The Potential Fate of Local Model Building

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CL, Hans Peter Nilles, Claudia Christine Stephan
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Motivation

- F-Theory: Vacua with general branes in type IIB string theory
- Exceptional symmetries available, so interesting for GUT model building (as generalisations of perturbative intersecting brane models)
- Usually, consider local models: Focus on brane stack or points within the stack and decouple bulk of the compactification manifold
- Advantages: Simple, hope for predictivity from genericity
- Some questions cannot be addressed, e.g. moduli stabilisation
- Obvious problem: Existence of global completion
- Consider $SU(5)$ GUT with matter parity to forbid proton decay
- Models very constrained locally – global completion impossible
1 F-Theory GUT Model Building

2 Local $SU(5)$ GUT with Matter Parity

3 Matter Parity in Local Models

4 Semilocal Embedding

5 Conclusion
• Aim of String Phenomenology: Find (semi) realistic particle physics models from string theory, e.g. the MSSM or SUSY GUTs in four dimensions, required properties include gauge group, matter content, Yukawa couplings, proton stability . . .

• String-derived GUTs generically include exceptional symmetry groups (though not as gauge groups in four dimensions): $E_8 \times E_8$ heterotic string, F-Theory GUTs

• Generally, nontrivial pattern of gauge symmetries and fields localised on different subspaces of compactification space allows for GUT models: Fixed tori and points for orbifold twisted sectors; branes and their intersections for type II theories
F-Theory and Model Building

F-Theory: Nonperturbative formulation of type IIB theory, geometrises the brane physics with two extra auxiliary dimensions – allows for e.g. $E_8$ symmetries (cf. duality to heterotic string)

[Vafa 96; Bershadsky et al. 96; Morrison, Vafa 96; …]

Applications of F-theory to phenomenological GUT model building:
Started by [Donagi, Wijnholt ’08; Beasley, Heckman, Vafa ’08], soon lot of work towards constructing GUTs addressing spectral covers, flavour, cosmology, gauge coupling unification, exotics, …

[Donagi, Wijnholt; Heckman, Vafa et al.; Marsano et al.; Watari et al.; Blumenhagen et al.; Grimm et al.; Dudas, Palti; Choi; …]

Usually, local models (bottom-up) considered: Corresponds to decoupling gravity, focusing on gauge theory sector
Stack of $N$ D7 branes $\leadsto U(N)$ gauge theory

Intersection with another stack of $M$ branes: Along the intersection, localised bifundamental matter: Locally, $N + M$ branes, so $U(N + M)$ symmetry. Matter representation can be inferred from adjoint decomposition

\[
U(N + M) \longrightarrow U(N) \times U(M)
\]

\[
(N + M)^2 \longrightarrow (N^2, 1) \oplus (1, M^2) \oplus (N, \bar{M}) \oplus (\bar{N}, M)
\]

Inclusion of O7 planes: Also allows for $SO(2N)$ gauge groups and two-index antisymmetric representations – again representations can be inferred from decomposition of adjoint of higher local symmetry group

Model building problems: For $SU(5)$ GUTs, top quark Yukawa coupling not possible, and no spinors of $SO(10)$ – both requires exceptional local symmetries
F-Theory

Type IIB has more 7-branes: \((p, q)\) branes

7-Branes induce \(SL(2, \mathbb{Z})\) transformations of axiodilaton ↔ complex structure of (auxiliary) torus

Torus varies ↦ elliptic fibration over base \(B_3\), described by Weierstraß model

\[
y^2 = x^3 + f(x)z^4 + g(x)z^6, \quad (x, y, z) \in \mathbb{P}_{(2,3,1)}
\]

with \(f\) and \(g\) functions on the base (sections in certain line bundles)

Brane positions: Torus degenerates, i.e. discriminant \(\Delta = 4f^3 + 27g^2\) vanishes: Complex codimension one, i.e. eight-dimensional worldvolume

Type of brane (i.e. gauge symmetry) determined by vanishing orders of \(f\), \(g\) and \(\Delta\) – ADE classification of singularities

[Vafa ’96] [Kodaira ’60s]
Matter is localised on curves of local symmetry enhancement – to study this, reformulate Weierstraß model to Tate form (locally) by shifting the coordinates:

\[ y^2 = x^3 + a_5xyz + a_4x^2z^2 + a_3yz^3 + a_2xz^4 + a_0z^6 \]

\( \leadsto \) refined Kodaira classification in terms of vanishing orders of the \( a_i \) and \( \Delta \)

To engineer desired GUT group \( G_{\text{GUT}} \), assume brane stack \( S \) locally given by equation \( w = 0 \), then choose the \( a_i = b_iw \) appropriate power

Effective restriction to \( S \): Consider only the \( b_i \) in the following – sections of certain line bundles on \( S \leadsto \text{ALE fibration over } S \)
Brane intersections: Symmetry enhancements $G_{GUT} \rightarrow G_{\Sigma}$ on matter curves $\Sigma$ within $S$ – signalled by vanishing of $b_i$ (or combinations of $b_i$)

Localised matter: Determined by decomposition of adjoint of $G_{\Sigma}$:

$$G_{\Sigma} \rightarrow G_{GUT} \times H$$

$$\text{ad } G_{\Sigma} \rightarrow (\text{ad } G_{GUT}, 1) \oplus (1, \text{ad } H) \oplus \bigoplus (R_{G_{GUT}}, R_H)$$
Symmetry Enhancements: Matter Curves

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localised matter

For example, enhancements

- $SU(5) \longrightarrow SU(6) \quad \leadsto \quad \text{localised } 5$
- $SU(5) \longrightarrow SO(10) \quad \leadsto \quad \text{localised } 10$
- $SO(10) \longrightarrow E_6 \quad \leadsto \quad \text{localised } 16$

Matter is six-dimensional, so comes in hypermultiplets, corresponding to two $\mathcal{N} = 1$ chiral multiplets – four-dimensional zero modes determined by flux along $\Sigma$
Finally, matter curves meet in points $\Rightarrow$ further symmetry enhancement to $G_P$

Intersection of curves $\Sigma_{1,2,3}$ with localised matter representations $R_{\Sigma_{1,2,3}}$ leads to Yukawa couplings from triple adjoint interaction of $G_P$:

$$(\text{ad } G_P)^3 = R_{\Sigma_1} R_{\Sigma_2} R_{\Sigma_3} + \cdots$$
For F-Theory GUTs, different degrees of locality:

- **Global** model: Specify full compactification space (CY fourfold): Includes all branes, fluxes, obeys consistency conditions, can stabilise moduli etc.
  
  [Blumenhagen et al.; Grimm et al.; Marsano et al.; ...]

- **Semilocal** model: Focus on the GUT surface (brane stack) $S$ and matter curves within $S$: Decouples bulk of compactification space, certain consistency conditions included
  
  [Hayashi et al.; Donagi, Wijnholt; Grimm, Weigand; Marsano et al.; Dudas, Palti; CL, Nilles, Stephan; ...]

- **Local** model: Consider only points within $S$ where matter curves intersect and interactions are localised: Simple, and hope for predictivity because any good global model must contain good local model and bulk physics decoupled. Certain questions cannot be answered, and actual existence of global completion is not guaranteed.
  
  [Donagi, Wijnholt; Heckman, Vafa et al.; Watari et al.; ...]
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2 Local $SU(5)$ GUT with Matter Parity

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5 Conclusion
• $SU(5)$ GUT on two-complex-dimensional surface $S$ in the base of elliptically fibred CY fourfold $X$, locally given by $w = 0$
• Elliptic fibration over $S$ described by Tate model (scaled $z \rightarrow 1$)

$$y^2 = x^3 + a_5 \, xy + a_4 \, x^2 + a_3 \, y + a_2 \, x + a_0$$

$b_k$: sections in certain line bundles on $S$
GUT Surface, Matter Curves

*SU(5)* GUT on two-complex-dimensional surface $S$ in the base of elliptically fibred CY fourfold $X$, locally given by $w = 0$

*Elliptic fibration over* $S$ *described by Tate model (scaled* $z \rightarrow 1$)

\[ y^2 = x^3 + b_5 \, x y + b_4 w \, x^2 + b_3 w^2 y + b_2 w^4 x + b_0 w^5 \]

$b_k$: sections in certain line bundles on $S$
• SU(5) GUT on two-complex-dimensional surface $S$ in the base of elliptically fibred CY fourfold $X$, locally given by $w = 0$
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$b_k$: sections in certain line bundles on $S$
• Discriminant becomes

$$\Delta = w^5 \left( b_5^4 P + w b_5^2 (8 b_4 P + b_5 R) + O(w^2) \right)$$

$P, R$: polynomials in the $b_k$
GUT Surface, Matter Curves

- SU(5) GUT on two-complex-dimensional surface $S$ in the base of elliptically fibred CY fourfold $X$, locally given by $w = 0$
- Elliptic fibration over $S$ described by Tate model (scaled $z \to 1$)

$$y^2 = x^3 + b_5 \, x y + b_4 \, w \, x^2 + b_3 \, w^2 \, y + b_2 \, w^4 \, x + b_0 \, w^5$$

$b_k$: sections in certain line bundles on $S$

- Discriminant becomes

$$\Delta = w^5 \left( b_5^4 P + w \, b_5^2 \left( 8b_4 \, P + b_5 \, R \right) + O(w^2) \right)$$

$P, R$: polynomials in the $b_k$

- Locally, SU(5) is enhanced

  to $SU(6)$: $P = 0 \implies$ localised $5$

  to $SO(10)$: $b_5 = 0 \implies$ localised $10$
Yukawa Couplings

- 6D Matter in hypermultiplets – 4D zero modes determined by flux (via index theorem)
- $U(1)$ flux keeps full multiplets, hypercharge flux can split multiplets – doublet-triplet splitting
  \[[\text{BHV, DW}]\]
- For $SU(5)$ GUT require $SO(12)$ enhancement for down-type Yukawas,

\[
(66)^3 \supset \bar{5}_{H_d} \bar{5}_M 10_M
\]

and $E_6$ enhancement for up-type,

\[
(78)^3 \supset 5_{H_u} 10_M 10_M
\]
Point of $E_8$

- Need $E_6$ and $SO(12)$ enhancements for up- and down-type Yukawas
- CKM matrix: Favourably, these points coincide (no geometric suppression of quark mixing) $\rightsquigarrow E_7$
- For PMNS matrix: Further enhancement to $E_8$ (but we do not consider neutrinos in the following)
- Hence: One single Yukawa “point of $E_8$”, all interactions localised here
  - Allows for higher interaction terms – Froggatt–Nielsen type masses using GUT singlets
- Simple and potentially predictive: All (superpotential) interactions determined by group theory, geometric data can be largely ignored
**Gauge Theory Description**

- Consider SYM theory on worldvolume of $S$: $E_8$ GUT, broken to $SU(5)$ by adjoint Higgs (parameterises brane motion)
- Actually, rank-preserving breaking

$$E_8 \longrightarrow (SU(5) \times SU(5)_\perp) \longrightarrow SU(5) \times U(1)^4$$

- Extra $U(1)$'s generically massive in F-Theory by geometric Stueckelberg effect, but this cannot be analysed in local model – $U(1)$'s remain as global selection rules
  [Grimm, Weigand]

- Higgs field varies over $S$ – matter curves now visible as vanishing loci of Higgs eigenvalues, e.g.

$$\langle \Phi \rangle \sim \begin{pmatrix} z \\ -z \end{pmatrix}$$
$E_8$ Higgs

$E_8 \rightarrow SU(5) \times SU(5)_{\perp}$

$248 \rightarrow (24, 1) \oplus (1, 24) \oplus [(10, 5) \oplus (5, \overline{10}) \oplus \text{c.c.}]$

$Higgs \quad \Phi \sim \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{pmatrix} \in (1, 24), \quad \sum_i t_i = 0$

Connection to Tate model: Deformed $E_8$ singularity,

$y^2 = x^3 + b_0 w^5 \quad \rightarrow \quad y^2 = x^2 + b_0 \prod (w - t_i)$

$\bowtie$ the $b_k$ are symmetric polynomials in the $t_i$ of order $k$, no $b_1$ because of tracelessness
$t_i$ are eigenvalues in the $5$ of $SU(5)_{\perp}$, i.e.

$$\Phi e_i = t_i e_i$$

$\sim 10$ of $SU(5)_{\perp}$ spanned by $e_i \wedge e_j, \; i \neq j$, with eigenvalue $t_i + t_j$

Representations of $SU(5) \times SU(5)_{\perp}$ appear as $(10, 5) \oplus (5, 10)$

$\sim$ in terms of reps of visible $SU(5)$, matter curves are given by

$$t_i = 0 \quad \text{localised } 10$$
$$-t_i - t_j = 0 \quad \text{localised } 5$$
$$t_i - t_j = 0 \quad \text{localised } 1$$

$t_i$ double as charges: For gauge-invariant terms, $t_i$ must sum to zero (possibly using $\sum_i t_i = 0$) – realises $U(1)^4 \subset SU(5)_{\perp}$ selection rules
The $b_k$ in the Tate model are symmetric polynomials in the $t_i$
$\Rightarrow$ Invariant under permutations of the $t_i$

Interpretation: Self-intersection, locally distinct-looking branes are the same

Heavy top requires coupling $5_{H_u} \cdot 10_{\text{top}} \cdot 10_{\text{top}}$
$\sim$ (at least) $\mathbb{Z}_2$ monodromy $t_1 \leftrightarrow t_2$

Fixes top and up-type Higgs curve: $10_{\text{top}} \sim \{t_1, t_2\}$, $5_{H_u} \sim -t_1 - t_2$
Reduces $SU(5)_{\perp}$ to lower rank
**SU(5) GUT**

Unify SM gauge group $G_{\text{SM}} = SU(3) \times SU(2) \times U(1) \rightarrow SU(5)$, decomposition of $SU(5)$ fields into SM representations:

\[
\begin{align*}
\text{Gauge bosons:} & \quad 24 \sim G \oplus W \oplus B_Y \oplus X \oplus Y \\
\text{Matter:} & \quad 10_M \sim Q \oplus u^c \oplus e^c \\
& \quad \overline{5}_M \sim d^c \oplus L \\
\text{Higgs:} & \quad 5_{H_u} \sim H_u \oplus T_u \\
& \quad \overline{5}_{H_d} \sim H_d \oplus T_d
\end{align*}
\]

Higgs triplets and $X$, $Y$ bosons need to be very heavy for proton stability.

Break $SU(5)$ by hypercharge flux – topological condition to avoid breaking hypercharge (not available in heterotic models): Flux needs to be globally trivial.
Yukawa Couplings

Good couplings: Quark and lepton masses, weak-scale $\mu$ term

$$W_{\text{good}} = \mu 5_{H_u} 5_{H_d} + Y_{u} 5_{H_u} 10_{M} 10_{M} + Y_{d} 5_{H_d} 5_{M} 10_{M}$$
Yukawa Couplings

Good couplings: Quark and lepton masses, weak-scale $\mu$ term

$$W_{\text{good}} = \mu \bar{5}_{H_u} 5_{H_d} + Y_u \bar{5}_{H_u} 10_M 10_M + Y_d 5_{H_d} \bar{5}_{M} 10_M$$

Bad couplings: Baryon and lepton number violating operators

$$W_{\text{bad}} = \beta \bar{5}_{H_u} 5_M + \lambda \bar{5}_M 5_M 10_M$$
$$+ W^1 10_M 10_M 10_M \bar{5}_M + W^2 10_M 10_M 10_M \bar{5}_{H_d}$$
$$+ W^3 \bar{5}_M 5_M 5_{H_u} 5_{H_u} + W^4 \bar{5}_M 5_{H_d} 5_{H_u} 5_{H_u}$$
$$K_{\text{bad}} = K^1 10_M 10_M 5_M + K^2 \bar{5}_{H_u} \bar{5}_{H_u} 10_M$$

Coefficients can contain singlet VEVs, suppressed by $M_{\text{GUT}}$ [Conlon, Palti]
Yukawa Couplings

Good couplings: Quark and lepton masses, weak-scale $\mu$ term

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$$+ W^1 10_M 10_M 10_M \bar{5}_M + W^2 10_M 10_M 10_M \bar{5}_{H_d}$$

$$+ W^3 \bar{5}_M \bar{5}_M 5_{H_u} 5_{H_u} + W^4 \bar{5}_M \bar{5}_{H_d} 5_{H_u} 5_{H_u}$$

$$K_{\text{bad}} = K^1 10_M 10_M 5_M + K^2 \bar{5}_{H_u} \bar{5}_{H_u} 10_M$$

Coefficients can contain singlet VEVs, suppressed by $M_{\text{GUT}}$ [Conlon, Palti]

Some terms related by interchange $\bar{5}_{H_d} \leftrightarrow \bar{5}_M$
Various discrete symmetries help for proton stability. Compatibility with $SU(5)$ implies $\mathbb{Z}_2$ “matter parity” which distinguishes Higgs and matter:

$$
\begin{array}{c|cc}
P_M & 5_{H_u}, \bar{5}_{H_d} & 10_M, \bar{5}_M \\
+1 & \text{Forbids all baryon and lepton number violating operators except} \\
-1 & \\
\end{array}
$$

Forbids all baryon and lepton number violating operators except

$$W^1 10_M 10_M 10_M \bar{5}_M \quad \text{and} \quad W^3 \bar{5}_M \bar{5}_M 5_{H_u} 5_{H_u}$$

$W^3$ generates neutrino masses (Weinberg operator), can be tolerated if suppression scale high enough (but will not be generated, so ignore from now on)

$W^1$ is very strongly constrained ($W^1 \supset QQQL, \bar{u}\bar{u}\bar{d}\bar{e}, \ldots$) – forbid this by clever choice of matter curves (i.e. $U(1)$s)
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Model Requirements

For the local model we require

- $P_M$ defined at the point of $E_8$
- heavy top quark (i.e. rank-one up-type Yukawa matrix at tree level)
- No dim-5 proton decay (the $W^1$ operator forbidden at all orders)
- Masses for all quarks and leptons after switching on VEVs
  (down-type Yukawa matrix can be rank-zero or one, but not rank-two)
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Local model building freedom: Freely choose

- Monodromy (at least $\mathbb{Z}_2$)
- Assignment of matter and Higgs zero modes to curves
- Singlet VEVs (for matter parity even singlets)
- Assume: Allowed terms generated with order-one coefficients
Matter Parity

Define $\mathbb{Z}_2$ matter parity in terms of the $t_i$ (i.e. as subgroup of $SU(5)_\perp$):

$$P_M = (-1)^{c_it_i}, \quad c_i = 0, 1 \quad \text{(defined mod 2)}$$

- Monodromy $t_1 \leftrightarrow t_2$ requires $c_1 = c_2 = 1$ so $10_{\text{top}}$ is odd
- Up-type masses always allowed once gauge invariant
- Down-type masses give constraint:

\[
\begin{array}{cccc}
\bar{5}_{H_d} & \bar{5}_M & 10_M \\
\text{charge} & t_i + t_j & t_k + t_l & t_m
\end{array}
\]
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$$\begin{array}{ccc}
\bar{5}_{H_d} & \bar{5}_M & 10_M \\
\text{charge} & t_i + t_j & t_k + t_l & t_m \\
c_i t_i & 0 \text{ or } 2 & 1 & 1
\end{array}$$

Gauge invariant iff all $t_i$ distinct – can only be matter parity even if even number of $c_i = 1$ (singlets have charge $t_i - t_j$, so don’t change the argument)

- Note: $W^1$ operator has same charge structure
Two Possibilities

Hence, two possible definitions of matter parity:

**Case I:** \( P_M = (-1)^{t_1 + t_2 + t_3 + t_4} \)

**Case II:** \( P_M = (-1)^{t_1 + t_2} \)

Now analyse matter, Higgs and VEV assignment for both cases: \( 10_{\text{top}} \) and \( 5_{H_u} \) already fixed, need to distribute remaining matter and \( 5_{H_d} \) according to their matter parity.

Main restriction: Forbid \( W^1 \), but allow down-type Yukawas.
## Case I: Matter and VEV Assignment

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<tr>
<th>Matter 10 Curves</th>
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<tr>
<td>$10_1$</td>
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<td>$10_2$</td>
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<th>Matter 5 Curves</th>
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<tr>
<td>$5_3$</td>
<td>$-t_{1,2} - t_5$</td>
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<tr>
<td>$5_5$</td>
<td>$-t_3 - t_5$</td>
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<td>$5_6$</td>
<td>$-t_4 - t_5$</td>
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<tr>
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<td>$1_1$</td>
<td>$\pm (t_{1,2} - t_3)$</td>
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<tr>
<td>$1_2$</td>
<td>$\pm (t_{1,2} - t_4)$</td>
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<td>$1_4$</td>
<td>$\pm (t_3 - t_4)$</td>
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<td>$1_7$</td>
<td>$t_1 - t_2$</td>
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- $W^1$ without singlets:
  - $10_1 10_1 10_2 5_6$,
  - $10_1 10_1 10_3 5_5$,
  - $10_1 10_2 10_3 5_3$
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- \( W¹ \) without singlets:
  - \( 10₁ 10₁ 10₂ \bar{5}_₆ \),
  - \( 10₁ 10₁ 10₃ \bar{5}_₅ \),
  - \( 10₁ 10₂ 10₃ \bar{5}_₃ \)

\( \rightsquigarrow \) no matter on \( 10₂, 5₅ \)
### Case I: Matter and VEV Assignment

**Matter 10 Curves**

| 10₁ | $t_{1,2}$ | top |
| 10₂ | $t_3$     | no matter |
| 10₃ | $t_4$     | matter |

**Matter 5 Curves**

| 5₃  | $-t_{1,2} - t_5$ | matter |
| 5₅  | $-t_3 - t_5$     | no matter |
| 5₆  | $-t_4 - t_5$     | matter |

**Even Singlet Curves**

| 1₁  | $\pm (t_{1,2} - t_3)$ | + |
| 1₂  | $\pm (t_{1,2} - t_4)$ | + |
| 1₄  | $\pm (t_3 - t_4)$     | + |
| 1₇  | $t_1 - t_2$           | + |

- \( W^1 \) without singlets:
  - \( 10_110_110_2\bar{5}_6 \),
  - \( 10_110_110_3\bar{5}_5 \),
  - \( 10_110_210_3\bar{5}_3 \)

  \( \sim \) no matter on \( 10_2, 5_5 \)

- \( W^1 \) with singlets:
  - e.g. \( 10_110_110_3\bar{5}_61_4 \),
  - \( 10_110_110_3\bar{5}_31_1 \)
Case I: Matter and VEV Assignment

### Matter 10 Curves

<table>
<thead>
<tr>
<th>10₁</th>
<th>( t_{1,2} )</th>
<th>top</th>
</tr>
</thead>
<tbody>
<tr>
<td>10₂</td>
<td>( t_3 )</td>
<td>no matter</td>
</tr>
<tr>
<td>10₃</td>
<td>( t_4 )</td>
<td>matter</td>
</tr>
</tbody>
</table>

### Matter 5 Curves

<table>
<thead>
<tr>
<th>5₃</th>
<th>(- t_{1,2} - t_5 )</th>
<th>matter</th>
</tr>
</thead>
<tbody>
<tr>
<td>5₅</td>
<td>(- t_3 - t_5 )</td>
<td>no matter</td>
</tr>
<tr>
<td>5₆</td>
<td>(- t_4 - t_5 )</td>
<td>matter</td>
</tr>
</tbody>
</table>

### Even Singlet Curves

<table>
<thead>
<tr>
<th>1₁</th>
<th>( \pm (t_{1,2} - t_3) )</th>
<th>no VEV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1₂</td>
<td>( \pm (t_{1,2} - t_4) )</td>
<td>VEV</td>
</tr>
<tr>
<td>1₄</td>
<td>( \pm (t_3 - t_4) )</td>
<td>no VEV</td>
</tr>
<tr>
<td>1₇</td>
<td>( t_1 - t_2 )</td>
<td>VEV</td>
</tr>
</tbody>
</table>

- \( W¹ \) without singlets:
  
  \[
  10_1 10_1 10_2 5_6 , \\
  10_1 10_1 10_3 5_5 , \\
  10_1 10_2 10_3 5_3
  \]

  \( \leadsto \) no matter on \( 10_2, 5_5 \)

- \( W¹ \) with singlets:

  e.g. \( 10_1 10_1 10_3 5_6 1_4 , \)

  \[
  10_1 10_1 10_3 5_3 1_1
  \]

  \( \leadsto \) no VEVs for \( 1_1, 1_4 \)

  (because of \( t_3 \))
Case I: Down-Type Higgs

<table>
<thead>
<tr>
<th>Higgs-like $\bar{5}$ Curves</th>
<th>Down-type Yukawas</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{5}_{H_u}$</td>
<td>$-t_1 - t_2$</td>
</tr>
<tr>
<td>$\bar{5}_1$</td>
<td>$-t_{1,2} - t_3$</td>
</tr>
<tr>
<td>$\bar{5}_2$</td>
<td>$-t_{1,2} - t_4$</td>
</tr>
<tr>
<td>$\bar{5}_4$</td>
<td>$-t_3 - t_4$</td>
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Case I: Down-Type Higgs

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<th>Higgs-like $\mathbf{5}$ Curves</th>
<th>Down-type Yukawas</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{5}_{Hu}$ $-t_1 - t_2$</td>
<td>No masses at tree level or with singlets</td>
</tr>
<tr>
<td>$\bar{5}<em>1$ $-t</em>{1,2} - t_3$</td>
<td></td>
</tr>
<tr>
<td>$\bar{5}<em>2$ $-t</em>{1,2} - t_4$</td>
<td>No masses at tree level or with singlets</td>
</tr>
<tr>
<td>$\bar{5}_4$ $-t_3 - t_4$</td>
<td></td>
</tr>
</tbody>
</table>

- Down-type Higgs needs a factor of $t_3$ to allow for Yukawa couplings (at any order)
Case I: Down-Type Higgs

<table>
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<tr>
<td>$\bar{5}_{H_u}$ $-t_1 - t_2$</td>
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</tr>
<tr>
<td>$\bar{5}<em>1$ $-t</em>{1,2} - t_3$</td>
<td>either rank-two Yukawa matrix, or no up-type masses with singlets</td>
</tr>
<tr>
<td>$\bar{5}<em>2$ $-t</em>{1,2} - t_4$</td>
<td>No masses at tree level or with singlets</td>
</tr>
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<td>$\bar{5}_4$ $-t_3 - t_4$</td>
<td></td>
</tr>
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- Down-type Higgs needs a factor of $t_3$ to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two
Case I: Down-Type Higgs

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</tr>
<tr>
<td></td>
<td>No masses at tree level or with singlets</td>
</tr>
<tr>
<td></td>
<td>$\mu$ term</td>
</tr>
<tr>
<td>$\bar{5}_1$</td>
<td>$-t_{1,2} - t_3$</td>
</tr>
<tr>
<td></td>
<td>either rank-two Yukawa matrix, or no up-type masses with singlets</td>
</tr>
<tr>
<td>$\bar{5}_2$</td>
<td>$-t_{1,2} - t_4$</td>
</tr>
<tr>
<td></td>
<td>No masses at tree level or with singlets</td>
</tr>
<tr>
<td>$\bar{5}_4$</td>
<td>$-t_3 - t_4$</td>
</tr>
</tbody>
</table>

- Down-type Higgs needs a factor of $t_3$ to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two
- String-scale $\mu$ term for both Higgses on one curve
### Case I: Down-Type Higgs

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<th>Down-type Yukawas</th>
</tr>
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<tbody>
<tr>
<td>$\bar{5}_{H_u}$ $-t_1 - t_2$</td>
<td>No masses at tree level or with singlets $\mu$ term</td>
</tr>
<tr>
<td>$\bar{5}<em>1$ $-t</em>{1,2} - t_3$</td>
<td>either rank-two Yukawa matrix, or no up-type masses with singlets</td>
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<td>No masses at tree level or with singlets</td>
</tr>
<tr>
<td>$\bar{5}_4$ $-t_3 - t_4$</td>
<td>Rank-one Yukawa matrix, bottom quark heavy</td>
</tr>
</tbody>
</table>

- Down-type Higgs needs a factor of $t_3$ to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two
- String-scale $\mu$ term for both Higgses on one curve
- $\bar{5}_4 = \bar{5}_{H_d}$ is unique choice, tree-level coupling $\bar{5}_{H_d} 10_{\text{top}} \bar{5}_3$
Case I: Yukawas and CKM

- Example Assignment: Third generation on $10_1$ and $\bar{5}_3$, light generations on $10_3$ and $\bar{5}_6$
- Higgses: $\bar{5}_{H_u}$ and $\bar{5}_4$, only $\langle 1_2 \rangle \sim \epsilon$ required at first order
- Ignore $1_7$ and $O(1)$ coefficients
- Yukawa matrices (schematically):

\[
Y^u \sim Y^d \sim \begin{pmatrix}
\epsilon^2 & \epsilon^2 & \epsilon \\
\epsilon^2 & \epsilon^2 & \epsilon \\
\epsilon & \epsilon & 1
\end{pmatrix}
\]

- CKM matrix:

\[
V_{\text{CKM}} \sim \begin{pmatrix}
1 & 1 & \epsilon \\
1 & 1 & \epsilon \\
\epsilon & \epsilon & 1
\end{pmatrix}
\]

- Masses and mixings possible though not a great fit
- Degeneracy because three generations come from two curves
\[ P_M = (-1)^{t_1 + t_2} \]

\( \Leftrightarrow \) split t’s into \( t_{\text{odd}} = \{t_1, t_2\} \) and \( t_{\text{even}} = \{t_3, t_4, t_5\} \)

- Symmetric setup, possible monodromy acting on \( t_{\text{even}} \)
- \( 10_{\text{top}} \) is the unique matter \( 10 \) curve
- Down-type Higgs unique (up to relabeling)
- Matter-parity even singlets do not mix \( t_{\text{odd}} \) and \( t_{\text{even}} \)
- \( W^1 \) operator cannot be generated: Charge \( 4t_{\text{odd}} + t_{\text{even}} \) cannot be compensated by matter-parity even singlets
- Three possible matter \( 5 \) curves (charges \( t_{\text{odd}} - t_{\text{even}} \)): model building choice
- Different choices of singlet VEVs possible, achieve masses and mixing
Local Model Summary

- Already locally, rather constrained model: Only two possible definitions of matter parity at the point of $E_8$
- In both cases, assignments of matter and Higgses is unique or strongly constrained
- Restrictions mainly from forbidding $W^1$ while allowing for down-type masses
- $W^3$ operator (neutrino masses) is not generated in any case
- Masses for all matter fields and CKM mixing possible
- Involves choices of zero modes and VEVs by hand – these cannot be calculated in the local framework
Semilocal Approach

Now *semilocal* picture: Consider GUT surface $S$, using spectral cover approach. 
Main aim: Find homology classes of matter curves which allow to find the flux restrictions and thus the zero mode spectrum.

Two types of fluxes:

- $U(1) \subset SU(5)_{\perp}$ fluxes on matter curves (from the transverse branes): Determines (chiral) 4D zero modes for full GUT multiplets. These are still free parameters up to anomaly cancellation requirements.

- Hypercharge flux on $S$ (globally trivial so hypercharge stays unbroken): Restrictions to matter curves splits $SU(5)$ multiplets; homological relations between matter curves lead to relations between the splittings.
Spectral Cover

Spectral cover: Hypersurface in projective threefold

\[ \mathbb{P}(K_S \oplus \mathcal{O}_S) \]

with homogeneous coordinates \( U : V \) given by spectral equation for \( \Phi \). Because of \( \mathbb{Z}_2 \) monodromy, spectral equation must factorise:

\[
0 = b_0 U^5 + b_2 U^3 V^2 + b_3 U^2 V^3 + b_4 U V^4 + b_5 V^5 \\
= (a_1 V^2 + a_2 U V + a_3 U^2) (a_4 V + a_7 U) (a_5 V + a_8 U) (a_6 V + a_9 U)
\]

\( b_k \) are sections in line bundles with Chern classes
\( \eta - kc_1 = (6 - k) c_1(S) + c_1(N_{S/X}) \). This determines the bundles for the \( a_m \), and in turn for the matter curves, in terms of three unspecified line bundles \( \chi_{7,8,9} \).

Involves particular solution of \( b_1 = 0 \) constraint – might not be most general one?
Fluxes and Zero Modes

$U(1)$ fluxes: Given by integers $M_5$, $M_{10}$. Free up to consistency conditions

$$\sum M_{10} + \sum M_5 = 0,$$
$$M_{101} = -(M_{51} + M_{52} + M_{53})$$

Hypercharge flux must be globally trivial, hence

$$0 = F_Y \cdot c_1 = F_Y \cdot \eta \implies \sum_{10} F_Y = \sum_{5} F_Y = 0$$

Restrictions to matter curves given by $N_Y = F_Y \cdot$ (homology class). For curve with flux numbers $M$ and $N_Y$, zero modes given by

$$10 \quad (3, 2) : M_{10} \quad (3, 1) : M_5$$
$$\quad (\bar{3}, 1) : M_{10} - N_Y \quad (1, 2) : M_5 + N_Y$$
$$\quad (1, 1) : M_{10} + N_Y$$
### 10 Curves

<table>
<thead>
<tr>
<th></th>
<th>$M$</th>
<th>$N_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10(_1)</td>
<td>$-(M_{51} + M_{52} + M_{53})$</td>
<td>$\tilde{N}$</td>
</tr>
<tr>
<td>10(_2)</td>
<td>$M_{102}$</td>
<td>$N_7$</td>
</tr>
<tr>
<td>10(_3)</td>
<td>$M_{103}$</td>
<td>$N_8$</td>
</tr>
<tr>
<td>10(_4)</td>
<td>$M_{104}$</td>
<td>$N_9$</td>
</tr>
</tbody>
</table>

### 5 Curves

<table>
<thead>
<tr>
<th></th>
<th>$M$</th>
<th>$N_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5(_{H_u})</td>
<td>$M_{5_{H_u}}$</td>
<td>$\tilde{N}$</td>
</tr>
<tr>
<td>5(_1)</td>
<td>$M_{51}$</td>
<td>$-\tilde{N}$</td>
</tr>
<tr>
<td>5(_2)</td>
<td>$M_{52}$</td>
<td>$-\tilde{N}$</td>
</tr>
<tr>
<td>5(_3)</td>
<td>$M_{53}$</td>
<td>$-\tilde{N}$</td>
</tr>
<tr>
<td>5(_4)</td>
<td>$M_{54}$</td>
<td>$N_7 + N_8$</td>
</tr>
<tr>
<td>5(_5)</td>
<td>$M_{55}$</td>
<td>$N_7 + N_9$</td>
</tr>
<tr>
<td>5(_6)</td>
<td>$M_{56}$</td>
<td>$N_8 + N_9$</td>
</tr>
</tbody>
</table>

- Three free parameters $N_{7,8,9}$ for the hypercharge flux, corresponding to three unspecified line bundles
- $\tilde{N} = N_7 + N_8 + N_9$
- Split some 5 curves $\Rightarrow$ split some 10 curves

[Marsano et al.; Dudas, Palti]
Case I: Matter Sector is Fine

- Doublet-triplet splitting for Higgses ($\tilde{N} \neq 0$) inevitably splits $10_{\text{top}}$ and at least one more $10$ curve.
- However, splitting of matter multiplets is OK as long as there are three generations of zero modes in the end, i.e. other $10$ curve must have “opposite” split.
- Matter on $10_1, 10_3, 5_3$ and $5_6$, so to have full net generations, we require $N_7 = N_9 = 0 \Rightarrow$ only $N_8$ left free.
- No exotics from $10$’s and remaining matter-like $5$ curve can be satisfied by choosing appropriate $M$’s.

Thus, satisfactory matter sector can be engineered easily.
Case I: Higgs Sector is not Fine

- Higgs sector:

<table>
<thead>
<tr>
<th></th>
<th>(3, 1)</th>
<th>(1, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5_{Hu}$</td>
<td>$M_{5_{Hu}}$</td>
<td>$M_{5_{Hu}} + N_8$</td>
</tr>
<tr>
<td>$5_1$</td>
<td>$M_{5_1}$</td>
<td>$M_{5_1} - N_8$</td>
</tr>
<tr>
<td>$5_2$</td>
<td>$M_{5_2}$</td>
<td>$M_{5_2} - N_8$</td>
</tr>
<tr>
<td>$5_4$</td>
<td>$M_{5_4}$</td>
<td>$M_{5_4} + N_8$</td>
</tr>
</tbody>
</table>

- We can pairwise decouple unwanted triplets from $5_{Hu}$ and $5_2$, and from $5_1$ and $5_4$ by coupling to VEV for $1_2$

- However:

$$\#(\text{doublets from } 5_{Hu}, 5_2) = \#(\text{triplets from } 5_{Hu}, 5_2)$$

- Problem persists even when allowing exotics from the matter sector
- Separately, down-type Higgs on $5_4$ cannot be realised
Case II: Again not Fine

- Only one matter $\mathbf{10}$ curve, split with same parameter as up-type Higgs
- Higgs triplets cannot be decoupled by even matter parity singlets
- No exotic matter $\Rightarrow$ no doublet-triplet splitting
- Ignoring matter sector: Still cannot achieve pair of Higgs doublets
Case II: Again not Fine

- Only one matter $10$ curve, split with same parameter as up-type Higgs
- Higgs triplets cannot be decoupled by even matter parity singlets
- No exotic matter $\Rightarrow$ no doublet-triplet splitting
- Ignoring matter sector: Still cannot achieve pair of Higgs doublets

Upshot: In both cases, proper doublet-triplet splitting in Higgs sector does not work, even when allowing for exotics from the matter sector – both models cannot be realised already in semilocal setup!
Contents

1 F-Theory GUT Model Building

2 Local $SU(5)$ GUT with Matter Parity

3 Matter Parity in Local Models

4 Semilocal Embedding

5 Conclusion
Conclusions

- Analysed F-Theory GUT at “point of $E_8$” and in semilocal approach
- Goal: Find a locally defined matter parity to ensure proton stability
- Local model is already very constrained: Two cases only
- In semilocal embedding, doublet-triplet splitting cannot be realised in either case

=>$\rightarrow$ Models fail first step towards realisation
Conclusions

- Analysed F-Theory GUT at “point of $E_8$” and in semilocal approach
- Goal: Find a locally defined matter parity to ensure proton stability
- Local model is already very constrained: Two cases only
- In semilocal embedding, doublet-triplet splitting cannot be realised in either case

$\Rightarrow$ Models fail first step towards realisation

- Predictivity of local point in question – Crucial model features required to have nonlocal origin?
- Possible loopholes:
  - Assumed GUT breaking by hypercharge flux – different breaking mechanisms?
  - Non-diagonal Higgs fields ( “T-Branes”, “Gluing Morphisms”) might help to get rid of exotics

[Cecotti et al.; Donagi, Wijnholt]