# General Warped Solution in 6d Supergravity

Christoph Lüdeling

DESY Hamburg

DPG-Frühjahrstagung Teilchenphysik

H. M. Lee, CL, JHEP **01**(2006) 062 [arXiv:hep-th/0510026]

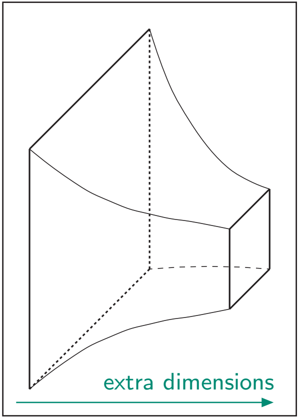
# Contents

# Warped Geometries

- Warped $4 + d$-dimensional geometries of the form

$$\mathrm{d}s_{4+d}^2 = W^2(y)\mathrm{d}s_4^2 + \mathrm{d}s_d^2$$

  can generate large hierarchies between branes located at different points in the extra dimensions
- See Randall-Sundrum-Scenario in five dimensions or KKLT string compactifications



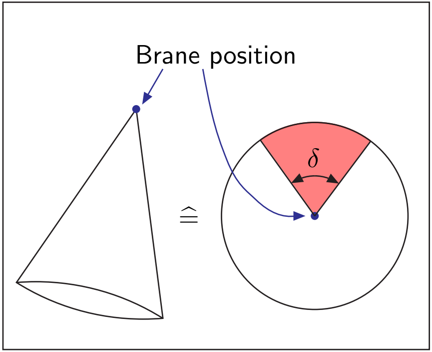
# Conical Branes

- Codimension-two branes are special: They are conical branes, i.e. the codimensional metric is of the form

$$ds_2^2 \propto d\rho^2 + \beta^2 \rho^2 d\theta^2$$

with a deficit angle

$$\delta = 2\pi\,(1-\beta)$$




- The curvature is finite up to a $\delta$-function at the brane position
- The brane curvature is independent of the brane tension ("self-tuning")
- Cosmic strings also are codimension-two objects with deficit angle

# The Model

Ingredients:

- 6d supergravity: Gravity & tensor Multiplet $(G_{MN}, \Psi_M, \chi, B_{MN}, \Phi)$
- Gauged $U(1)_R$-symmetry: vector multiplet $(A_M, \lambda)$
- 4d branes with tensions $\Lambda_i$

## Action: Bulk and branes

$$S_{\text{bulk}} = \int \mathrm{d}^6 X \sqrt{-G} \left\{ \tfrac{1}{2} R - \tfrac{1}{2} \partial_M \Phi \partial^M \Phi - \tfrac{1}{4} e^{-\Phi} F_{MN} F^{MN} - 2g^2 e^{\Phi} + \text{2-form} + \text{fermions} \right\}$$

Potential

$$S_{\text{branes}} = -\sum_i \int \mathrm{d}^4 x_i \sqrt{-g_i}\, \Lambda_i$$

# Ansatz

Ansatz: Warped background solution with 4d maximal symmetry (i.e. de Sitter, Minkowski or anti-de Sitter space):

$$\begin{aligned} ds^2 &= W^2(y)\tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu + \hat{g}_{mn}dy^m dy^n \\ R_{\mu\nu}(\tilde{g}) &= 3\lambda\tilde{g}_{\mu\nu} \\ F_{mn} &= \sqrt{\hat{g}}\,\epsilon_{mn}F(y)\,, \quad F_{\mu\nu} = F_{\mu m} = 0 \\ \Phi &= \Phi(y) \\ H_{MNP} &= 0\,, \quad \text{fermions} = 0 \end{aligned}$$

- Regular solution and compact internal space $\Rightarrow \lambda = 0$, i.e. Minkowski space is the unique maximally symmetric solution [Gibbons et al. '03]
- Dilaton: $\Phi = \Phi_0 - 2\ln W$
- Gauge flux $F(y) = f e^{\Phi} W^{-4} = f e^{\Phi_0} W^{-6}$

# Warp Factor

- For the warp factor, rewrite metric in terms of complex coordinate $z = y_5 + \mathrm{i} y_6$ as ($W \equiv e^B$)

$$\mathrm{d}s^2 = e^{2B(z,\bar{z})} \left( \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{2A(z,\bar{z})} \mathrm{d}z \mathrm{d}\bar{z} \right)$$

- ($zz$)-component of Einstein equation is easily solved up to a holomorphic function:

$$\overline{\partial} \left( e^{-2A} \overline{\partial} B \right) = 0 \quad \Rightarrow \quad \text{holomorphic function } V(z) = e^{-2A} \overline{\partial} B$$

- Choice of $V(z)$ determines warp factor

# Warp Factor $\leftrightarrow$ $V(z)$

- Unwarped solution $\Leftrightarrow$ $B =$ const. $\Leftrightarrow$ $V = 0$ and $f^2 = 4g^2$

  [Aghababaie et al. '03, Redi 04]
- $V \neq 0$ $\Rightarrow$ ordinary differential equation for the warp factor

  [Chodos, Poppitz '99]

$$\frac{\mathrm{d}W}{\mathrm{d}\zeta} = -\gamma^2 \frac{W^4 - 2v + u^2 W^{-4}}{2W^3} = \frac{P(W)}{W^3}$$

with new real coordinate and parameters

$$\zeta = \tfrac{1}{2} \int^z \frac{\mathrm{d}\omega}{V(\omega)} + \text{c.c.}\,, \quad \gamma^2 = \tfrac{1}{4} e^{\Phi_0} g^2\,,\ u^2 = \frac{f^2}{4g^2}\,,\ v$$

$\zeta$ might be only locally defined.
- Warp factor is independent of the "imaginary counterpart" of $\zeta$

$$\theta = \tfrac{i}{2} \int^z \frac{\mathrm{d}\omega}{V(\omega)} - \text{c.c.}$$

# Solution

This can be integrated to give

$$\frac{\left(W^4(\zeta) - W_-^4\right)^{W_-^4}}{\left(W_+^4 - W^4(\zeta)\right)^{W_+^4}} = \exp\left\{2\gamma^2\left(W_+^4 - W_-^4\right)(\zeta - \zeta_0)\right\}$$

with the roots of $P(W)$

$$W_\pm^4 = v \pm \sqrt{v^2 - u^2}$$

- Warp factor bounded in the range $W_- \leq W \leq W_+$
- Reality of warp factor gives constraints $v^2 \geq u^2\,, \quad v > 0$
- Extrema $W_\pm$ reached for $\zeta \to \pm\infty$, correspond to conical singularities
- Four real integration constants: $f$, $\Phi_0$, $v$ and $\zeta_0$
- For $g^2 \to 0$, i.e. no potential, the warp factor is unbounded

# General Metric

The general metric finally is

$$\mathrm{d}s^2 = W^2(z,\bar{z})\,\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \frac{1}{2\,|V(z)|}\frac{P(W)}{W^2}\mathrm{d}z\mathrm{d}\bar{z}$$

- Constraint on $V(z)$: For well-behaved (i.e. single-valued and free from singularities up to $\delta$-functions) warp factor and higher curvature invariants, $V$ can have only simple zeroes or poles
- Zeroes and poles of $V(z)$:
  - Simple zero $V(z) \propto (z - z_0)/c$ with real $c$ leads to conical brane with deficit angle

    $$2\pi\left(1 - \gamma^2\,|c|\,W_\pm^{-4}\left(W_+^4 - W_-^4\right)\right) \quad \text{for } c \lessgtr 0$$

  - Simple pole $V(z) \propto 1/(z - z_0)$ leads to brane with fixed deficit angle $-2\pi$
- Behaviour at infinity: For $V \propto z^n$ with $n > 2$, there will another brane with fixed deficit angle $2\pi(2-n)$ at $z = \infty$

# Example: Two Branes

Simple ansatz:

$$V(z) = -\frac{z}{c}\,, \quad c \text{ real and positive}$$

- Globally well-defined change of coordinates

$$\zeta = -\tfrac{1}{2} c \ln |z|^2 \qquad\qquad \theta = -\frac{1}{2\mathrm{i}} c \ln \frac{z}{\bar{z}}$$

- Conical branes at $z = 0$ and $z = \infty$ with warp factors $W_\pm \Rightarrow$ warped rugby
- Warp factor ($\mathrm{d}\eta = c^{-1} W^{-4} \mathrm{d}\zeta$)

$$W^4(\eta) = \tfrac{1}{2}\left(W_+^4 + W_-^4\right) + \tfrac{1}{2}\left(W_+^4 - W_-^4\right) \tanh\left[\left(W_+^4 - W_-^4\right)\gamma^2 c\eta\right]$$

  interpolates between $W_+$ and $W_-$
- Warp factor does not depend on $\theta \rightsquigarrow$ axial symmetry in extra dimensions

# Flux Quantisation and Unwarped Limit

- For compact extra dimensions, flux is quantised, in this case

$$\frac{W_+^4 - W_-^4}{W_+^4 W_-^4} f = \frac{8n}{g} \frac{e^{-\Phi_0}}{c} \quad \rightsquigarrow \quad (2\pi - \Lambda_+)(2\pi - \Lambda_-) = (2\pi n)^2$$

- The parameter $c$ can be absorbed by a rescaling of $\theta$, two parameters are fixed by matching of brane tensions to deficit angles $\rightsquigarrow$ One undetermined modulus remains.
- For the unwarped limit, take $c$ to infinity while keeping

$$k = c \left( W_+^4 - W_-^4 \right)$$

finite $\rightsquigarrow$ unwarped rugby, two branes with same deficit angle $2\pi \left(1 - \gamma^2 W_+^{-4} k\right)$. This is consistent with brane conditions and keeps the Planck mass finite.

# Example: Many Branes

For a multi-brane solution, take a similar ansatz:

$$V = \frac{1}{c} \prod_{i=1}^{N} (z - z_i)$$

- For single-valued warp factor, $c$ and all $z_i$ have to be real
- $N$ branes with warp factor $W_+$ or $W_-$, depending on the sign of

$$a_i = c \prod_{j \neq i} \frac{1}{z_i - z_j}$$

- Additional brane at $z = \infty$ with fixed brane tension $\Lambda^\infty = 2\pi\,(2 - N)$
- After flux quantisation and brane tension matching, still one undetermined modulus
- Planck mass and unwarped limit OK

# Conclusions and Outlook

- We have presented the general warped solution of 6d supergravity with 4d maximal symmetry
- Important properties depend on a free holomorphic function
  - Linear function: Recover known two-brane solutions
  - Function with many zeros gives multi-brane solutions. However, fixed-tension brane required in this case
- One undetermined modulus in simple cases, one fine-tuning relation between brane tensions
- To do:
  - Systematic study of different functions, in particular elliptic (doubly periodic) functions for torus geometry in extra dimensions
  - Generalisations: Modulus stabilisation, time-dependent solutions