixed point, including restriction to unbroken subgroup at fixed point f
- $Y_{2,f}$ is gauge-invariant two-form $\Rightarrow Y_{2,f} \propto F_f$: Field strength of anomalous $U(1)$ at fixed point f

Factorisation is a Strong Constraint

- In our model, there are two inequivalent fixed points ($\rightsquigarrow$ two independent local anomaly polynomials I_0, I_1) with six and eight $U(1)$ factors
  - I_{T^2} has 160 coefficients ($\mathrm{tr}\, R^4$, $\mathrm{tr}\, F_A^2 \,\mathrm{tr}\, F_B^2$, $F_u F_v F_w F_x$, $\ldots$)
  - I_0 and I_1 have 82 and 178 coefficients, respectively
  - $\Rightarrow$ 420 coefficients
- Since Y_4, $Y_{2,0}$ and $Y_{2,1}$ have only 33 free parameters, the factorisation involves 387 extra conditions!
- String Theory and modular invariance conditions on shifts and Wilson lines guarantee anomaly cancellation
- Check of calculated spectrum
- Determination of local anomalous $U(1)$'s

Anomalous $U(1)$'s and FI Terms

- At each fixed point, a FI term is generated for the anomalous $U(1)$,

$$\xi_f = \frac{M_{\mathrm{P}}^2}{192\pi^2} \operatorname{tr} t_f^{\mathrm{an}}$$

- Here t_f^{an} is the generator of the anomalous $U(1)$, and the trace is given by

$$\operatorname{tr} t_f^{\mathrm{an}} = \sum_{\text{brane } i} q_i^{\mathrm{an}} + \frac{1}{4} \sum_{\text{surviving bulk } i} q_i^{\mathrm{an}}$$

- Sum of local anomalous $U(1)$'s weighted by their FI terms gives anomalous $U(1)$ in 4d limit, this $U(1)$ is broken by Green–Schwarz mechanism

D-Flatness

- $D = 0$ SUSY condition for non-anomalous gauge symmetries is satisfied by finding gauge invariant monomial $I = \phi_1^{n_1} \cdots \phi_N^{n_N}$ and setting

$$\frac{\langle \phi_1 \rangle}{\sqrt{n_1}} = \frac{\langle \phi_2 \rangle}{\sqrt{n_2}} = \cdots = \frac{\langle \phi_N \rangle}{\sqrt{n_N}} = \overline{\phi}$$

- Each independent monomial correspond to a D-flat direction
- The FI term modifies the D-term condition for the anomalous $U(1)$:

$$D_{\text{an}} = \sum_i q_i^{\text{an}} \left|\phi_i\right|^2 + \xi$$

- This requires a monomial with negative anomalous charge (for $\xi > 0$) and stabilises the flat directions at $\overline{\phi} = \xi / Q_{\text{an}}$
- These VEVs are large, $\langle \phi \rangle \sim M_{\text{P}}/(10 \ldots 100)$, so higher order terms in superpotential expansion need not be very suppressed

Vacuum Selection

- Choice of vacuum restricted by phenomenological constraints, e.g.
  - Gauge group $G = G_{\text{SM}} \times G_{\text{hidden}}$
  - Decoupling of exotic states via VEV mass terms
  - Yukawa couplings for quarks and leptons, in particular heavy top
  - Light Higgses
- In our model, the vacuum breaks R parity, so proton decay is a problem
- Models with R-parity or some variant exist in "Mini-Landscape"

  [Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter 06, 07]
- F-flatness and D-flatness generically can be satisfied simultaneously

Field Localisation due to FI Terms

- Different FI terms at different fixed points have important effects on field configuration:
- Background profile for extra-dimensional components of gauge field, including localised field strength

  [Groot Nibbelink, Nilles, Olechowski 02;Lee, Nilles, Zucker 04]
- This in turn implies localisation of charged bulk scalars at or away from fixed points, i.e. VEVs are not constant in the extra dimension
- Four-dimensional cancellation of FI terms generically involves twisted and untwisted sector field VEVs
  - Non-constant untwisted sector field might fail to cancel FI terms
  - Large VEVs for localised fields is related to blow-up of orbifold singularities
- Furthermore, profiles of bulk fields may cause warping and hence break supersymmetry

Summary

- Anomaly cancellation is a nontrivial check of the spectrum and determines the anomalous $U(1)$'s
- FI terms induced by anomalous $U(1)$'s need to be cancelled:
  - In four-dimensional limit, D-flatness can be satisfied by finding appropriate monomials, and phenomenologically attractive vacuum configurations are known
  - Local cancellation of FI terms much more involved
- VEV profiles and backreaction on geometry need to be investigated
- Simple orbifold GUT description might not be applicable