Local SU(5) GUT from the Heterotic String

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W. Buchmüller, CL, J. Schmidt: JHEP 0709:113 (arXiv:0707.1651) and work in progress

Local GUTs

- GUT: Attractive features:
  - Gauge coupling unification with supersymmetry
  - Matter unification into larger multiplets
  - Appealing sequence $SU(3) \times SU(2) \times U(1) \subset SU(5) \subset SO_{10} \subset E_8$
- Drawbacks in 4d GUTS
  - Large Higgs representations required
  - Doublet–triplet–splitting
  - Yukawa couplings do not unify
- Drawbacks can be addressed in higher-dimensional orbifold GUTs
  - GUT group broken locally by boundary conditions
  - Higgses arise as split bulk multiplets
  - Yukawa coupling unification can be avoided

Heterotic Orbifold Compactification

- String compactifications provide UV completion and anomaly freedom
- Procedure:
  - Choose a torus with discrete isometry (“twist”) with fixed points
  - Mod out by this isometry, fixed points become singularities
  - Fixing boundary conditions at fixed points requires embedding the twist into gauge group and choosing Wilson lines
- Gauge symmetry reduced at fixed points (but rank usually preserved)
- States localised at fixed points (twisted sectors): Fixed by choice of twist and Wilson lines
- Models with MSSM in 4d knwon (cf. talk by H.P. Nilles)
- Intermediate GUTs provide more structure and better intuition

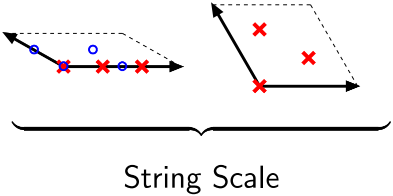
[Kobayashi, Raby, Zhang; Buchmüller, Hamaguchi, Lebedev, Ratz; Kim, Kim, Kyae; Förste, Nilles, Vaudrevange, Wingerter, Ramos-Sanchez,. . .]

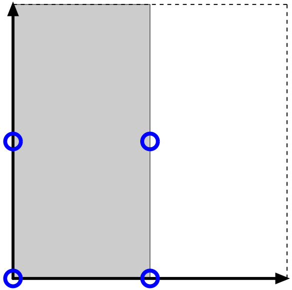
The Model: Geometry

[Buchmüller, Hamaguchi, Lebedev, Ratz]

- Torus: $G_2 \times SU(3) \times SO(4)$ root lattice, $\mathbb{Z}_{6-II} = \mathbb{Z}_3 \times \mathbb{Z}_2$ twist:

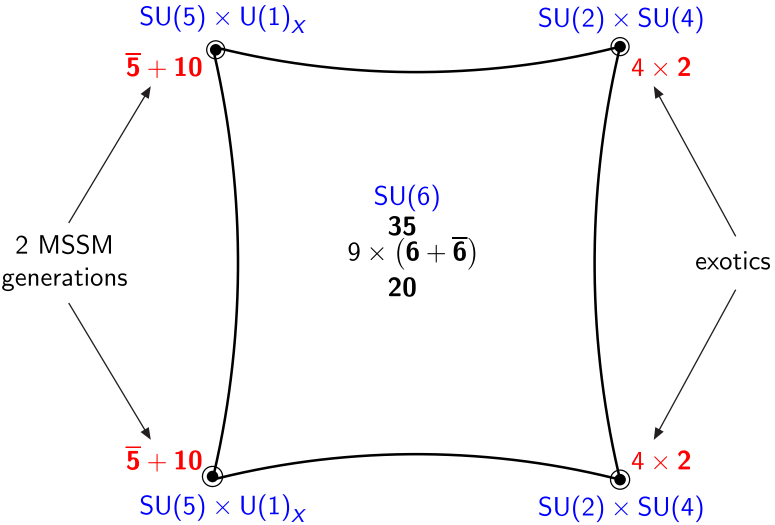
[Kobayashi,Raby,Zhang]





- Obtain effective 6D Theory on $T^2/\mathbb{Z}_2$ orbifold
- Internal zero modes and $\mathbb{Z}_3$ twisted states show up as bulk states, $\mathbb{Z}_2$ twisted states are localised at orbifold fixed points

The Model: Effective $T^2/\mathbb{Z}_2$ Orbifold



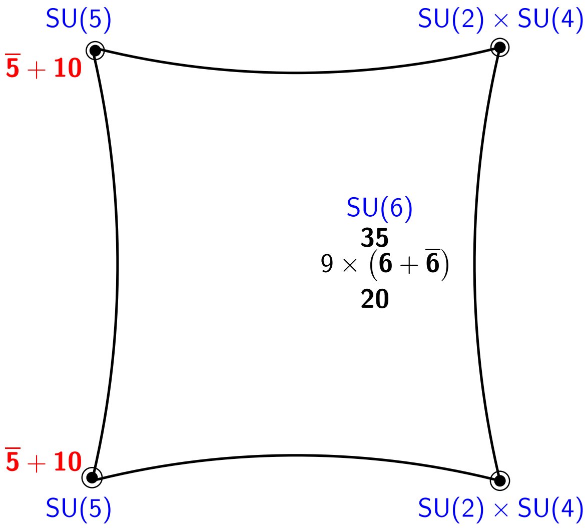
Local SU(5) GUT

- Local GUT: At fixed points, boundary conditions break bulk gauge group to smaller groups,

$$\text{in our case:} \quad \mathrm{SU}(6) \longrightarrow \begin{cases} \mathrm{SU}(5) \times \mathrm{U}(1)_X \\ \mathrm{SU}(2) \times \mathrm{SU}(4) \end{cases}$$

- In zero mode spectrum, only the intersection of local groups survives, which is $G_{SM} = SU(3) \times SU(2) \times U(1)$
- Localised fields come in complete multiplets of local GUT group
- Due symmetry breaking, bulk fields form split multiplets
- Due to higher symmetry, decoupling of exotics much more transparent that in four-dimensional limit, e.g. several $\left(\mathbf{5}, \overline{\mathbf{5}}\right)$ pairs from twisted sectors T_2 and T_4 can decoupled in one step

Projection



- On branes, SUSY is broken to $\mathcal{N} = 1$
- Bulk Matter: Hypermultiplets, split as $H = (H_L, H_R)$ into chiral multiplet
- Bulk vector multiplets split as $V = (A, \phi)$ into vector and chiral multiplets
- Only one $\mathcal{N} = 1$ multiplet survives projection

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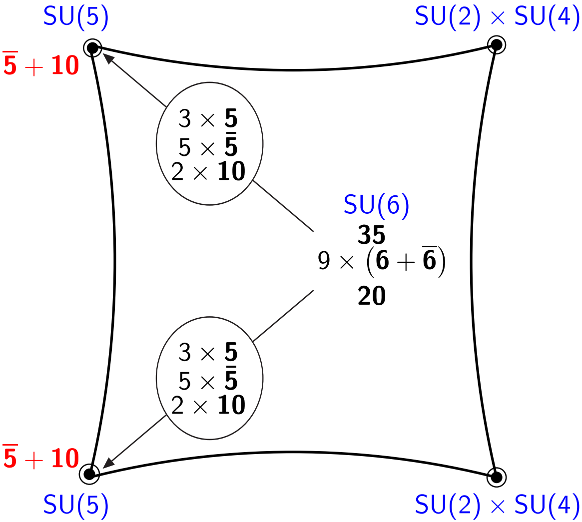

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Decoupling: Example of Gauge–Higgs Unification

- Choice of Higgs and matter fields not unique
- Several pairs of $\mathbf{5}+\bar{\mathbf{5}}$ and most exotics decoupled easily
- Remaining $\mathbf{5}$'s and $\bar{\mathbf{5}}$'s:

Bulk: / Zero modes:	$\mathbf{5}$	$\mathbf{5}_1$	$\bar{\mathbf{5}}_0^c$	$\bar{\mathbf{5}}$	$\bar{\mathbf{5}}_1$	$\bar{\mathbf{5}}_2$	$\mathbf{5}_0^c$	$\mathbf{5}_2^c$
SU(3) × SU(2)	$(1,\mathbf{2})$	$(1,\mathbf{2})$	$(\mathbf{3},1)$	$(1,\mathbf{2})$	$(1,\mathbf{2})$	$(\bar{\mathbf{3}},1)$	$(\bar{\mathbf{3}},1)$	$(1,\mathbf{2})$
$\mathrm{U}(1)_{B-L}$	0	0	$-\frac{2}{3}$	0	0	$-\frac{1}{3}$	$\frac{2}{3}$	-1
MSSM content	H_u				H_d	d_3		l_3

$2\times(\bar{\mathbf{5}}+\mathbf{10})$ generations on the branes
$2\times(\bar{\mathbf{5}}+\mathbf{10})$ generations in the bulk
$\mathbf{5}+\bar{\mathbf{5}}$ Higgses in the bulk

Split Multiplets

- Bulk generations:

$$\bar{\mathbf{5}}_{(3)} = \left(\bar{\mathbf{3}}, 1\right) + (1, \mathbf{2}) \qquad \mathbf{10}_{(3)} = (\mathbf{3}, \mathbf{2}) + \left(\bar{\mathbf{3}}, 1\right) + (1, 1)$$
$$\bar{\mathbf{5}}_{(4)} = \left(\bar{\mathbf{3}}, 1\right) + (1, \mathbf{2}) \qquad \mathbf{10}_{(4)} = (\mathbf{3}, \mathbf{2}) + \left(\bar{\mathbf{3}}, 1\right) + (1, 1)$$

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One generation remains, avoiding SU(5) mass relations
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Orbifold projection solves doublet–triplet–splitting

Yukawa Couplings

$$W = C^{(u)}_{(ij)} \mathbf{5}_u \mathbf{10}_{(i)} \mathbf{10}_{(j)} + C^{(d)}_{(ij)} \mathbf{5}_d \bar{\mathbf{5}}_{(i)} \mathbf{10}_{(j)}$$

$$C^{(u)}_{(ij)} = \begin{pmatrix} a_1 & 0 & a_2 & a_3 \\ 0 & a_1 & a_2 & a_3 \\ a_2 & a_2 & 0 & g \\ a_3 & a_3 & g & a_4 \end{pmatrix}, \qquad C^{(d)}_{ij} = \begin{pmatrix} 0 & 0 & b_1 & b_2 \\ 0 & 0 & b_1 & b_2 \\ b_3 & b_3 & b_4 & 0 \\ b_5 & b_5 & b_6 & b_5^2 \end{pmatrix}$$

$$a_1 = \langle Y_0^c \bar{Y}_0^c S_1 S_3 \rangle, \qquad a_2 = \langle \left(\bar{Y}_0^c S_1\right)^2 S_5 \rangle, \qquad a_3 = \langle Y_0^c \bar{Y}_0^c S_1 S_3 S_5 \rangle,$$

$$a_4 = \langle Y_0^c \bar{Y}_0^c S_1 S_3 (S_5)^2 \rangle,$$

$$b_1 = \langle Y_0 \bar{Y}_1 (S_5)^3 (S_7)^2 \rangle, \qquad b_2 = \langle X_1^c \bar{Y}_2^c U_1^c S_7 \rangle, \qquad b_3 = \langle X_1^c \bar{Y}_1 S_3 (S_5 S_7)^2 \rangle,$$

$$b_4 = \langle (X_1^c)^2 \bar{Y}_1 U_1^c S_4 S_7 \rangle, \qquad b_5 = \langle S_5 \rangle, \qquad b_6 = \langle (X_1^c)^2 Y_1 S_1 S_7 \rangle$$

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$$W = Y^u_{ij} h_u u^c_i q_j + Y^d_{ij} h_d d^c_i q_j + Y^l_{ij} h_d l_i e^c_j$$

$$Y^u_{ij} = \begin{pmatrix} a_1 & 0 & a_3 \\ 0 & a_1 & a_3 \\ a_2 & a_2 & g \end{pmatrix}, \quad Y^d_{ij} = \begin{pmatrix} 0 & 0 & b_2 \\ 0 & 0 & b_2 \\ b_5 & b_5 & b_7 \end{pmatrix}, \quad Y^l_{ij} = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_1 \\ b_3 & b_3 & b_4 \end{pmatrix}$$

Anomalies

- Anomaly cancellation by Green–Schwarz mechanism requires factorisation of bulk and localised anomaly polynomials, $I_8 = X_4 Y_4$ and $I_6^f = X_4^f Y_2$
- $\mathcal{O}(400)$ conditions, but guaranteed by string theory (and modular invariance conditions on twist vectors and Wilson lines): Check of spectrum
- Anomalous U(1)'s induce localised FI terms at each fixed point:

$$\xi_0 = 148 \left(\frac{g M_{\mathrm{P}}^2}{384\pi^2} \right) \delta^{(2)}(z - z_0) \qquad \xi_1 = 80 \left(\frac{g M_{\mathrm{P}}^2}{384\pi^2} \right) \delta^{(2)}(z - z_1)$$

- Sum of localised anomalous U(1)'s gives 4d anomalous U(1) and FI term
- Contribution to the salar potential $\rightsquigarrow$ VEVs of charged fields, $\langle\phi\rangle \sim M_{\mathrm{GUT}}$
- Huge VEVs, so higher order terms in superpotential expansion not very suppressed

Vacuum Selection

- Choice of vacuum: Choice of singlet VEVs for mass terms
- Choice of vacuum restricted by phenomenological constraints, e.g.
  - Gauge group $G = G_{\text{SM}} \times G_{\text{hidden}}$
  - Decoupling of exotic states via VEV mass terms
  - Yukawa couplings for quarks and leptons, in particular heavy top
  - Light Higgses, i.e. no μ-term
- $\mathrm{U}(1)_X$ can be broken to $\mathbb{Z}_2$ matter parity
- Extensive scans of "Mini-Landscape"

  [Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter 06, 07]
- F-flatness and D-flatness generically can be satisfied simultaneously

Field Localisation due to FI Terms

- Different FI terms at different fixed points have important effects on field configuration:
- Background profile for extra-dimensional components of gauge field, including localised field strength

  [Groot Nibbelink, Nilles, Olechowski 02;Lee, Nilles, Zucker 04]
- This in turn implies localisation of charged bulk scalars at or away from fixed points, i.e. VEVs are not constant in the extra dimension
- Four-dimensional cancellation of FI terms generically involves twisted and untwisted sector field VEVs
  - Non-constant untwisted sector field might fail to cancel FI terms
  - Large VEVs for localised fields is related to blow-up of orbifold singularities
- Furthermore, profiles of bulk fields may cause warping and hence break supersymmetry
- VEVs of fields without zero modes possible
- Work in progress...

Outlook

- Constructed local 6D GUT from the heterotic string
- Doublet–triplet splitting achieved easily, SU(5) mass relations avoided due to split bulk multiplets
- More symmetry in 6D $\rightsquigarrow$ simple decoupling of unwanted states
- Supersymmetric vacuum: four-dimensional *D*-term vanishes
- Open Questions:
  - Stabilisation of moduli, in particular, size of two-dimensional torus
  - Profiles of bulk fields due to localised FI terms
  - Blowup/resolution of singularities, generalisation to K3 internal space