The Potential Fate of Local Model Building

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Motivation and Outline

- F-Theory Model Building: Generalisation of type IIB intersecting branes
- Usually, consider local models: Focus on brane stack or points within the stack and decouple bulk of the compactification manifold
- Advantage: Simple, physics basically fixed by symmetry
- Obvious question: Existence of global completion

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- Obvious question: Existence of global completion
- GUT models need to address proton stability
- Dimension-four proton decay: Forbidden by matter parity or variants – should be defined locally
- Dimension-five proton decay: Use zero mode assignment, i.e. additional $U(1)$ symmetries present in the setup

Contents

String Phenomenology

1. Find realistic particle physics models in string theory:
   - Gauge group (standard model or GUT)
   - Matter content (chiral spectrum, doublet-triplet splitting, absence of light exotics)
   - Proton stability
   - Fermion masses and mixings
   - Spontaneously broken $\mathcal{N} = 1$ SUSY in four dimensions
2. Look for imprints of string theory in low-energy physics:
   - Mediation schemes, patterns of soft masses
   - Exotics below GUT/Planck scale
   - Thresholds, gauge coupling unification

GUTs and Strings

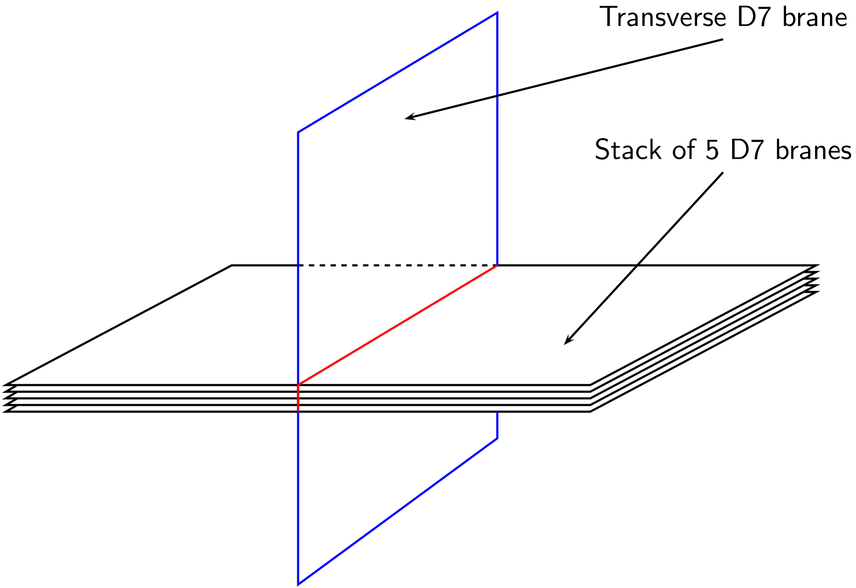
Promising paths:

- $E_8 \times E_8$ heterotic string on orbifolds or smooth Calabi–Yaus
  - Global models, i.e. full compactification space is specified
  - Gauge fields live in bulk, matter in bulk or on lower-dimensional subspaces
- Type II theories with intersecting branes $\rightsquigarrow$ F-theory
  - Mostly local models, i.e. focus on branes and “decouple” bulk
  - Gauge fields on branes, matter on intersections of branes

General features:

- Exceptional symmetry groups (though not as gauge groups in four dimensions)
- Nontrivial pattern of gauge and matter fields localised on different subspaces of compactification space

Cartoon: Intersecting Branes in Perturbative Type IIB



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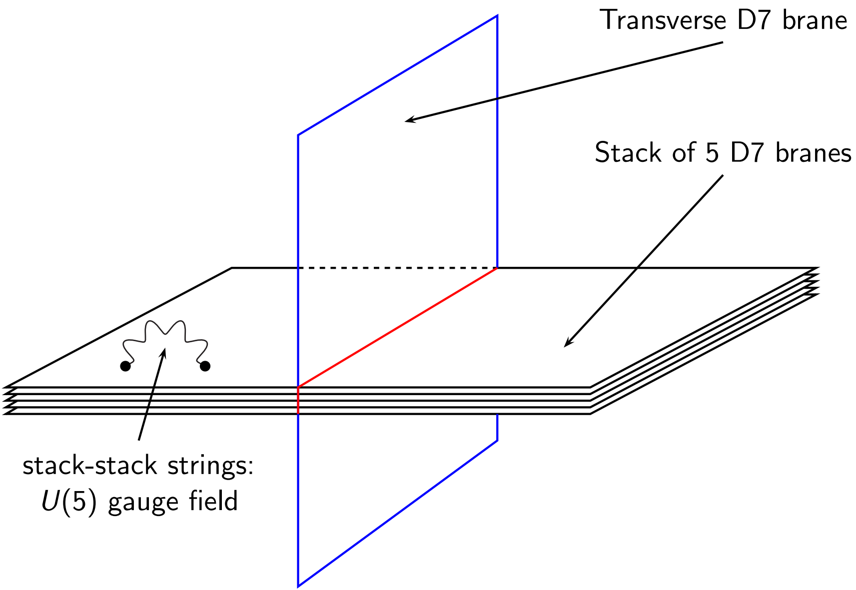

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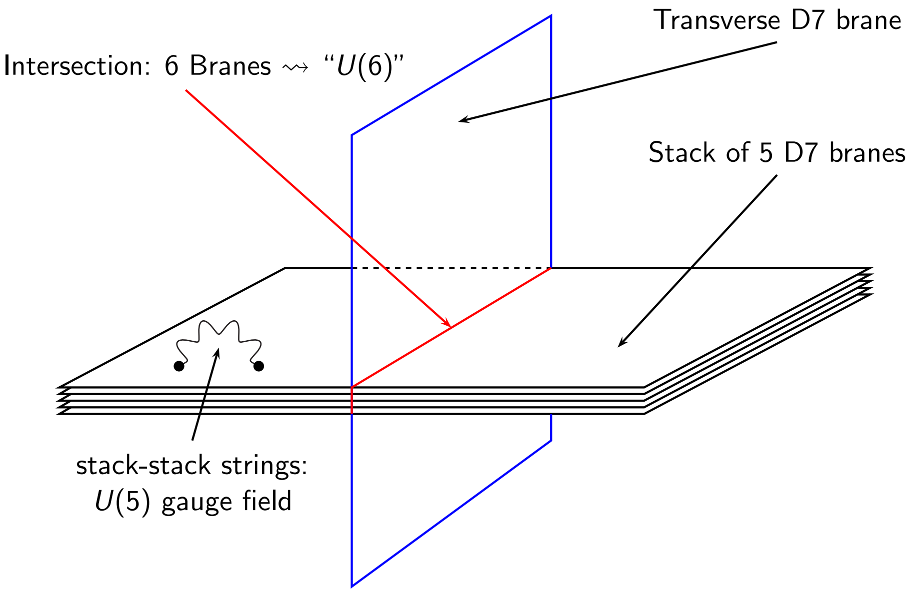

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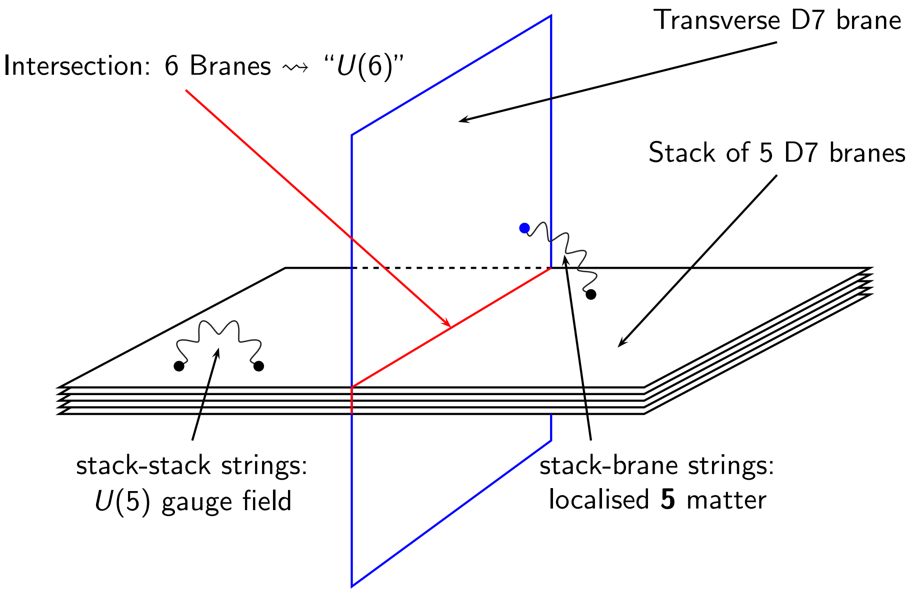

Intersecting Branes: Matter

- Stacks of branes carry gauge theory
- Strings between stacks become massless at intersection ("matter curve") – massless matter
- To infer representation: Consider (auxiliary) higher symmetry group on intersection (not a gauge group!) and decompose adjoint

[Katz, Vafa]

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- Example: Stack of N D7 branes $\rightsquigarrow U(N)$ gauge theory
- Intersection with another stack of M D7 branes: On the intersection, symmetry becomes $U(N+M)$
- Decomposition of adjoint:

$$(\mathbf{N}+\mathbf{M})^2 \longrightarrow \mathbf{N}^2 \oplus \mathbf{M}^2 \oplus (\mathbf{N}, \bar{\mathbf{M}}) \oplus (\bar{\mathbf{N}}, \mathbf{M})$$

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Intersecting Branes: Matter

- Stacks of D7 branes and their intersections: $U(N)$ gauge groups, bifundamental matter
- Include O7 planes: Realise $SO(2N)$ gauge groups and two-index antisymmetric representations, e.g. **10** of $SU(5)$
- Matter curves six-dimensional – matter still in hypermultiplets, i.e. nonchiral
- Chiral four-dimensional spectrum: Determined by flux $\mathcal{F}$ along matter curve, e.g.

$$\mathbf{5}\text{ matter curve} \longrightarrow \#_{\mathbf{5}} = \int \mathcal{F} \quad \text{zero modes in 4D}$$

- $\int \mathcal{F}$ can be negative, so **5** matter curve can give **5**'s or $\bar{\mathbf{5}}$'s in 4D

Intersecting Branes: Yukawas

- Intersection of matter curves in a point: Even higher symmetry group (again not a gauge group)
- Triple adjoint interaction of this group $\rightsquigarrow$ Yukawa couplings between the representations on the matter curves

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- $\curvearrowright$ Hierarchy:
  - 10D Bulk: Closed string modes (supergravity)
  - 8D branes: Gauge fields
  - 6D curves: Localised matter
  - 4D points: Yukawa couplings

Problems

- Possible symmetry groups: $U(N)$, $SO(2N)$ and $Sp(2N)$
- Matter representations: Bifundamental, two-index antisymmetric
- For $SU(5)$ GUTs: diagonal $U(1) \subset U(5)$ forbids top quark Yukawa coupling
- For $SO(10)$ GUTs: no spinors available

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- For $SO(10)$ GUTs: no spinors available
- Both require local E_6 enhancement:

$$\begin{aligned} SU(5) \text{ Yukawas:} \qquad & (\mathbf{78})^3 \supset \mathbf{10}\,\mathbf{10}\,\mathbf{5} \\ SO(10) \text{ Spinors:} \qquad & \mathbf{78} \longrightarrow \mathbf{45} + \mathbf{1} + \mathbf{16} + \overline{\mathbf{16}} \end{aligned}$$

- Type IIB string theory has more general (p, q) branes – cannot be treated perturbatively
- Nicely realised in F-theory

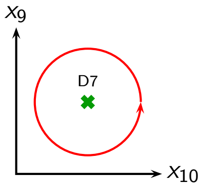
F-Theory: Axiodilaton Monodromy

Type IIB contains complex scalar field: axiodilaton

$$\tau = C_0 + \mathrm{i}e^{-\phi}$$

Going around a single D7 brane at $z = 0$, τ undergoes monodromy

$$\tau \longrightarrow \tau + 1$$



$\curvearrowright$ at brane position τ diverges as

$$\tau \sim \ln z$$

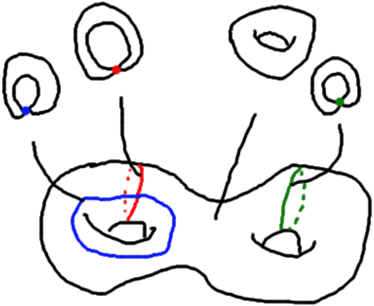
For more general branes, monodromy is in $SL(2, \mathbb{Z})$:

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d}$$

F-Theory: Extra Torus

[Vafa 96]

Key idea of F-Theory: $SL(2,\mathbb{Z})$ is also symmetry of torus complex structure $\rightsquigarrow$ describe variation of τ by *auxiliary* torus over every point of compactification space B_6: elliptic fibration
Brane positions and types encoded in torus singularities



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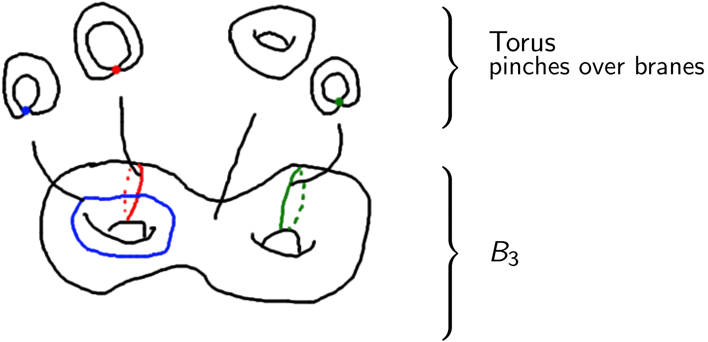

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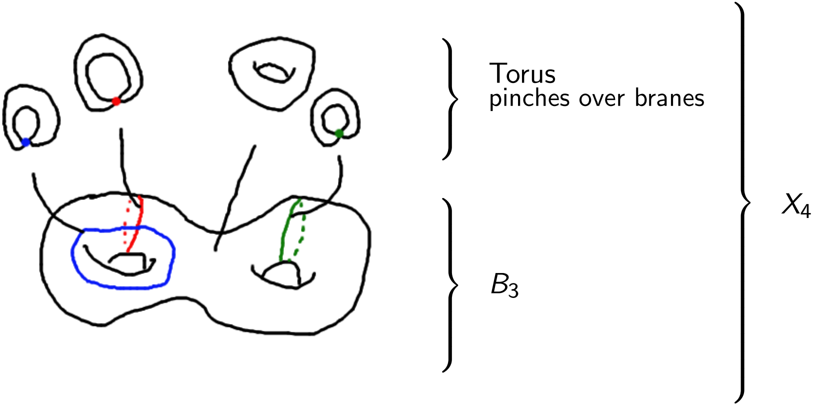

F-Theory: Elliptic Fibration

- Complex three-dimensional base manifold B_3: Compactification space of type IIB becomes base of fibration
- Over each point of B_3, take two complex coordinates x, y and one equation which cuts out a torus:

$$y^2 = x^3 + a_5 xy + a_4 x^2 + a_3 y + a_2 x + a_0$$

(Weierstraß model in "Tate form")

- The a_k are functions on B_3 $\rightsquigarrow$ torus varies over base
- Brane positions $\Leftrightarrow$ Torus degenerates $\Leftrightarrow$ Discriminant vanishes:

$$\Delta = \text{polynomial in the } a_k = 0$$

F-Theory: Tate Model

[Bershadsky et al. 96]

- Generically, several branes: Discriminant factorises as $\Delta = \Delta_1 \cdot \cdots \cdot \Delta_n$
- Type of each brane, i.e. gauge symmetry, determined by vanishing orders of the a_k and Δ (cf. ADE classification of singularities)

[Kodaira '60s]

- Intersections with other branes $\Leftrightarrow$ Local symmetry enhancement $\Leftrightarrow$ Locally Δ and a_k vanish to higher order: Matter curves, Yukawa points
- Much of intersecting brane intuition carries over:
  - Dimensions: $\Delta_1 = 0$ cuts out 8D space (brane), $\Delta_1 = \Delta_2 = 0$ give 6D matter curve etc.
  - Representations can be inferred from (fiducial) higher local symmetries (see, however, [Esole, Yau; Marsano, Schäfer-Nameki])

Global, Semilocal, Local

For F-Theory GUTs, different degrees of locality:

- *Global* model: Specify full compactification space (CY fourfold): Includes all branes, fluxes, obeys consistency conditions, can stabilise moduli etc. [Blumenhagen et al.; Grimm et al.; Marsano et al.;...]
- *Semilocal* model: Focus on the GUT surface (brane stack) S and matter curves within S: Decouples bulk of compactification space, certain consistency conditions included

  [Hayashi et al.; Donagi, Wijnholt; Grimm, Weigand; Marsano et al.; Dudas, Palti; CL, Nilles, Stephan;...]
- *Local* model: Consider only points within S where matter curves intersect and interactions are localised: Simple, hope for predictivity. Certain questions cannot be answered, and actual existence of global completion is not guaranteed.

  [Donagi, Wijnholt; Heckman, Vafa et al.; Watari et al.;...]

Contents

Local $SU(5)$ GUT

To engineer $SU(5)$ GUT, take brane position locally given by coordinate $w = 0$ and choose Tate model appropriately:

$$y^2 = x^3 + a_5 xy + a_4 x^2 + a_3 y + a_2 x + a_0$$

a_k: functions on base

Local $SU(5)$ GUT

To engineer $SU(5)$ GUT, take brane position locally given by coordinate $w = 0$ and choose Tate model appropriately:

$$y^2 = x^3 + b_5\, xy + b_4 w\, x^2 + b_3 w^2\, y + b_2 w^4\, x + b_0 w^5$$

a_k: functions on base
b_k: functions on brane stack

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a_k: functions on base
b_k: functions on brane stack
Discriminant becomes

$$\Delta = w^5 \left(b_5^4 P + w\, b_5^2 \left(8 b_4 P + b_5 R\right) + \mathcal{O}\left(w^2\right)\right)$$

P, R: polynomials in the b_k

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P, R: polynomials in the b_k
Locally, $SU(5)$ is enhanced

to $SU(6)$:	$P = 0$	$\Rightarrow$	localised **5**
to $SO(10)$:	$b_5 = 0$	$\Rightarrow$	localised **10**

Yukawa Couplings

- Tate model for $SU(5)$ GUT localised at $w = 0$:

$$y^2 = x^3 + b_5\, xy + b_4 w\, x^2 + b_3 w^2\, y + b_2 w^4\, x + b_0 w^5$$

- Yukawa couplings require $SO(12)$ and E_6 enhancements:

$$(\mathbf{66})^3 \supset \bar{\mathbf{5}}_{H_d} \bar{\mathbf{5}}_M\, \mathbf{10}_M \qquad \Rightarrow b_3 = b_5 = 0$$

$$(\mathbf{78})^3 \supset \mathbf{5}_{H_u} \mathbf{10}_M\, \mathbf{10}_M \qquad \Rightarrow b_4 = b_5 = 0$$

- $\curvearrowright$ Matter spectrum and Yukawa couplings can be engineered in F-theory

Point of E_8

[Heckman, Tavanfar, Vafa]

- Need E_6 and $SO(12)$ enhancements for up- and down-type Yukawas
- CKM matrix: Favourably, these points coincide (no geometric suppression of quark mixing) $\rightsquigarrow E_7$
- For PMNS matrix: Further enhancement to E_8 (but we do not consider neutrinos in the following)
- Hence: One single Yukawa "point of E_8", all interactions localised here

$\curvearrowright$ Allows for higher interaction terms – Froggatt–Nielsen type masses using GUT singlets

- Simple and potentially predictive: All (superpotential) interactions determined by group theory, geometric data can be largely ignored

Gauge Theory Description

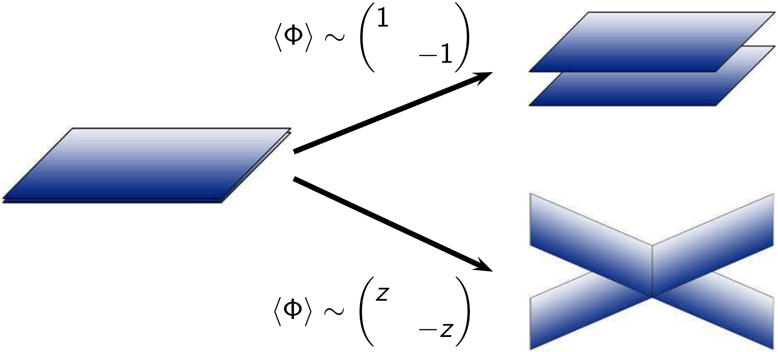
- $SU(5)$ GUT, variously enhanced (potentially) up to E_8
  $\leftrightarrow$ E_8 gauge theory variously broken, generically to $SU(5)$
- 8D super-Yang–Mills theory contains adjoint scalar field
  $\rightsquigarrow$ E_8-breaking Higgs
- Actually: rank-preserving breaking

$$E_8 \longrightarrow (SU(5) \times SU(5)_\perp) \longrightarrow SU(5) \times U(1)^4$$

- Extra $U(1)$'s generically massive in F-Theory by geometric Stueckelberg effect, but this cannot be analysed in local model – $U(1)$'s remain as *global selection rules* [Grimm, Weigand]
- Higgs field varies over S – matter curves now visible as vanishing loci of Higgs eigenvalues

Type IIB interpretation: Higgs as Brane Splitter

Adjoint Higgs field – parameterises brane motion:



$\langle\Phi\rangle \neq 0$: Masses for W bosons – correspond to strings between the branes.
Symmetry is (partially) restored locally where (parts of) $\langle\Phi\rangle = 0$

E_8 Higgs

$$E_8 \longrightarrow SU(5) \times SU(5)_\perp$$
$$\mathbf{248} \longrightarrow (\mathbf{24},\mathbf{1}) \oplus (\mathbf{1},\mathbf{24}) \oplus \left[(\mathbf{10},\mathbf{5}) \oplus \left(\mathbf{5},\overline{\mathbf{10}}\right) \oplus \text{c.c.}\right]$$

$$\text{Higgs} \quad \Phi \sim \begin{pmatrix} t_1 & & & & \\ & t_2 & & & \\ & & t_3 & & \\ & & & t_4 & \\ & & & & t_5 \end{pmatrix} \in (\mathbf{1},\mathbf{24}) \,, \quad \sum_i t_i = 0$$

Connection to Tate model: Deformed E_8 singularity,

$$y^2 = x^3 + b_0 w^5 \quad \longrightarrow \quad y^2 = x^2 + b_0 \prod (w - t_i)$$

$\curvearrowright$ the b_k are symmetric polynomials in the t_i of order k, no b_1 because of tracelessness

Matter Curves

t_i are eigenvalues in the **5** of $SU(5)_\perp$, i.e.

$$\Phi e_i = t_i e_i$$

$\curvearrowright$ **10** of $SU(5)_\perp$ spanned by $e_i \wedge e_j$, $i \neq j$, with eigenvalue $t_i + t_j$

Representations of $SU(5) \times SU(5)_\perp$ appear as $(\mathbf{10}, \mathbf{5}) \oplus \left(\mathbf{5}, \overline{\mathbf{10}}\right)$
$\curvearrowright$ in terms of $SU(5)$ reps, matter curves are given by

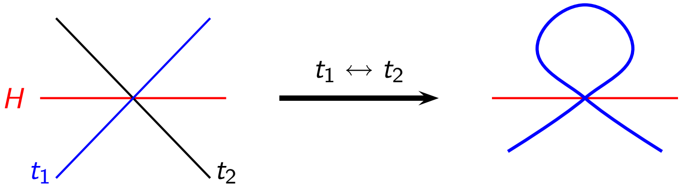
$$\begin{aligned} t_i &= 0 & &\text{localised } \mathbf{10} \\ -t_i - t_j &= 0 & &\text{localised } \mathbf{5} \\ t_i - t_j &= 0 & &\text{localised } \mathbf{1} \end{aligned}$$

t_i double as charges: For gauge-invariant terms, t_i must sum to zero (possibly using $\sum_i t_i = 0$) – realises $U(1)^4 \subset SU(5)_\perp$ selection rules

Monodromy

[Bershadsky et al.]

- The b_k in the Tate model are symmetric polynomials in the t_i
  $\Rightarrow$ Invariant under permutations of the t_i
- Interpretation: Self-intersection, locally distinct-looking branes are the same



- Heavy top requires coupling $\mathbf{5}_{H_u}\mathbf{10}_{\text{top}}\mathbf{10}_{\text{top}}$
  $\rightsquigarrow$ (at least) $\mathbb{Z}_2$ monodromy $t_1 \leftrightarrow t_2$
- Fixes top and up-type Higgs curve: $\mathbf{10}_{\text{top}} \sim \{t_1, t_2\}$, $\mathbf{5}_{H_u} \sim -t_1 - t_2$
- Reduces $SU(5)_\perp$ to lower rank

$SU(5)$ Breaking

Need to break $SU(5) \to G_{\text{SM}}$ and remove X, Y bosons and Higgs triplets from spectrum
Discrete Wilson lines and adjoint Higgses not available for S a del Pezzo surface!

$\curvearrowright$ Break $SU(5)$ by hypercharge flux

$$F_Y \sim \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$$

F_Y must be nontrivial on brane, but trivial in full compactification space to preserve hypercharge – mechanism not available in heterotic models!

Superpotential Couplings

Good couplings: Quark and lepton masses, weak-scale μ term

$$W_{\text{good}} = \mu\, \mathbf{5}_{H_u} \overline{\mathbf{5}}_{H_d} + Y_u \mathbf{5}_{H_u} \mathbf{10}_M \mathbf{10}_M + Y_d \overline{\mathbf{5}}_{H_d} \overline{\mathbf{5}}_M \mathbf{10}_M$$

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Bad couplings: Baryon and lepton number violating operators

$$\begin{aligned} W_{\text{bad}} &= \beta\, \mathbf{5}_{H_u}\overline{\mathbf{5}}_M + \lambda \overline{\mathbf{5}}_M\overline{\mathbf{5}}_M\mathbf{10}_M \\ &\quad + W^1\mathbf{10}_M\mathbf{10}_M\mathbf{10}_M\overline{\mathbf{5}}_M + W^2\mathbf{10}_M\mathbf{10}_M\mathbf{10}_M\overline{\mathbf{5}}_{H_d} \\ &\quad + W^3\overline{\mathbf{5}}_M\overline{\mathbf{5}}_M\mathbf{5}_{H_u}\mathbf{5}_{H_u} + W^4\overline{\mathbf{5}}_M\overline{\mathbf{5}}_{H_d}\mathbf{5}_{H_u}\mathbf{5}_{H_u} \\ K_{\text{bad}} &= K^1\mathbf{10}_M\mathbf{10}_M\mathbf{5}_M + K^2\overline{\mathbf{5}}_{H_u}\overline{\mathbf{5}}_{H_u}\mathbf{10}_M \end{aligned}$$

dim-3/4 (first line of W_{bad})

dim-5 (remaining lines)

Coefficients can contain singlet VEVs, suppressed by M_{GUT} [Conlon, Palti]

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dim-3/4

dim-5

Coefficients can contain singlet VEVs, suppressed by M_{GUT} [Conlon, Palti]

Some terms related by interchange $\overline{\mathbf{5}}_{H_d} \leftrightarrow \overline{\mathbf{5}}_M$

Matter Parity

[Dimopoulos, Raby, Wilczek; Ibanez, Ross; Dreiner, Luhn, Thormeier]

Various discrete symmetries for proton stability – compatibility with $SU(5)$ singles out $\mathbb{Z}_2$ "matter parity":

	$\mathbf{5}_{H_u}, \bar{\mathbf{5}}_{H_d}$	$\mathbf{10}_M, \bar{\mathbf{5}}_M$
P_M	$+1$	-1

Forbids all baryon and lepton number violating operators except

$$W^1 \mathbf{10}_M \mathbf{10}_M \mathbf{10}_M \bar{\mathbf{5}}_M \quad \text{and} \quad W^3 \bar{\mathbf{5}}_M \bar{\mathbf{5}}_M \mathbf{5}_{H_u} \mathbf{5}_{H_u}$$

W^3 (Weinberg operator),can be tolerated if suppression scale high enough (but will not be generated, so ignore from now on)
$W^1 \supset QQQL$, $\bar{u}\bar{u}\bar{d}\bar{e}$ extremely constrained – forbid this by clever choice of matter curves (i.e. $U(1)$s)

Contents

Model Requirements

For the local model we require

- P_M defined locally
- heavy top quark (i.e. rank-one up-type Yukawa matrix at tree level)
- No dim-5 proton decay (the W^1 operator forbidden at all orders)
- Masses for all quarks and leptons after switching on VEVs (down-type Yukawa matrix can be rank-zero or one, but not rank-two)

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Local model building freedom: Freely choose

- Monodromy (at least $\mathbb{Z}_2$)
- Assignment of matter and Higgs zero modes to curves
- Singlet VEVs (for matter parity even singlets)
- Assume: Allowed terms generated with order-one coefficients

Matter Parity

Define $\mathbb{Z}_2$ matter parity in terms of the t_i (i.e. as subgroup of $SU(5)_\perp$):

$$P_M = (-1)^{c_i t_i} \ , \quad c_i = 0, 1 \qquad (\text{defined} \mod 2)$$

- Monodromy $t_1 \leftrightarrow t_2$ requires $c_1 = c_2 = 1$ so $\mathbf{10}_{\text{top}}$ is odd
- Up-type masses always allowed once gauge invariant
- Down-type masses give constraint:

	$\bar{\mathbf{5}}_{H_d}$	$\bar{\mathbf{5}}_M$	$\mathbf{10}_M$
charge	$t_i + t_j$	$t_k + t_l$	t_m

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	$\bar{\mathbf{5}}_{H_d}$	$\bar{\mathbf{5}}_M$	$\mathbf{10}_M$
charge	$t_i + t_j$	$t_k + t_l$	t_m
$c_i t_i$	$0/2$	1	1

  Gauge invariant iff all t_i distinct – can only be matter parity even if even number of $c_i = 1$ (singlets have charge $t_i - t_j$, so don't change the argument)
- Note: W^1 operator has same charge structure

Two Possibilities

Hence, two possible definitions of matter parity:

Case I: $P_M = (-1)^{t_1+t_2+t_3+t_4}$

Case II: $P_M = (-1)^{t_1+t_2}$

Now analyse matter, Higgs and VEV assignment for both cases: $\mathbf{10}_{\text{top}}$ and $\mathbf{5}_{H_u}$ already fixed, need to distribute remaining matter and $\overline{\mathbf{5}}_{H_d}$ according to their matter parity

Main restriction: Forbid W^1, but allow down-type Yukawas

Case I: Matter and VEV Assignment

	Matter **10** Curves		
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	
$\mathbf{10}_3$	t_4	$-$	
	Matter **5** Curves		
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	
	Even Singlet Curves		
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	
$\mathbf{1}_7$	$t_1 - t_2$	$+$	

Case I: Matter and VEV Assignment

Matter $\mathbf{10}$ Curves			
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	
$\mathbf{10}_3$	t_4	$-$	
Matter $\mathbf{5}$ Curves			
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	
Even Singlet Curves			
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	
$\mathbf{1}_7$	$t_1 - t_2$	$+$	

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

Case I: Matter and VEV Assignment

Matter **10** Curves

$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	no matter
$\mathbf{10}_3$	t_4	$-$	matter

Matter **5** Curves

$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	matter
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	no matter
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	matter

Even Singlet Curves

$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$
$\mathbf{1}_7$	$t_1 - t_2$	$+$

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

$\rightsquigarrow$ no matter on $\mathbf{10}_2$, $\mathbf{5}_5$

Case I: Matter and VEV Assignment

Matter **10** Curves			
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	no matter
$\mathbf{10}_3$	t_4	$-$	matter
Matter 5 Curves			
$\mathbf{5}_3$	$-t_{1,2}-t_5$	$-$	matter
$\mathbf{5}_5$	$-t_3-t_5$	$-$	no matter
$\mathbf{5}_6$	$-t_4-t_5$	$-$	matter
Even Singlet Curves			
$\mathbf{1}_1$	$\pm(t_{1,2}-t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2}-t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3-t_4)$	$+$	
$\mathbf{1}_7$	t_1-t_2	$+$	

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

$\rightsquigarrow$ no matter on $\mathbf{10}_2$, $\mathbf{5}_5$

- W^1 with singlets:

e.g. $\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_6\mathbf{1}_4\,,$

$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_3\mathbf{1}_1$

Case I: Matter and VEV Assignment

Matter **10** Curves			
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	no matter
$\mathbf{10}_3$	t_4	$-$	matter
Matter 5 Curves			
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	matter
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	no matter
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	matter
Even Singlet Curves			
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	no VEV
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	VEV
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	no VEV
$\mathbf{1}_7$	$t_1 - t_2$	$+$	VEV

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

$\rightsquigarrow$ no matter on $\mathbf{10}_2$, $\mathbf{5}_5$

- W^1 with singlets:

e.g. $\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_6\mathbf{1}_4\,,$

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_3\mathbf{1}_1$$

$\rightsquigarrow$ no VEVs for $\mathbf{1}_1$, $\mathbf{1}_4$ (because of t_3)

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	
$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	
$\bar{\mathbf{5}}_2$	$-t_{1,2} - t_4$	
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$ ⚡	$-t_1 - t_2$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	
$\bar{\mathbf{5}}_2$ ⚡	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)

Case I: Down-Type Higgs

Higgs-like **5** Curves			Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$	⚡	$-t_1 - t_2$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_1$	⚡	$-t_{1,2} - t_3$	either rank-two Yukawa matrix, or no up-type masses with singlets
$\bar{\mathbf{5}}_2$	⚡	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_4$		$-t_3 - t_4$	

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two

Case I: Down-Type Higgs

Higgs-like **5** Curves	Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$ ⚡ $-t_1 - t_2$	No masses at tree level or with singlets μ term
$\bar{\mathbf{5}}_1$ ⚡ $-t_{1,2} - t_3$	either rank-two Yukawa matrix, or no up-type masses with singlets
$\bar{\mathbf{5}}_2$ ⚡ $-t_{1,2} - t_4$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_4$ $-t_3 - t_4$	

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two
- String-scale μ term for both Higgses on one curve

Case I: Down-Type Higgs

Higgs-like **5** Curves			Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$	⚡	$-t_1 - t_2$	No masses at tree level or with singlets μ term
$\bar{\mathbf{5}}_1$	⚡	$-t_{1,2} - t_3$	either rank-two Yukawa matrix, or no up-type masses with singlets
$\bar{\mathbf{5}}_2$	⚡	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_4$	✓	$-t_3 - t_4$	Rank-one Yukawa matrix, bottom quark heavy

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two
- String-scale μ term for both Higgses on one curve
- $\bar{\mathbf{5}}_4 = \bar{\mathbf{5}}_{H_d}$ is unique choice, tree-level coupling $\bar{\mathbf{5}}_{H_d}\mathbf{10}_{\text{top}}\bar{\mathbf{5}}_3$

Case I: Yukawas and CKM

- Example Assignment: Third generation on $\mathbf{10}_1$ and $\overline{\mathbf{5}}_3$, light generations on $\mathbf{10}_3$ and $\overline{\mathbf{5}}_6$
- Higgses: $\overline{\mathbf{5}}_{H_u}$ and $\overline{\mathbf{5}}_4$, only $\langle \mathbf{1}_2 \rangle \sim \epsilon$ required at first order
- Ignore $\mathbf{1}_7$ and $\mathcal{O}(1)$ coefficients
- Yukawa matrices (schematically):

$$Y^u \sim Y^d \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}$$

- CKM matrix:

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}$$

- Masses and mixings possible though not a great fit
- Degeneracy because three generations come from two curves

Case II

$$P_M = (-1)^{t_1+t_2}$$

$\rightsquigarrow$ split t's into $t_{\text{odd}} = \{t_1, t_2\}$ and $t_{\text{even}} = \{t_3, t_4, t_5\}$

- Symmetric setup, possible monodromy acting on t_{even}
- $\mathbf{10}_{\text{top}}$ is the unique matter $\mathbf{10}$ curve
- Down-type Higgs unique (up to relabeling)
- Matter-parity even singlets do not mix t_{odd} and t_{even}
- W^1 operator cannot be generated: Charge $4t_{odd} + t_{even}$ cannot be compensated by matter-parity even singlets
- Three possible matter $\bar{\mathbf{5}}$ curves (charges $t_{\text{odd}} - t_{\text{even}}$): model building choice
- Different choices of singlet VEVs possible, achieve masses and mixing

Local Model Summary

- Already locally, rather constrained model: Only two possible definitions of matter parity
- In both cases, assignments of matter and Higgses is unique or strongly constrained
- Restrictions mainly from forbidding W^1 while allowing for down-type masses
- W^3 operator (neutrino masses) is not generated in any case
- Masses for all matter fields and CKM mixing possible
- Involves choices of zero modes and VEVs by hand – these cannot be calculated in the local framework

Contents

Semilocal Approach

[Friedman, Morgan, Witten; Donagi, Wijnholt; Marsano at al.]

Now *semilocal* picture: Consider GUT surface S and fluxes, using spectral cover approach

Two types of fluxes (actually, both merge to G_4 flux in F-theory):

- $U(1) \subset SU(5)_\perp$ fluxes on matter curves (from the transverse branes): Determines (chiral) 4D zero modes for full GUT multiplets, free parameters up to anomaly cancellation requirements
- Hypercharge flux on S (globally trivial so hypercharge stays unbroken): Breaks $SU(5)$, restrictions to matter curves splits $SU(5)$

Aim: Find relations between homology classes of matter curves $\rightsquigarrow$ relation between flux restrictions and multiplet splittings

Spectral Cover

Spectral cover: Five-fold cover of S in projective threefold

$$\mathbb{P}(K_S \oplus \mathcal{O}_S)$$

with homogeneous coordinates $U : V$ given by spectral equation for Φ.
Because of $\mathbb{Z}_2$ monodromy, spectral equation must factorise:

$$\begin{aligned} 0 &= b_0 U^5 + b_2 U^3 V^2 + b_3 U^2 V^3 + b_4 U V^4 + b_5 V^5 \\ &= \left(a_1 V^2 + a_2 UV + a_3 U^2\right)\left(a_4 V + a_7 U\right)\left(a_5 V + a_8 U\right)\left(a_6 V + a_9 U\right) \end{aligned}$$

b_k are sections in certain line bundles on $S \Rightarrow$ line bundles for the a_i
$\Rightarrow$ homology classes of matter curves
Impose triviality of hypercharge flux $\rightsquigarrow$ solution contains three arbitrary line bundles

Involves particular solution of $b_1 = 0$ constraint – might not be most general one?

Fluxes and Zero Modes

$U(1)$ fluxes $M_{\mathbf{5}}$, $M_{\mathbf{10}}$: Free up to consistency conditions [Dudas, Palti]

Hypercharge flux on matter curves: $N_Y = F_Y \cdot (\text{homology class})$.

For curve with flux numbers M and N_Y, zero modes given by

$\mathbf{10}$ →
- $(\mathbf{3}, \mathbf{2}) : M_{\mathbf{10}}$
- $(\overline{\mathbf{3}}, \mathbf{1}) : M_{\mathbf{10}} - N_Y$
- $(\mathbf{1}, \mathbf{1}) : M_{\mathbf{10}} + N_Y$

$\mathbf{5}$ →
- $(\mathbf{3}, \mathbf{1}) : M_{\mathbf{5}}$
- $(\mathbf{1}, \mathbf{2}) : M_{\mathbf{5}} + N_Y$

Hypercharge flux must be globally trivial, hence no net "$SU(5)$ breaking chirality":

$$0 = F_Y \cdot c_1 = F_Y \cdot \eta \quad \rightsquigarrow \quad \sum_{\mathbf{5}} N_Y = \sum_{\mathbf{10}} N_Y = 0$$

Upshot

10 Curves

	M	N_Y
$\mathbf{10}_1$	$-(M_{\mathbf{5}_1} + M_{\mathbf{5}_2} + M_{\mathbf{5}_3})$	$-\widetilde{N}$
$\mathbf{10}_2$	$M_{\mathbf{10}_2}$	N_7
$\mathbf{10}_3$	$M_{\mathbf{10}_3}$	N_8
$\mathbf{10}_4$	$M_{\mathbf{10}_4}$	N_9

5 Curves

	M	N_Y
$\mathbf{5}_{H_u}$	$M_{\mathbf{5}_{H_u}}$	$\widetilde{N}$
$\mathbf{5}_1$	$M_{\mathbf{5}_1}$	$-\widetilde{N}$
$\mathbf{5}_2$	$M_{\mathbf{5}_2}$	$-\widetilde{N}$
$\mathbf{5}_3$	$M_{\mathbf{5}_3}$	$-\widetilde{N}$
$\mathbf{5}_4$	$M_{\mathbf{5}_4}$	$N_7 + N_8$
$\mathbf{5}_5$	$M_{\mathbf{5}_5}$	$N_7 + N_9$
$\mathbf{5}_6$	$M_{\mathbf{5}_6}$	$N_8 + N_9$

- Three free parameters $N_{7,8,9}$ for the hypercharge flux, corresponding to three unspecified line bundles
- $\widetilde{N} = N_7 + N_8 + N_9$
- Split some **5** curves $\Rightarrow$ split some **10** curves
  [Marsano et al.; Dudas, Palti]

Case I: Matter Sector is Fine

- Doublet-triplet splitting for Higgses ($\widetilde{N} \neq 0$) inevitably splits $\mathbf{10}_{\text{top}}$ and at least one more **10** curve (and at least one matter **5** curve)
- However, splitting of matter multiplets is OK as long as there are three generations of zero modes in the end, i.e. other **10** curve must have "opposite" split (hence cannot have three generations from one matter curve)
- Matter on $\mathbf{10}_1$, $\mathbf{10}_3$, $\mathbf{5}_3$ and $\mathbf{5}_6$, so to have full net generations, we require

$$N_7 = N_9 = 0 \quad \Rightarrow \quad \text{only } N_8 \text{ left free}$$

- No exotics from **10**'s and remaining matter-like **5** curve can be satisfied by choosing appropriate M's

↷ Satisfactory matter sector can be engineered easily

Case I: Higgs Sector is not Fine

- Higgs sector:

	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$
$\mathbf{5}_{H_u}$	$M_{\mathbf{5}_{H_u}}$	$M_{\mathbf{5}_{H_u}} + N_8$
$\mathbf{5}_1$	$M_{\mathbf{5}_1}$	$M_{\mathbf{5}_1} - N_8$
$\mathbf{5}_2$	$M_{\mathbf{5}_2}$	$M_{\mathbf{5}_2} - N_8$
$\mathbf{5}_4$	$M_{\mathbf{5}_4}$	$M_{\mathbf{5}_4} + N_8$

- We can pairwise decouple unwanted triplets from $\mathbf{5}_{H_u}$ and $\mathbf{5}_2$, and from $\mathbf{5}_1$ and $\mathbf{5}_4$ by coupling to VEV for $\mathbf{1}_2$
- However:

$$\#(\text{doublets from } \mathbf{5}_{H_u}, \mathbf{5}_2) = \#(\text{triplets from } \mathbf{5}_{H_u}, \mathbf{5}_2)$$

- Problem persists even when allowing exotics from the matter sector
- Separately, down-type Higgs on $\mathbf{5}_4$ cannot be realised

Case II: Again not Fine

- Only one matter $\mathbf{10}$ curve, split with same parameter as up-type Higgs
- Higgs triplets cannot be decoupled by even matter parity singlets
- No exotic matter $\Rightarrow$ no doublet-triplet splitting
- Ignoring matter sector: Still cannot achieve pair of Higgs doublets

Case II: Again not Fine

- Only one matter **10** curve, split with same parameter as up-type Higgs
- Higgs triplets cannot be decoupled by even matter parity singlets
- No exotic matter $\Rightarrow$ no doublet-triplet splitting
- Ignoring matter sector: Still cannot achieve pair of Higgs doublets

Upshot: In both cases, proper doublet-triplet splitting in Higgs sector does not work, even when allowing for exotics from the matter sector – both models cannot be realised already in semilocal setup!

Contents

Conclusions

- Analysed F-Theory GUT in local and semilocal approach
- Goal: Use locally defined matter parity and additional $U(1)$s to ensure proton stability
- Local model is already very constrained: Two cases only
- In semilocal embedding, doublet-triplet splitting cannot be realised in either case

$\rightsquigarrow$ Models fail first step towards realisation

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- Analysed F-Theory GUT in local and semilocal approach
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- Predictivity of local point in question – Crucial model features required to have nonlocal origin?
- GUT breaking by hypercharge flux seems too restrictive, also problems with exotics [Marsano et al.; Dudas, Palti]

Conclusions

- Analysed F-Theory GUT in local and semilocal approach
- Goal: Use locally defined matter parity and additional $U(1)$s to ensure proton stability
- Local model is already very constrained: Two cases only
- In semilocal embedding, doublet-triplet splitting cannot be realised in either case

$\rightsquigarrow$ Models fail first step towards realisation

- Predictivity of local point in question – Crucial model features required to have nonlocal origin?
- GUT breaking by hypercharge flux seems too restrictive, also problems with exotics [Marsano et al.; Dudas, Palti]
- Possible loopholes: Localised matter might be more subtle
  - Non-diagonal Higgs fields ("T-Branes", "Gluing Morphisms") [Cecotti et al.; Donagi, Wijnholt]
  - Relation to higher symmetry groups [Esole, Yau; Marsano, Schäfer-Nameki]