The Potential Fate of Local Model Building

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Motivation

- F-Theory: Vacua with general branes in type IIB string theory
- Exceptional symmetries available, so interesting for GUT model building (as generalisations of perturbative intersecting brane models)
- Usually, consider local models: Focus on brane stack or points within the stack and decouple bulk of the compactification manifold
- Advantages: Simple, hope for predictivity from genericity
- Some questions cannot be addressed, e.g. moduli stabilisation
- Obvious problem: Existence of global completion
- Consider $SU(5)$ GUT with matter parity to forbid proton decay
- Models very constrained locally – global completion impossible

Contents

GUTs and Strings

- Aim of String Phenomenology: Find (semi) realistic particle physics models from string theory, e.g. the MSSM or SUSY GUTs in four dimensions, required properties include gauge group, matter content, Yukawa couplings, proton stability ...
- String-derived GUTs generically include exceptional symmetry groups (though not as gauge groups in four dimensions): $E_8 \times E_8$ heterotic string, F-Theory GUTs
- Generally, nontrivial pattern of gauge symmetries and fields localised on different subspaces of compactification space allows for GUT models: Fixed tori and points for orbifold twisted sectors; branes and their intersections for type II theories

F-Theory and Model Building

F-Theory: Nonperturbative formulation of type IIB theory, geometrises the brane physics with two extra auxiliary dimensions – allows for e.g. E_8 symmetries (cf. duality to heterotic string)

[Vafa 96; Bershadsky et al. 96; Morrison, Vafa 96;. . .]

Applications of F-theory to phenomenological GUT model building: Started by [Donagi, Wijnholt '08; Beasley, Heckman, Vafa '08], soon lot of work towards constructing GUTs addressing spectral covers, flavour, cosmology, gauge coupling unification, exotics, . . .

[Donagi, Wijnholt; Heckman, Vafa et al.; Marsano et al.; Watari et al.; Blumenhagen et al.; Grimm et al.; Dudas, Palti; Choi;. . .]

Usually, local models (bottom-up) considered: Corresponds to decoupling gravity, focusing on gauge theory sector

Intuition: Intersecting Branes in Perturbative Type IIB

Stack of N D7 branes $\rightsquigarrow$ $U(N)$ gauge theory
Intersection with another stack of M branes: Along the intersection, localised bifundamental matter: Locally, $N + M$ branes, so $U(N + M)$ symmetry. Matter representation can be inferred from adjoint decomposition

$$\begin{aligned} U(N+M) &\longrightarrow U(N) \times U(M) \\ (N+M)^2 &\longrightarrow (N^2, 1) \oplus (1, M^2) \oplus \left(N, \bar{M}\right) \oplus \left(\bar{N}, M\right) \end{aligned}$$

Inclusion of O7 planes: Also allows for $SO(2N)$ gauge groups and two-index antisymmetric representations – again representations can be inferred from decomposition of adjoint of higher local symmetry group

Model building problems: For $SU(5)$ GUTs, top quark Yukawa coupling not possible, and no spinors of $SO(10)$ – both requires exceptional local symmetries

F-Theory

[Vafa '96]

Type IIB has more 7-branes: (p, q) branes
7-Branes induce $SL(2, \mathbb{Z})$ transformations of axiodilaton $\leftrightarrow$ complex structure of (auxiliary) torus
Torus varies $\rightsquigarrow$ elliptic fibration over base B_3, described by Weierstraß model

$$y^2 = x^3 + f\,x\,z^4 + g\,z^6\,, \qquad (x, y, z) \in \mathbb{P}_{(2,3,1)}$$

with f and g functions on the base (sections in certain line bundles)

Brane positions: Torus degenerates, i.e. discriminant $\Delta = 4f^3 + 27g^2$ vanishes: Complex codimensoion one, i.e. eight-dimensional worldvolume
Type of brane (i.e. gauge symmetry) determined by vanishing orders of f, g and Δ – ADE classification of singularities [Kodaira '60s]

Tate Model

[Bershadsky et al. '96]

Matter is localised on curves of local symmetry enhancement – to study this, reformulate Weierstraß model to Tate form (locally) by shifting the coordinates:

$$y^2 = x^3 + a_5 xyz + a_4 x^2 z^2 + a_3 yz^3 + a_2 xz^4 + a_0 z^6$$

$\rightsquigarrow$ refined Kodaira classification in terms of vanishing orders of the a_i and Δ

To engineer desired GUT group G_{GUT}, assume brane stack S locally given by equation $w = 0$, then choose the $a_i = b_i w^{\text{appropriate power}}$

Effective restriction to S: Consider only the b_i in the following – sections of certain line bundles on S $\curvearrowright$ ALE fibration over S

Symmetry Enhancements: Matter Curves

Brane intersections: Symmetry enhancements $G_{\mathrm{GUT}} \to G_\Sigma$ on matter curves Σ within S – signalled by vanishing of b_i (or combinations of b_i)
Localised matter: Determined by decomposition of adjoint of G_Σ:

$$G_\Sigma \longrightarrow G_{\mathrm{GUT}} \times H$$
$$\mathrm{ad}\, G_\Sigma \longrightarrow (\mathrm{ad}\, G_{\mathrm{GUT}}, \mathbf{1}) \oplus (\mathbf{1}, \mathrm{ad}\, H) \oplus \bigoplus (R_{G_{\mathrm{GUT}}}, R_H)$$

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For example, enhancements

$$\begin{aligned} SU(5) &\longrightarrow SU(6) & &\rightsquigarrow & &\text{localised } \mathbf{5} \\ SU(5) &\longrightarrow SO(10) & &\rightsquigarrow & &\text{localised } \mathbf{10} \\ SO(10) &\longrightarrow E_6 & &\rightsquigarrow & &\text{localised } \mathbf{16} \end{aligned}$$

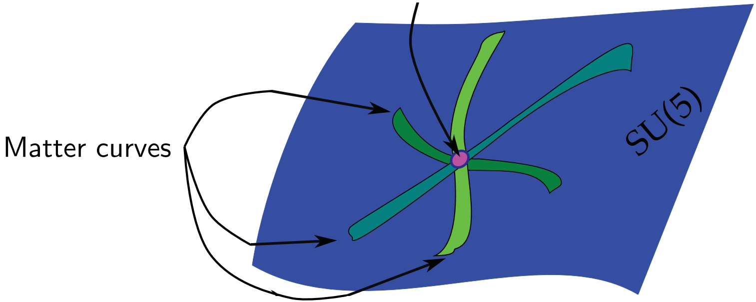
Matter is six-dimensional, so comes in hypermultiplets, corresponding to two $\mathcal{N} = 1$ chiral multiplets – four-dimensional zero modes determined by flux along Σ

Symmetry Enhancements: Yukawa Couplings

Finally, matter curves meet in points $\rightsquigarrow$ further symmetry enhancement to G_P

Intersection of curves $\Sigma_{1,2,3}$ with localised matter representations $R_{\Sigma_{1,2,3}}$ leads to Yukawa couplings from triple adjoint interaction of G_P:

$$(\text{ad } G_P)^3 = R_{\Sigma_1} R_{\Sigma_2} R_{\Sigma_3} + \cdots$$



[picture from Sakura Schäfer-Nameki]

Global, Semilocal, Local

For F-Theory GUTs, different degrees of locality:

- *Global* model: Specify full compactification space (CY fourfold): Includes all branes, fluxes, obeys consistency conditions, can stabilise moduli etc. [Blumenhagen et al.; Grimm et al.; Marsano et al.;. . .]
- *Semilocal* model: Focus on the GUT surface (brane stack) S and matter curves within S: Decouples bulk of compactification space, certain consistency conditions included [Hayashi et al.; Donagi, Wijnholt; Grimm, Weigand; Marsano et al.; Dudas, Palti; CL, Nilles, Stephan;. . .]
- *Local* model: Consider only points within S where matter curves intersect and interactions are localised: Simple, and hope for predictivity because any good global model must contain good local model and bulk physics decoupled. Certain questions cannot be answered, and actual existence of global completion is not guaranteed. [Donagi, Wijnholt; Heckman, Vafa et al.; Watari et al.;. . .]

Contents

GUT Surface, Matter Curves

[Conventions of Dudas, Palti]

- $SU(5)$ GUT on two-complex-dimensional surface S in the base of elliptically fibred CY fourfold X, locally given by $w = 0$
- Elliptic fibration over S described by Tate model (scaled $z \longrightarrow 1$)

$$y^2 = x^3 + a_5\, xy + a_4\, x^2 + a_3\, y + a_2\, x + a_0$$

b_k: sections in certain line bundles on S

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b_k: sections in certain line bundles on S

- Discriminant becomes

$$\Delta = w^5 \left(b_5^4 P + w\, b_5^2 \left(8 b_4 P + b_5 R\right) + \mathcal{O}\left(w^2\right)\right)$$

P, R: polynomials in the b_k

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P, R: polynomials in the b_k

- Locally, $SU(5)$ is enhanced

to $SU(6)$:	$P = 0$	$\Rightarrow$	localised **5**
to $SO(10)$:	$b_5 = 0$	$\Rightarrow$	localised **10**

Yukawa Couplings

- 6D Matter in hypermultiplets – 4D zero modes determined by flux (via index theorem)
- $U(1)$ flux keeps full multiplets, hypercharge flux can split multiplets – doublet-triplet splitting [BHV, DW]
- For $SU(5)$ GUT require $SO(12)$ enhancement for down-type Yukawas,

$$(\mathbf{66})^3 \supset \bar{\mathbf{5}}_{H_d} \bar{\mathbf{5}}_M \, \mathbf{10}_M$$

and E_6 enhancement for up-type,

$$(\mathbf{78})^3 \supset \mathbf{5}_{H_u} \mathbf{10}_M \, \mathbf{10}_M$$

Point of E_8

[Heckman, Tavanfar, Vafa]

- Need E_6 and $SO(12)$ enhancements for up- and down-type Yukawas
- CKM matrix: Favourably, these points coincide (no geometric suppression of quark mixing) $\rightsquigarrow E_7$
- For PMNS matrix: Further enhancement to E_8 (but we do not consider neutrinos in the following)
- Hence: One single Yukawa "point of E_8", all interactions localised here

$\curvearrowright$ Allows for higher interaction terms – Froggatt–Nielsen type masses using GUT singlets

- Simple and potentially predictive: All (superpotential) interactions determined by group theory, geometric data can be largely ignored

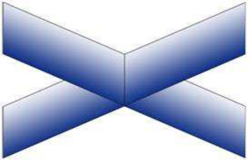
Gauge Theory Description

- Consider SYM theory on worldvolume of S: E_8 GUT, broken to $SU(5)$ by adjoint Higgs (parameterises brane motion)
- Actually, rank-preserving breaking

$$E_8 \longrightarrow (SU(5) \times SU(5)_\perp) \longrightarrow SU(5) \times U(1)^4$$

- Extra $U(1)$'s generically massive in F-Theory by geometric Stueckelberg effect, but this cannot be analysed in local model – $U(1)$'s remain as global selection rules [Grimm, Weigand]
- Higgs field varies over S – matter curves now visible as vanishing loci of Higgs eigenvalues, e.g.

$$\langle \Phi \rangle \sim \begin{pmatrix} z & \\ & -z \end{pmatrix} \longrightarrow$$



E_8 Higgs

$$E_8 \longrightarrow SU(5) \times SU(5)_\perp$$
$$\mathbf{248} \longrightarrow (\mathbf{24},\mathbf{1}) \oplus (\mathbf{1},\mathbf{24}) \oplus \left[(\mathbf{10},\mathbf{5}) \oplus \left(\mathbf{5},\overline{\mathbf{10}}\right) \oplus \text{c.c.}\right]$$

$$\text{Higgs} \quad \Phi \sim \begin{pmatrix} t_1 & & & & \\ & t_2 & & & \\ & & t_3 & & \\ & & & t_4 & \\ & & & & t_5 \end{pmatrix} \in (\mathbf{1},\mathbf{24})\ , \quad \sum_i t_i = 0$$

Connection to Tate model: Deformed E_8 singularity,

$$y^2 = x^3 + b_0 w^5 \quad \longrightarrow \quad y^2 = x^2 + b_0 \prod (w - t_i)$$

$\curvearrowright$ the b_k are symmetric polynomials in the t_i of order k, no b_1 because of tracelessness

Matter Curves

t_i are eigenvalues in the **5** of $SU(5)_\perp$, i.e.

$$\Phi e_i = t_i e_i$$

$\curvearrowright$ **10** of $SU(5)_\perp$ spanned by $e_i \wedge e_j$, $i \neq j$, with eigenvalue $t_i + t_j$

Representations of $SU(5) \times SU(5)_\perp$ appear as $(\mathbf{10}, \mathbf{5}) \oplus \left(\mathbf{5}, \overline{\mathbf{10}}\right)$
$\curvearrowright$ in terms of reps of visible $SU(5)$, matter curves are given by

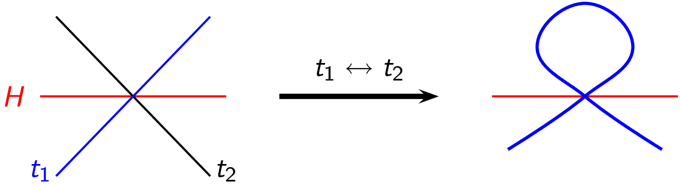
$$\begin{aligned} t_i &= 0 && \text{localised } \mathbf{10} \\ -t_i - t_j &= 0 && \text{localised } \mathbf{5} \\ t_i - t_j &= 0 && \text{localised } \mathbf{1} \end{aligned}$$

t_i double as charges: For gauge-invariant terms, t_i must sum to zero (possibly using $\sum_i t_i = 0$) – realises $U(1)^4 \subset SU(5)_\perp$ selection rules

Monodromy

[Marsano et al.]

- The b_k in the Tate model are symmetric polynomials in the t_i
  $\Rightarrow$ Invariant under permutations of the t_i
- Interpretation: Self-intersection, locally distinct-looking branes are the same



- Heavy top requires coupling $\mathbf{5}_{H_u}\mathbf{10}_{\text{top}}\mathbf{10}_{\text{top}}$
  $\rightsquigarrow$ (at least) $\mathbb{Z}_2$ monodromy $t_1 \leftrightarrow t_2$
- Fixes top and up-type Higgs curve: $\mathbf{10}_{\text{top}} \sim \{t_1, t_2\}$, $\mathbf{5}_{H_u} \sim -t_1 - t_2$
- Reduces $SU(5)_\perp$ to lower rank

$SU(5)$ GUT

Unify SM gauge group $G_{\mathrm{SM}} = SU(3) \times SU(2) \times U(1) \longrightarrow SU(5)$, decomposition of $SU(5)$ fields into SM representations:

$$\begin{array}{llcl}
\text{Gauge bosons:} & \mathbf{24} & \sim & G \oplus W \oplus B_Y \oplus X \oplus Y \\
\text{Matter:} & \mathbf{10}_M & \sim & Q \oplus u^c \oplus e^c \\
 & \bar{\mathbf{5}}_M & \sim & d^c \oplus L \\
\text{Higgs:} & \mathbf{5}_{H_u} & \sim & H_u \oplus T_u \\
 & \bar{\mathbf{5}}_{H_d} & \sim & H_d \oplus T_d
\end{array}$$

Higgs triplets and X, Y bosons need to be very heavy for proton stability

Break $SU(5)$ by hypercharge flux – topological condition to avoid breaking hypercharge (not available in heterotic models): Flux needs to be globally trivial

Yukawa Couplings

Good couplings: Quark and lepton masses, weak-scale μ term

$$W_{\text{good}} = \mu\, \mathbf{5}_{H_u} \overline{\mathbf{5}}_{H_d} + Y_u \mathbf{5}_{H_u} \mathbf{10}_M \mathbf{10}_M + Y_d \overline{\mathbf{5}}_{H_d} \overline{\mathbf{5}}_M \mathbf{10}_M$$

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Bad couplings: Baryon and lepton number violating operators

$$\begin{aligned} W_{\text{bad}} &= \beta\, \mathbf{5}_{H_u} \bar{\mathbf{5}}_M + \lambda \bar{\mathbf{5}}_M \bar{\mathbf{5}}_M \mathbf{10}_M && \text{dim-3/4} \\ &\quad + W^1 \mathbf{10}_M \mathbf{10}_M \mathbf{10}_M \bar{\mathbf{5}}_M + W^2 \mathbf{10}_M \mathbf{10}_M \mathbf{10}_M \bar{\mathbf{5}}_{H_d} \\ &\quad + W^3 \bar{\mathbf{5}}_M \bar{\mathbf{5}}_M \mathbf{5}_{H_u} \mathbf{5}_{H_u} + W^4 \bar{\mathbf{5}}_M \bar{\mathbf{5}}_{H_d} \mathbf{5}_{H_u} \mathbf{5}_{H_u} \\ K_{\text{bad}} &= K^1 \mathbf{10}_M \mathbf{10}_M \mathbf{5}_M + K^2 \bar{\mathbf{5}}_{H_u} \bar{\mathbf{5}}_{H_u} \mathbf{10}_M \end{aligned} \quad \Bigg\} \text{dim-5}$$

Coefficients can contain singlet VEVs, suppressed by M_{GUT} [Conlon, Palti]

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dim-3/4

dim-5

Coefficients can contain singlet VEVs, suppressed by M_{GUT} [Conlon, Palti]

Some terms related by interchange $\bar{\mathbf{5}}_{H_d} \leftrightarrow \bar{\mathbf{5}}_M$

Matter Parity

[Dimopoulos, Raby, Wilczek; Ibanez, Ross; Dreiner, Luhn, Thormeier]

Various discrete symmetries help for proton stability. Compatibility with $SU(5)$ implies $\mathbb{Z}_2$ "matter parity" which distinguishes Higgs and matter:

	$\mathbf{5}_{H_u}, \bar{\mathbf{5}}_{H_d}$	$\mathbf{10}_M, \bar{\mathbf{5}}_M$
P_M	$+1$	-1

Forbids all baryon and lepton number violating operators except

$$W^1 \mathbf{10}_M \mathbf{10}_M \mathbf{10}_M \bar{\mathbf{5}}_M \quad \text{and} \quad W^3 \bar{\mathbf{5}}_M \bar{\mathbf{5}}_M \mathbf{5}_{H_u} \mathbf{5}_{H_u}$$

W^3 generates neutrino masses (Weinberg operator), can be tolerated if suppression scale high enough (but will not be generated, so ignore from now on)

W^1 is very strongly constrained ($W^1 \supset QQQL, \bar{u}\bar{u}\bar{d}\bar{e}, \dots$) – forbid this by clever choice of matter curves (i.e. $U(1)$s)

Contents

Model Requirements

For the local model we require

- P_M defined *at the point of* E_8
- heavy top quark (i.e. rank-one up-type Yukawa matrix at tree level)
- No dim-5 proton decay (the W^1 operator forbidden at all orders)
- Masses for all quarks and leptons after switching on VEVs (down-type Yukawa matrix can be rank-zero or one, but not rank-two)

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Local model building freedom: Freely choose

- Monodromy (at least $\mathbb{Z}_2$)
- Assignment of matter and Higgs zero modes to curves
- Singlet VEVs (for matter parity even singlets)
- Assume: Allowed terms generated with order-one coefficients

Matter Parity

Define $\mathbb{Z}_2$ matter parity in terms of the t_i (i.e. as subgroup of $SU(5)_\perp$):

$$P_M = (-1)^{c_i t_i} \;, \quad c_i = 0, 1 \qquad (\text{defined} \mod 2)$$

- Monodromy $t_1 \leftrightarrow t_2$ requires $c_1 = c_2 = 1$ so $\mathbf{10}_{\text{top}}$ is odd
- Up-type masses always allowed once gauge invariant
- Down-type masses give constraint:

	$\bar{\mathbf{5}}_{H_d}$	$\bar{\mathbf{5}}_M$	$\mathbf{10}_M$
charge	$t_i + t_j$	$t_k + t_l$	t_m

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- Down-type masses give constraint:

	$\bar{\mathbf{5}}_{H_d}$	$\bar{\mathbf{5}}_M$	$\mathbf{10}_M$
charge	$t_i + t_j$	$t_k + t_l$	t_m
$c_i t_i$	0 or 2	1	1

  Gauge invariant iff all t_i distinct – can only be matter parity even if even number of $c_i = 1$ (singlets have charge $t_i - t_j$, so don't change the argument)
- Note: W^1 operator has same charge structure

Two Possibilities

Hence, two possible definitions of matter parity:

Case I: $P_M = (-1)^{t_1+t_2+t_3+t_4}$

Case II: $P_M = (-1)^{t_1+t_2}$

Now analyse matter, Higgs and VEV assignment for both cases: $\mathbf{10}_{\text{top}}$ and $\mathbf{5}_{H_u}$ already fixed, need to distribute remaining matter and $\overline{\mathbf{5}}_{H_d}$ according to their matter parity

Main restriction: Forbid W^1, but allow down-type Yukawas

Case I: Matter and VEV Assignment

	Matter **10** Curves		
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	
$\mathbf{10}_3$	t_4	$-$	
	Matter **5** Curves		
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	
	Even Singlet Curves		
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	
$\mathbf{1}_7$	$t_1 - t_2$	$+$	

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$\mathbf{10}_3$	t_4	$-$	
Matter **5** Curves			
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	
Even Singlet Curves			
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	
$\mathbf{1}_7$	$t_1 - t_2$	$+$	

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$

$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

Case I: Matter and VEV Assignment

Matter **10** Curves			
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	no matter
$\mathbf{10}_3$	t_4	$-$	matter
Matter **5** Curves			
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	matter
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	no matter
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	matter
Even Singlet Curves			
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	
$\mathbf{1}_7$	$t_1 - t_2$	$+$	

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

$\rightsquigarrow$ no matter on $\mathbf{10}_2$, $\mathbf{5}_5$

Case I: Matter and VEV Assignment

Matter **10** Curves			
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	no matter
$\mathbf{10}_3$	t_4	$-$	matter
Matter 5 Curves			
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	matter
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	no matter
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	matter
Even Singlet Curves			
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	
$\mathbf{1}_7$	$t_1 - t_2$	$+$	

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

$\rightsquigarrow$ no matter on $\mathbf{10}_2$, $\mathbf{5}_5$

- W^1 with singlets:

e.g. $\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_6\mathbf{1}_4\,,$

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_3\mathbf{1}_1$$

Case I: Matter and VEV Assignment

Matter **10** Curves			
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	no matter
$\mathbf{10}_3$	t_4	$-$	matter
Matter **5** Curves			
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	matter
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	no matter
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	matter
Even Singlet Curves			
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	no VEV
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	VEV
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	no VEV
$\mathbf{1}_7$	$t_1 - t_2$	$+$	VEV

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

$\rightsquigarrow$ no matter on $\mathbf{10}_2$, $\mathbf{5}_5$

- W^1 with singlets:

e.g. $\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_6\mathbf{1}_4\,,$
$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_3\mathbf{1}_1$

$\rightsquigarrow$ no VEVs for $\mathbf{1}_1$, $\mathbf{1}_4$ (because of t_3)

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	
$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	
$\bar{\mathbf{5}}_2$	$-t_{1,2} - t_4$	
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	
$\bar{\mathbf{5}}_2$	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	either rank-two Yukawa matrix, or no up-type masses with singlets
$\bar{\mathbf{5}}_2$	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	No masses at tree level or with singlets μ term
$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	either rank-two Yukawa matrix, or no up-type masses with singlets
$\bar{\mathbf{5}}_2$	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two
- String-scale μ term for both Higgses on one curve

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
$\overline{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	No masses at tree level or with singlets μ term
$\overline{\mathbf{5}}_1$	$-t_{1,2} - t_3$	either rank-two Yukawa matrix, or no up-type masses with singlets
$\overline{\mathbf{5}}_2$	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\overline{\mathbf{5}}_4$	$-t_3 - t_4$	Rank-one Yukawa matrix, bottom quark heavy

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two
- String-scale μ term for both Higgses on one curve
- $\overline{\mathbf{5}}_4 = \overline{\mathbf{5}}_{H_d}$ is unique choice, tree-level coupling $\overline{\mathbf{5}}_{H_d}\mathbf{10}_{\text{top}}\overline{\mathbf{5}}_3$

Case I: Yukawas and CKM

- Example Assignment: Third generation on $\mathbf{10}_1$ and $\overline{\mathbf{5}}_3$, light generations on $\mathbf{10}_3$ and $\overline{\mathbf{5}}_6$
- Higgses: $\overline{\mathbf{5}}_{H_u}$ and $\overline{\mathbf{5}}_4$, only $\langle \mathbf{1}_2 \rangle \sim \epsilon$ required at first order
- Ignore $\mathbf{1}_7$ and $\mathcal{O}(1)$ coefficients
- Yukawa matrices (schematically):

$$Y^u \sim Y^d \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}$$

- CKM matrix:

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}$$

- Masses and mixings possible though not a great fit
- Degeneracy because three generations come from two curves

Case II

$$P_M = (-1)^{t_1+t_2}$$

$\rightsquigarrow$ split t's into $t_{\text{odd}} = \{t_1, t_2\}$ and $t_{\text{even}} = \{t_3, t_4, t_5\}$

- Symmetric setup, possible monodromy acting on t_{even}
- $\mathbf{10}_{\text{top}}$ is the unique matter $\mathbf{10}$ curve
- Down-type Higgs unique (up to relabeling)
- Matter-parity even singlets do not mix t_{odd} and t_{even}
- W^1 operator cannot be generated: Charge $4t_{odd} + t_{even}$ cannot be compensated by matter-parity even singlets
- Three possible matter $\bar{\mathbf{5}}$ curves (charges $t_{\text{odd}} - t_{\text{even}}$): model building choice
- Different choices of singlet VEVs possible, achieve masses and mixing

Local Model Summary

- Already locally, rather constrained model: Only two possible definitions of matter parity at the point of E_8
- In both cases, assignments of matter and Higgses is unique or strongly constrained
- Restrictions mainly from forbidding W^1 while allowing for down-type masses
- W^3 operator (neutrino masses) is not generated in any case
- Masses for all matter fields and CKM mixing possible
- Involves choices of zero modes and VEVs by hand – these cannot be calculated in the local framework

Contents

Semilocal Approach

[Friedman, Morgan, Witten; Donagi, Wijnholt]

Now *semilocal* picture: Consider GUT surface S, using spectral cover approach

Main aim: Find homology classes of matter curves which allow to find the flux restrictions and thus the zero mode spectrum.

Two types of fluxes:

- $U(1) \subset SU(5)_\perp$ fluxes on matter curves (from the transverse branes): Determines (chiral) 4D zero modes for full GUT multiplets. These are still free parameters up to anomaly cancellation requirements.
- Hypercharge flux on S (globally trivial so hypercharge stays unbroken): Restrictions to matter curves splits $SU(5)$ multiplets; homological relations between matter curves lead to relations between the splittings.

Spectral Cover

Spectral cover: Hypersurface in projective threefold

$$\mathbb{P}(K_S \oplus \mathcal{O}_S)$$

with homogeneous coordinates $U : V$ given by spectral equation for Φ. Because of $\mathbb{Z}_2$ monodromy, spectral equation must factorise:

$$\begin{aligned} 0 &= b_0 U^5 + b_2 U^3 V^2 + b_3 U^2 V^3 + b_4 U V^4 + b_5 V^5 \\ &= \left(a_1 V^2 + a_2 UV + a_3 U^2\right)\left(a_4 V + a_7 U\right)\left(a_5 V + a_8 U\right)\left(a_6 V + a_9 U\right) \end{aligned}$$

b_k are sections in line bundles with Chern classes $\eta - kc_1 = (6-k)\, c_1(S) + c_1\left(N_{S/X}\right)$. This determines the bundles for the a_m, and in turn for the matter curves, in terms of three unspecified line bundles $\chi_{7,8,9}$.

Involves particular solution of $b_1 = 0$ constraint – might not be most general one?

Fluxes and Zero Modes

$U(1)$ fluxes: Given by integers $M_{\mathbf{5}}$, $M_{\mathbf{10}}$. Free up to consistency conditions [Dudas, Palti; Marsano]

$$\sum M_{\mathbf{10}} + \sum M_{\mathbf{5}} = 0\,, \qquad M_{\mathbf{10}_1} = -\left(M_{\mathbf{5}_1} + M_{\mathbf{5}_2} + M_{\mathbf{5}_3}\right)$$

Hypercharge flux must be globally trivial, hence

$$0 = F_Y \cdot c_1 = F_Y \cdot \eta \quad \rightsquigarrow \quad \sum_{\mathbf{5}} F_Y = \sum_{\mathbf{10}} F_Y = 0$$

Restrictions to matter curves given by $N_Y = F_Y \cdot$ (homology class). For curve with flux numbers M and N_Y, zero modes given by

$\mathbf{10} \to$
- $(\mathbf{3},\mathbf{2}) : M_{\mathbf{10}}$
- $(\overline{\mathbf{3}},\mathbf{1}) : M_{\mathbf{10}} - N_Y$
- $(\mathbf{1},\mathbf{1}) : M_{\mathbf{10}} + N_Y$

$\mathbf{5} \to$
- $(\mathbf{3},\mathbf{1}) : M_{\mathbf{5}}$
- $(\mathbf{1},\mathbf{2}) : M_{\mathbf{5}} + N_Y$

Upshot

10 Curves

	M	N_Y
$\mathbf{10}_1$	$-(M_{\mathbf{5}_1}+M_{\mathbf{5}_2}+M_{\mathbf{5}_3})$	$-\widetilde{N}$
$\mathbf{10}_2$	$M_{\mathbf{10}_2}$	N_7
$\mathbf{10}_3$	$M_{\mathbf{10}_3}$	N_8
$\mathbf{10}_4$	$M_{\mathbf{10}_4}$	N_9

5 Curves

	M	N_Y
$\mathbf{5}_{H_u}$	$M_{\mathbf{5}_{H_u}}$	$\widetilde{N}$
$\mathbf{5}_1$	$M_{\mathbf{5}_1}$	$-\widetilde{N}$
$\mathbf{5}_2$	$M_{\mathbf{5}_2}$	$-\widetilde{N}$
$\mathbf{5}_3$	$M_{\mathbf{5}_3}$	$-\widetilde{N}$
$\mathbf{5}_4$	$M_{\mathbf{5}_4}$	N_7+N_8
$\mathbf{5}_5$	$M_{\mathbf{5}_5}$	N_7+N_9
$\mathbf{5}_6$	$M_{\mathbf{5}_6}$	N_8+N_9

- Three free parameters $N_{7,8,9}$ for the hypercharge flux, corresponding to three unspecified line bundles
- $\widetilde{N}=N_7+N_8+N_9$
- Split some **5** curves $\Rightarrow$ split some **10** curves
  [Marsano et al.; Dudas, Palti]

Case I: Matter Sector is Fine

- Doublet-triplet splitting for Higgses ($\widetilde{N} \neq 0$) inevitably splits $\mathbf{10}_{\text{top}}$ and at least one more **10** curve
- However, splitting of matter multiplets is OK as long as there are three generations of zero modes in the end, i.e. other **10** curve must have "opposite" split
- Matter on $\mathbf{10}_1$, $\mathbf{10}_3$, $\mathbf{5}_3$ and $\mathbf{5}_6$, so to have full net generations, we require

$$N_7 = N_9 = 0 \quad \Rightarrow \quad \text{only } N_8 \text{ left free}$$

- No exotics from **10**'s and remaining matter-like **5** curve can be satisfied by choosing appropriate M's

↷ Satisfactory matter sector can be engineered easily

Case I: Higgs Sector is not Fine

- Higgs sector:

	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$
$\mathbf{5}_{H_u}$	$M_{\mathbf{5}_{H_u}}$	$M_{\mathbf{5}_{H_u}} + N_8$
$\mathbf{5}_1$	$M_{\mathbf{5}_1}$	$M_{\mathbf{5}_1} - N_8$
$\mathbf{5}_2$	$M_{\mathbf{5}_2}$	$M_{\mathbf{5}_2} - N_8$
$\mathbf{5}_4$	$M_{\mathbf{5}_4}$	$M_{\mathbf{5}_4} + N_8$

- We can pairwise decouple unwanted triplets from $\mathbf{5}_{H_u}$ and $\mathbf{5}_2$, and from $\mathbf{5}_1$ and $\mathbf{5}_4$ by coupling to VEV for $\mathbf{1}_2$
- However:

$$\#(\text{doublets from } \mathbf{5}_{H_u}, \mathbf{5}_2) = \#(\text{triplets from } \mathbf{5}_{H_u}, \mathbf{5}_2)$$

- Problem persists even when allowing exotics from the matter sector
- Separately, down-type Higgs on $\mathbf{5}_4$ cannot be realised

Case II: Again not Fine

- Only one matter $\mathbf{10}$ curve, split with same parameter as up-type Higgs
- Higgs triplets cannot be decoupled by even matter parity singlets
- No exotic matter $\Rightarrow$ no doublet-triplet splitting
- Ignoring matter sector: Still cannot achieve pair of Higgs doublets

Case II: Again not Fine

- Only one matter **10** curve, split with same parameter as up-type Higgs
- Higgs triplets cannot be decoupled by even matter parity singlets
- No exotic matter ⇒ no doublet-triplet splitting
- Ignoring matter sector: Still cannot achieve pair of Higgs doublets

Upshot: In both cases, proper doublet-triplet splitting in Higgs sector does not work, even when allowing for exotics from the matter sector – both models cannot be realised already in semilocal setup!

Contents

Conclusions

- Analysed F-Theory GUT at "point of E_8" and in semilocal approach
- Goal: Find a locally defined matter parity to ensure proton stability
- Local model is already very constrained: Two cases only
- In semilocal embedding, doublet-triplet splitting cannot be realised in either case

$\rightsquigarrow$ Models fail first step towards realisation

Conclusions

- Analysed F-Theory GUT at "point of E_8" and in semilocal approach
- Goal: Find a locally defined matter parity to ensure proton stability
- Local model is already very constrained: Two cases only
- In semilocal embedding, doublet-triplet splitting cannot be realised in either case

$\rightsquigarrow$ Models fail first step towards realisation

- Predictivity of local point in question – Crucial model features required to have nonlocal origin?
- Possible loopholes:
  - Assumed GUT breaking by hypercharge flux – different breaking mechanisms?
  - Non-diagonal Higgs fields ("T-Branes", "Gluing Morphisms") might help to get rid of exotics [Cecotti et al.; Donagi, Wijnholt]