Fixing D7 Branes by Fluxes in F–Theory

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based on:
work in progress with A. Braun, A. Hebecker and R. Valandro and
A. Braun, A. Hebecker, H. Triendl [arXiv:0801.2163]

Motivation

- Type IIB orientifolds are one of the best-understood string setups (e.g for moduli stabilisation, warped compactifications and hierarchies, string cosmology)
- Drawback: Particle physics model building not very explicit yet, as opposed to e.g. heterotic orbifolds
- Aim: Develop tools for model building, in particular configurations of D7 branes their stabilisation
- Appropriate framework: F-Theory [Vafa;Sen]
- Simplest example: $K3 \times K3$

Contents

F-Theory and Type IIB

Consider Type IIB compactifications (on manifold B) with D7-branes (8dim obj).

- D7 is charged under the axiodilation $\Rightarrow \tau = C_0 + ie^{\phi}$ has non-trivial monodromy T when going around a D7.
- T is one of the two generators of $SL(2,\mathbb{Z})$-sym group of IIB:

$$\left.\begin{array}{ll} T: & \tau \mapsto \tau + 1 \\ S: & \tau \mapsto -\frac{1}{\tau} \end{array}\right\} \Rightarrow \text{ similar to torus complex structure}$$

  $\rightsquigarrow$ interpret axiodilaton as complex structure of axiliary torus
- O7 planes: Monodromy (fibre involution) $\cdot\, T^{-4}$
- τ is a field varying over $B \Rightarrow$ one torus (elliptic curve) for each point of $B \Rightarrow$ Elliptic fibration over the base B: manifold Y_8.

F-Theory and Type IIB

Working definition: F-theory: 12d theory compactified on elliptically fibred manifold Y_8 resulting in type IIB on the base B, i.e. the two additional dimensions are auxiliary

F-theory background: both geometric background (B) and D7 positions are encoded in geometry of Y_8.

- At the brane positions, $\tau \longrightarrow i\infty$:
  $\Rightarrow$ The D7's are at points of B where fibre degenerates
  $\Rightarrow$ Stacks of branes: Singularities of Y_8
- F-theory allows to automatically include obstructions on D7 motions, without extra constraints. [Braun,Hebecker,Triendl]
- In the following, we concentrate on the weak coupling limit, where some complication do not occur [Sen]

Example: Type IIB Orientifold on $\mathbb{R}^{1,7} \times T^2/\mathbb{Z}_2$

Consider type IIB on $\mathbb{R}^{1,7} \times T^2$, orientifolded by $(-1)^F \Omega_p \sigma$, where

$$\sigma : z \mapsto -z \qquad \text{is the involution of the } T^2$$

- 4 singularities of $T^2/\mathbb{Z}_2$ = four O7 planes
- Each O7 carries four D7's. $\Rightarrow$ Gauge group $SO(8)^4$.
- F-theory background: T^2 fibration over $\mathbb{CP}^1 = S^2$ ($K3$) with four $SO(8)$-singularities where the the fibre degenerates.
- O7 planes have monodromy T^{-4}, so axiodilaton is constant
- If we move some D7, the fibration changes, and we get a different gauge group

F-Theory/M-Theory Duality

For calculations, it is advantageous to consider duality between F- and M-theory:

- Consider M-theory on $CY_3 \times T^2$ ($\rightsquigarrow$ 3d effective theory)
- Compactification on one S^1 gives type IIA on $CY_3 \times S^1_A$
- T duality along the S^1_A gets us to type IIB on $CY_3 \times S^1_B$ with inverse radius, $R_B = 1/R_A$
- In the limit of $R_B \to \infty$, we recover type IIB on CY_3, i.e. four flat dimensions $\rightsquigarrow$ F-theory — on the M-theory side, this means taking the torus volume to vanish
- Hence, we can think of F-theory as being dual to M-theory on an elliptically fibred CY four-fold with vanishing fibre volume

[Vafa;Schwarz;Aspinwall;Gukov,Vafa,Witten]

F-Theory/M-Theory Duality

Straightforward map of moduli spaces on both sides:
Moduli space of Y_8, excluding the fibre volume

$\Rightarrow$ We can use the M-theory language to investigate F-theory!

For moduli stabilisation by fluxes, the procedure is:

- Take F-theory background and add type IIB fluxes (G_3 and F_2)
- Map to M-theory (G_4 flux), minimise the moduli potential
- Map back to F-theory

And now for...

Branes and Cycles

- Gauge enhancement occurs on brane stacks, i.e. when branes move on top of each other
- Single branes induce singularities of the fibration, stacks of branes cause singularities of the compactification space
- Singularities can be classified by ADE type
  $\rightsquigarrow$ ADE gauge groups, i.e. $SU(k)$, $SO(2k)$, E_6, E_7, E_8
- Brane distane can be measured by volume of certain cycles between the branes

$\Rightarrow$ Stacks occur when these cycles shrink to zero size

Remarks on K3

- Only Calabi–Yau two-fold (i.e. four real dimensions)
- $H^2(K3, \mathbb{R})$ is a 22-dim space. The intersection metric is

$$M(v, w) \equiv \int_{K3} v \wedge w \qquad \curvearrowright \qquad H^2(K3, \mathbb{R}) \cong \mathbb{R}^{3,19}$$

- Hyperkähler manifold: Kähler form j and holomorphic two-form ω fixed by three timelike two-forms ω_i, $M(\omega_i, \omega_j) = \delta_{ij}$ and the overall volume ν via

$$\omega = \omega_1 + \mathrm{i}\omega_2 \,, \qquad\qquad j = \sqrt{2\nu}\,\omega_3$$

- Metric is invariant under $SO(3)$ rotation of the ω_i

$\Rightarrow$ Geometry given by timelike 3-plane $\langle \omega_i \rangle \in \mathbb{R}^{3,19}$

$\Rightarrow$ Volume of any cycle is measured by projection on the three-plane (times $\sqrt{\nu}$)

K3: Cycles

- We can choose an integral basis of $H^2(K3, \mathbb{R})$ such that

$$M = U \oplus U \oplus U \oplus (-E_8) \oplus (-E_8) \ , \quad \text{where } U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

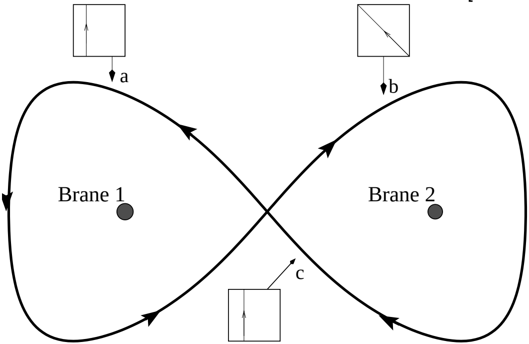
- The U blocks contain the timelike directions $\rightsquigarrow$ the ω_i need to have components in these blocks
- Elliptically fibred $K3$: Require integral cycles B and F (base and fibre)
  - with intersection matrix $\begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix}$, equivalent to a U block
  - and $M(B, \omega) = M(F, \omega) = 0$

$\Rightarrow$ One ω_i must be entirely in this U block, two $\perp$ to it

$\Rightarrow$ Remaining freedom: Two-plane in $\mathbb{R}^{2,18}$

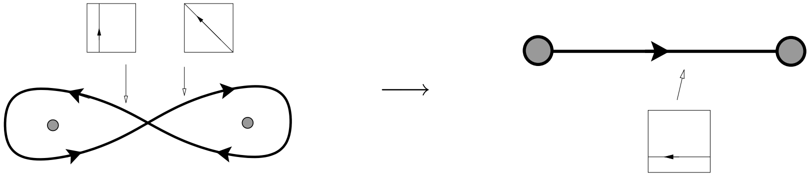
Cycles between D-Branes on $T^2/\mathbb{Z}_2$

[Braun, Hebecker, Triendl]

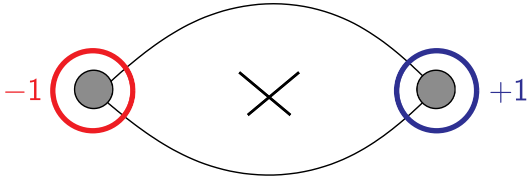


- One leg in the base, one in the fibre torus
- Monodromy $\tau \to \tau + 1$ twists fibre leg when going around a D-brane
- They are topologically a sphere $\leftrightarrow$ self-intersection -2.

- These cycles can be collapsed to a line in the base:

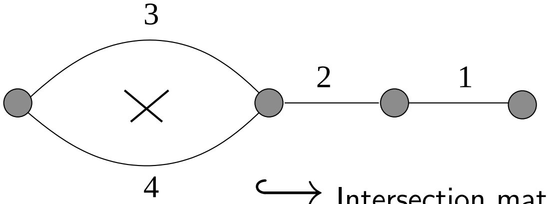


- Two cycles meeting at a brane have intersection 1
- O7 plane monodromy includes involution of the fibre, hence two cycles encircling an O7 plane ($\times$) do not intersect:



Shrinking Cycles and Gauge Enhancement

Intersection matrix of shrinking cycles determines gauge group:
Consider e.g. orientifold: One O7, four D7s



$\hookrightarrow$ Intersection matrix $\begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & -2 \end{pmatrix}$

$\Rightarrow$ Cartan matrix of $SO(8)$: If these cycles shrink to zero size, the resulting gauge group will be $SO(8)$

Note: Only cycles with self-intersection -2 produce singularities when shrinking

Cycles at the $SO(8)$ Singularity

Complex structure ω can be expanded in the vectors E_I and e^i, e_i of the $E_8 \times E_8$ and U-blocks with coefficients $W_I = W_I^1 + uW_I^2$, u, s

- Cf. Wilson line breaking in heterotic theory: W^1, W^2 act like Wilson lines on a T^2, i.e. one-to-one correspondence between surviving roots and shrinking cycles
- Take $W^1 = (0^4, \frac{1}{2}^4, 0^4, \frac{1}{2}^4)$, $W^2 = (1, 0^7, 1, 0^7)$

$\Rightarrow$ 16 shrinking cycles in 4 sets, each with intersection matrix D_4

$\Rightarrow$ $SO(8)^4$ singularity

Cycles at the $SO(8)$ Singularity

By chosing new basis elements α and β, we can give an expansion of ω adapted to the $SO(8)^4$ point:

$$\omega = \frac{\alpha}{2} + u\,e_2 + s\,\frac{\beta}{2} - \left(u\,s - \frac{z^2}{2}\right) e_1 + z_I \hat{E}_I$$

- $z_I \longrightarrow$ brane positions, $z_I = 0$ is $SO(8)$ point
- u and s: Complex structure of base, i.e. position of O7's and (constant) axiodilaton
- s: Complex structure of fibre torus, i.e. axiodilaton

$\hookrightarrow$ Explicit mapping between ω and the positions of the branes.

And now for...

Fluxes

To fix a certain D7 configuration, we want to stabilise the corresponding point in moduli space by fluxes:

- Fluxes: Background values of p-form field strengths
- In type IIB, there are
  - three-form flux G_3 on the whole space
  - two-form flux F_2 of brane gauge theories
- In F-theory, these are combined into four-form flux G_4 (from M-theory)
- Consistency conditions on flux choice:
  - Flux quantisation requires flux to be integral
  - Tadpole cancellation condition

$$\int_M G_4 \wedge G_4 \leq \frac{\chi}{24}$$

Fluxes on $K3 \times K3$

- Aim: determine explicitly the flux stabilizing a desired D7 configuration.
- The first example we study is $Y = K3 \times \widetilde{K3}$

  [Lüst,Mayr,Reffert,Stieberger; Görlich,Kachru,Tripathy,Trivedi; Aspinwall,Kallosh; Dasgupta,Rajesh,Sethi]
- Toy model (no intersections, hence no chiral matter)
- To find the moduli potential we use the language of M-theory and then we map back the results
- M-theory on $K3 \times \widetilde{K3}$: four-form flux $G = G^{I\Lambda}\eta_I \wedge \tilde{\eta}_\Lambda$ with two legs on each $K3$ (because of 4d Lorentz invariance)

  $\hookrightarrow$ We can associate with G two homomorphisms:

$$G : H^2(K3) \to H^2(\widetilde{K3}) \qquad G^a : H^2(\widetilde{K3}) \to H^2(K3)$$

where G^a is defined by $M[w, G^a\tilde{v}] = \widetilde{M}[G\,w, \tilde{v}]$.

Flux Potential on $K3 \times K3$

Same starting point as [Haack,Louis]:

$$V = \frac{1}{4\mathcal{V}^3}\left(\int_{K3\times\widetilde{K3}} G \wedge *G - \frac{l_M^6 \chi}{12}\right)$$

- $\mathcal{V}$: Overall volume of $K3 \times \widetilde{K3}$
- V depends on the metric moduli:
  - Metric is fixed by ω_i and $\tilde{\omega}_j$ (up to $SO(3)$ rotations), plus the two volumes
  - Inequivalent $\{\omega_i\}$ are given by different way of putting a 3-plane into $\mathbb{R}^{3,19}$ $\rightarrow$ 57 moduli
  - Total number of moduli is $2 \times (57 + 1)$ $(\delta\omega_i, \delta\tilde{\omega}_j, \nu, \widetilde{\nu})$
- $K3 \times \widetilde{K3}$ is not a proper CY_4, so some differences

Flux Potential on $K3 \times K3$

Flux potential in terms of the ω_i, $\tilde{\omega}_i$ and volumes:

$$V = -\frac{1}{2(\nu \cdot \widetilde{\nu})^3} \left(\sum_j \| G\, \tilde{\omega}_j \|_\perp^2 + \sum_i \| G^a \omega_i \|_{\widetilde{\perp}}^2 \right)$$

Here $\|v\|_\perp^2 = \left\| \left(v - \sum_i \left(v \cdot \omega_i \right) \omega_i \right) \right\|^2$ is the norm of the part of v orthogonal to the three-plane

- Positive definite potential
- Manifestly symmetric under $SO(3)$
- Minima at $V = 0$:

$$G\, \tilde{\omega}_j \in \langle \omega_1, \omega_2, \omega_3 \rangle \qquad G^a \omega_i \in \langle \tilde{\omega}_1, \tilde{\omega}_2, \tilde{\omega}_3 \rangle$$

- ν and $\widetilde{\nu}$ are unfixed, flat directions (when $V = 0$)

Existence of $V = 0$ Minima

There exist a minimum iff $\exists$ changes of bases $P, \widetilde{P} \in SO(3,19)$ s.t.

$$P^{-1} G \widetilde{P} = G_d \equiv \begin{pmatrix} a_3 & 0 \\ 0 & c_{19} \end{pmatrix}$$

a_3 and c_{19} diagonal matrices with eigenvalues a_i, c_q.

- The first 3 vectors of the new bases are $+$-norm and orthogonal. We call them ω_i and $\tilde{\omega}_j$.
- $\tilde{\omega}_j$ are mapped to ω_i by G and *vice versa*.
- Not every matrix is diagonalizable by two $SO(n,m)$ transformations:

G bi-diagonalisable $\Longleftrightarrow$ $G^a G$ is diagonalisable with all $\lambda_i > 0$.

$\hookrightarrow$ There are fluxes that give $V > 0$ without points where $V = 0$!

Flat Directions?

Assume $\{\omega_i, \tilde{\omega}_j\}$ is absolute minimum of the potential $\leadsto$ are all moduli stabilised?
$\{\omega_i, \tilde{\omega}_j\}$ isolated minimum $\Leftrightarrow \nexists$ continuous deformations $\{\delta\omega_i, \delta\tilde{\omega}_j\}$ s.t.

$$G\left(\tilde{\omega}_j + \delta\tilde{\omega}_j\right) \in \langle(\omega_i + \delta\omega_i)\rangle \qquad G^a(\omega_i + \delta\omega_i) \in \langle(\tilde{\omega}_j + \delta\tilde{\omega}_j)\rangle$$

Define $\delta\tilde{\omega}_j = \tilde{\beta}_j^q \tilde{u}_q$ and $\delta\omega_i = \beta_i{}^p u_p$ (with $\tilde{u}, u$ orthogonal to $\tilde{\omega}_j, \omega_i$).

$$G\left(\tilde{\omega}_k + \delta\tilde{\omega}_k\right) = a_i(\omega_i + \delta\omega_i) + \left(c_p\tilde{\beta}_k{}^p - a_k\beta_i{}^p\right) u_p$$
$$G^a(\omega_h + \delta\omega_h) = a_h(\tilde{\omega}_j + \delta\tilde{\omega}_j) + \left(c_q\beta_k{}^q - a_h\tilde{\beta}_j{}^q\right) \tilde{u}_q$$

Complicated linear system!

Flat Directions?

Can be rewritten in matrix form as

$$\left(\begin{array}{ccc|ccc} c_{19} & & & & & \\ & c_{19} & & & -a_3 \otimes \mathbb{1}_{19} & \\ & & c_{19} & & & \\ \hline & & & c_{19} & & \\ & -a_3 \otimes \mathbb{1}_{19} & & & c_{19} & \\ & & & & & c_{19} \end{array}\right) \begin{pmatrix} \tilde{\beta}_1 \\ \tilde{\beta}_2 \\ \tilde{\beta}_3 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = 0 \quad \forall \beta, \tilde{\beta}$$

Isolated minimum requires the determinant of this matrix to vanish. By determinant condensation, this is equivalent to the condition that $\nexists (i, q)$ such that $a_i = c_q$:

$\Rightarrow$ If $a_i \neq c_q \; \forall i, q$, then all the moduli (except $\nu, \widetilde{\nu}$) are stabilised!

(Non-)Supersymmetric Minima

- A vacuum is supersymmetric if G is [Becker-Becker]
  - primitive wrt $J = j + \tilde{j}$, i.e. $J \wedge G = 0$
  - (2,2) wrt to the c.s. $\Omega = \omega \wedge \tilde{\omega}$, i.e. $\Omega \wedge G = 0$
- This translates into the conditions:
  - $G\,\tilde{\omega}_3 = 0$ and $G^a \omega_3 = 0 \Rightarrow a_3 = 0$;
  - $\omega \cdot G\,\tilde{\omega} = 0 \Rightarrow a_1 = a_2$.

Minima at $V = 0$ can be:

- $\mathcal{N} = 2$ when $a_1 = a_2 = a_3 = 0$;
- $\mathcal{N} = 1$ when $a_3 = 0$ and $a_1 = a_2 \neq 0$;
- $\mathcal{N} = 0$ otherwise.

F-theory Restrictions

We use the M-th language to fix moduli by fluxes. To apply this to F-theory, we require:

- $\widetilde{K3}$ must be elliptically fibred.
- Three-form flux G_3 and two-form fluxes F_2 in F-th: 4-form flux G in M-theory with one leg on the fibre and one on the base of $\widetilde{K3}$.

In M-theory this translates to putting zero flux on one U-block. Then:

- $\widetilde{K3}$ is fixed to be elliptically fibered, with fibre and base in the U-block.
- the size of the fibre is not fixed (modulus in M-theory, but not in F-theory): we can do the F-theory limit.
- One modulus of lower $K3$ remains unfixed

Symmetry Breaking by Fluxes

- $U(1)$ fluxes in brane gauge theory can break gauge groups (but preserve rank)
- Fixing moduli can break Cartan generators: Fixed scalar modulus becomes component of 4d gauge field in F-theory limit
- Consistent with IIB analysis: Kähler moduli get charged, stabilisation induces anomaly which breaks the $U(1)$

[Haack,Krefl,Lüst,van Proeyen,Zagermann]

- We can engineer desired subgroup of the group inferred form the singularities alone
- Work in progress...

Fixing a given configuration

Aim: find the flux that fixes a given configuration of branes.

- Positions of branes encoded in the c.s. $\tilde{\omega}$ of $\widetilde{K3}$.
- Gauge enhancement $\leftrightarrow$ brane stacks $\leftrightarrow$ shrunken cycles $\leftrightarrow$ G has block-diagonal structure
- Method:

- Choose basis of cycles to shrink $\Rightarrow$ determines block-diagonal structure
- In blocks, find matrices with different positive eigenvalues in positive and negative norm subspaces

Flux Constraints

The flux is subject to strong constraints:

- Positive eigenvalues (to have a minimum)
- Integrality $G^{I\Lambda} \in \mathbb{Z}$
- Tadpole cancellation $\frac{1}{2}\int G \wedge G = \frac{1}{2}\operatorname{tr} G^a G = \frac{\chi}{24} = 24$

The last two constraints pose the hardest problem, since $\operatorname{tr} G^a G$ is generically too large, and we need to scan a large number of matrices.

Can stabilise $SO(8)^4$ point, and we can move branes off their stacks, i.e. get $SO(6) \times U(1) \times SO(8)^3$, $SO(4) \times SU(2) \times SO(8)^3$.

And now for...

Conclusions

- We have a nice geometric picture of D7 brane motion
- We studied the stablilisation of D7 configurations by fluxes in M-theory
- We found explicit conditions for the existence of minima and absence of flat directions
- Translation to F-theory $\Rightarrow$ recipe to find fluxes that stabilise a desired situation

- Open problem: Numerical scan of matrices is very time–consuming
- Outlook: Generalise to elliptically fibred four-folds to get physically more realistic models (intersecting branes, chiral fermions, ...)