(Non-)Universal Anomalies and Discrete Symmetries from the Heterotic String

Christoph Lüdeling
bctp and PI, University of Bonn

Planck 2012, Warsaw

CL, Fabian Ruehle, Clemens Wieck
PRD **85** [arXiv:1203.5789] and work in progress

Motivation

- MSSM superpotential contains (potentially) bad terms:

$$W_{\text{bad}} \supset \mu H_u H_d + QLd^c + u^c d^c d^c + LLe^c + QQQL + u^c u^c d^c e^c + \cdots$$

- Forbid/constrain these operators by (discrete) symmetries, (matter parity, proton hexality, $\mathbb{Z}_4^R$,...) [many people here]
- In string constructions, discrete symmetries can arise as remnants of gauge and internal Lorentz symmetries

Motivation

- MSSM superpotential contains (potentially) bad terms:

$$W_{\text{bad}} \supset \mu H_u H_d + QLd^c + u^c d^c d^c + LLe^c + QQQL + u^c u^c d^c e^c + \cdots$$

- Forbid/constrain these operators by (discrete) symmetries, (matter parity, proton hexality, $\mathbb{Z}_4^R$,...) [many people here]
- In string constructions, discrete symmetries can arise as remnants of gauge and internal Lorentz symmetries
- Discuss anomalies in such models (bottom-up) and the appearance of discrete symmetries in the $E_8 \times E_8$ heterotic string
- Simple example: Consider $\mathbb{Z}_3$ orbifold, its blowup and GLSM realisation

Contents

Anomalies in MSSM Extensions

Anomalies

- determined by chiral spectrum
- given in terms of anomaly polynomial I_6 (via descent equations) [Wess, Zumino '71; Stora '84; Alvarez-Gaumé, Ginsparg '84]
- insensitive to continuous deformations

Anomalies in MSSM Extensions

Anomalies

- determined by chiral spectrum
- given in terms of anomaly polynomial I_6 (via descent equations) [Wess, Zumino '71; Stora '84; Alvarez-Gaumé, Ginsparg '84]
- insensitive to continuous deformations

Require:

- $U(1)_A^2 - U(1)_Y$ anomaly should vanish
- for $\mathbb{Z}_N$ symmetries, only $G^2 - \mathbb{Z}_N$ meaningful

[Ibanez, Ross '91; Banks, Dine '92; Araki et al. '08]

Green–Schwarz Mechanism

[Green, Schwarz '84]

Cancel transformation of measure with variation of action – requires

a) factorisation of anomaly polynomial, $I_6 = X_4 Y_2$, i.e. $Y_2 = F_2 = \mathrm{d}A_1$

b) axion field a with shift gauge transformation

$$\delta a = -Y_0^{(1)} = -\lambda$$

Green–Schwarz Mechanism

[Green, Schwarz '84]

Cancel transformation of measure with variation of action – requires

a) factorisation of anomaly polynomial, $I_6 = X_4 Y_2$, i.e. $Y_2 = F_2 = \mathrm{d}A_1$

b) axion field a with shift gauge transformation

$$\delta a = -Y_0^{(1)} = -\lambda$$

c) axion coupling to X_4

$$S_{\mathrm{GS}} = \int a X_4 + \cdots \quad \Rightarrow \quad \delta S_{\mathrm{GS}} = -\int I_4^{(1)}$$

- Needs shift and coupling
- Dualise a to two-form B_2: exchange $Y_2 \leftrightarrow X_4$, shift$\leftrightarrow$coupling
- Generalisation: sum of factorised anomalies $I_6 = \sum_a Y^a X^a$ is cancelled by set of axions

Green–Schwarz Axion

$Y_2 = \mathrm{d}A_A$ is field strength of the "anomalous $U(1)$", axion transforms with a shift

$$\Longrightarrow S_{a,\mathrm{kin}} = \int \frac{1}{2} \left|\mathrm{d}a + A_A\right|^2$$

Green–Schwarz Axion

$Y_2 = \mathrm{d}A_A$ is field strength of the "anomalous $U(1)$", axion transforms with a shift

$$\Longrightarrow S_{a,\mathrm{kin}} = \int \frac{1}{2} \left|\mathrm{d}a + A_A\right|^2$$

- Axion kinetic term gives Stueckelberg mass term for $U(1)_A \Rightarrow$ anomalous $U(1)$s massive, but remain as (perturbative) selection rules
- X_4 contains field strengths of other gauge group factors G_i, weighted with (arbitrary) anomaly coefficients:

$$X_4 = A_{\mathrm{grav}-U(1)} \,\mathrm{tr}\, R^2 + \sum_i A_{G_i^2-U(1)_A} \,\mathrm{tr}\, F_i^2$$

- Special case: If anomaly is cancelled by Kalb–Ramond b_2 $\Rightarrow X_4$ is reduction of 10D X_4^{uni}, and universal axion a_0 couples universally to all gauge groups

Anomaly Coefficients: MSSM with extra $U(1)$ or $\mathbb{Z}_N$

Take MSSM with additional $U(1)_X$, charges $q_Q, \dots, q_{H_d}$
$\rightsquigarrow$ Anomaly coefficients generically not universal

Impose e.g.

- allowed Yukawa couplings,
- $U(1)_X$ is flavour-blind and commutes with $SU(5)$ (but assume doublet-triplet splitting, i.e. no Higgs triplets),
- may or may not be an R symmetry (i.e. $R = 0$ or $R = 1$)

$$A_{SU(3)^2-U(1)_X} = 3\,(3q_{10} + q_{\overline{5}})\,-6\,,$$
$$A_{SU(2)^2-U(1)_X} = 2\,(3q_{10} + q_{\overline{5}})\,-6\,,$$
$$A_{U(1)_Y^2-U(1)_X} = 2\,(3q_{10} + q_{\overline{5}})\,-9\,.$$

Universality from Underlying GUT?

- Unbroken GUT: Anomaly coefficients universal (duh)
- After GUT breaking: Depends on mechanism!
- 4D: Breaking by VEVs removes vector-like states (under unbroken group)
  $\rightsquigarrow$ no change to anomalies
- VEVs are continuous deformations

Universality from Underlying GUT?

- Unbroken GUT: Anomaly coefficients universal (duh)
- After GUT breaking: Depends on mechanism!
- 4D: Breaking by VEVs removes vector-like states (under unbroken group)
  $\rightsquigarrow$ no change to anomalies
- VEVs are continuous deformations
- Higher-dimensional models: 4D chiral spectrum formed by zero modes of internal Dirac operators
- Number of zero modes depends on discrete choices: boundary conditions, fluxes $\rightsquigarrow$ change in anomalies

Example: Broken $SU(5)$

Anomalies get contributions from matter, gauginos and Higgses:

	Matter	gauginos		Higgses	
	$\bar{\mathbf{5}}, \mathbf{10}$	SM	X, Y	$\mathbf{3}$'s	$\mathbf{2}$'s
A_5	$= \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}} - 4R\right)$	$+5R$		$+C_H$	
A_3	$= \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}} - 4R\right)$	$+3R$	$+2R$	$+C_H$	
A_2	$= \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}} - 4R\right)$	$+2R$	$+3R$		$+C_H$
A_1	$= \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}} - 4R\right)$		$+5R$	$+\frac{2}{5}C_H$	$+\frac{3}{5}C_H$

$(C_H = \frac{1}{2}\left(q_{H_u} + q_{H_d} - 2R\right))$

- For unbroken $SU(5)$, $A_3 = A_2 = A_1 = A_5$

Example: Broken $SU(5)$

Anomalies get contributions from matter, gauginos and Higgses:

Matter $\bar{\mathbf{5}}, \mathbf{10}$	gauginos SM	gauginos X, Y	Higgses $\mathbf{3}$'s	Higgses $\mathbf{2}$'s
$A_5 = \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}} - 4R\right)$	$+5R$		$+C_H$	
$A_3 = \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}} - 4R\right)$	$+3R$	$+2R$	$+C_H$	
$A_2 = \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}} - 4R\right)$	$+2R$	$+3R$		$+C_H$
$A_1 = \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}} - 4R\right)$		$+5R$	$+\frac{2}{5}C_H$	$+\frac{3}{5}C_H$

$\left(C_H = \frac{1}{2}\left(q_{H_u} + q_{H_d} - 2R\right)\right)$

- For unbroken $SU(5)$, $A_3 = A_2 = A_1 = A_5$
- $SU(5)$ breaking removes X, Y gauginos $\rightsquigarrow$ non-universality for R symmetries

Example: Broken $SU(5)$

Anomalies get contributions from matter, gauginos and Higgses:

Matter $\overline{\mathbf{5}}, \mathbf{10}$	gauginos SM	gauginos X, Y	Higgses $\mathbf{3}$'s	Higgses $\mathbf{2}$'s
$A_5 = \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\overline{\mathbf{5}}} - 4R\right)$	$+5R$		$+C_H$	
$A_3 = \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\overline{\mathbf{5}}} - 4R\right)$	$+3R$	$+2R$	$+C_H$	
$A_2 = \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\overline{\mathbf{5}}} - 4R\right)$	$+2R$	$+3R$		$+C_H$
$A_1 = \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\overline{\mathbf{5}}} - 4R\right)$		$+5R$	$+\frac{2}{5}C_H$	$+\frac{3}{5}C_H$

$\left(C_H = \frac{1}{2}\left(q_{H_u} + q_{H_d} - 2R\right)\right)$

- For unbroken $SU(5)$, $A_3 = A_2 = A_1 = A_5$
- $SU(5)$ breaking removes X, Y gauginos $\rightsquigarrow$ non-universality for R symmetries
- doublet-triplet splitting $\rightsquigarrow$ generic non-universality

More Possibilities, e.g. for $G_{\mathrm{SM}} \times \mathbb{Z}_N$

Operator				
$(LH_u)^2$				
$H_u H_d$				
LH_u				
$\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}}$				
$\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\overline{\mathbf{5}}$				
$\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,H_d$				
$LH_u H_d H_u$				

More Possibilities, e.g. for $G_{SM} \times \mathbb{Z}_N$

Operator	SM Yukawas	Weinberg Op.		
$(LH_u)^2$	$4R - 4q_{10} + 2q_{\bar{5}}$	$2R$		
$H_u H_d$	$4R - 3q_{10} - q_{\bar{5}}$	$5R - 5q_{10} + k\frac{N}{2}$		
LH_u	$2R - 2q_{10} + q_{\bar{5}}$	$R + k\frac{N}{2}$		
$\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}$	$q_{10} + 2q_{\bar{5}}$	$-2R + 5q_{10}$		
$\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\bar{\mathbf{5}}$	$3q_{10} + q_{\bar{5}}$	$-R + 5q_{10} + k\frac{N}{2}$		
$\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,H_d$	$4R + 2q_{10} - q_{\bar{5}}$	$3R + k\frac{N}{2}$		
$LH_u H_d H_u$	$6R - 5q_{10}$	$6R - 5q_{10}$		

More Possibilities, e.g. for $G_{\mathrm{SM}} \times \mathbb{Z}_N$

Operator	SM Yukawas	Weinberg Op.	$\mathbb{Z}_6^X$	
$(LH_u)^2$	$4R - 4q_{\mathbf{10}} + 2q_{\bar{\mathbf{5}}}$	$2R$	0	
$H_u H_d$	$4R - 3q_{\mathbf{10}} - q_{\bar{\mathbf{5}}}$	$5R - 5q_{\mathbf{10}} + k\frac{N}{2}$	4	
LH_u	$2R - 2q_{\mathbf{10}} + q_{\bar{\mathbf{5}}}$	$R + k\frac{N}{2}$	3	
$\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}$	$q_{\mathbf{10}} + 2q_{\bar{\mathbf{5}}}$	$-2R + 5q_{\mathbf{10}}$	5	
$\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\bar{\mathbf{5}}$	$3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}}$	$-R + 5q_{\mathbf{10}} + k\frac{N}{2}$	2	
$\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,H_d$	$4R + 2q_{\mathbf{10}} - q_{\bar{\mathbf{5}}}$	$3R + k\frac{N}{2}$	3	
$LH_u H_d H_u$	$6R - 5q_{\mathbf{10}}$	$6R - 5q_{\mathbf{10}}$	1	

$\rightsquigarrow$ forbid all bad terms e.g. by non-R $\mathbb{Z}_6^X$ with

$$q_{\mathbf{10}} = 1\,, \qquad q_{\bar{\mathbf{5}}} = 5\,, \qquad q_{H_u} = 4\,, \qquad q_{H_d} = 0$$

$\mathbb{Z}_2$ matter parity as subgroup

More Possibilities, e.g. for $G_{\text{SM}} \times \mathbb{Z}_N$

Operator	SM Yukawas	Weinberg Op.	$\mathbb{Z}_6^X$	$SO(10)$
$(LH_u)^2$	$4R - 4q_{\mathbf{10}} + 2q_{\bar{\mathbf{5}}}$	$2R$	0	$2R$
$H_u H_d$	$4R - 3q_{\mathbf{10}} - q_{\bar{\mathbf{5}}}$	$5R - 5q_{\mathbf{10}} + k\frac{N}{2}$	4	0
LH_u	$2R - 2q_{\mathbf{10}} + q_{\bar{\mathbf{5}}}$	$R + k\frac{N}{2}$	3	R
$\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}$	$q_{\mathbf{10}} + 2q_{\bar{\mathbf{5}}}$	$-2R + 5q_{\mathbf{10}}$	5	$3R$
$\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\bar{\mathbf{5}}$	$3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}}$	$-R + 5q_{\mathbf{10}} + k\frac{N}{2}$	2	$4R$
$\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,H_d$	$4R + 2q_{\mathbf{10}} - q_{\bar{\mathbf{5}}}$	$3R + k\frac{N}{2}$	3	$3R$
$LH_u H_d H_u$	$6R - 5q_{\mathbf{10}}$	$6R - 5q_{\mathbf{10}}$	1	R

$\rightsquigarrow$ forbid all bad terms e.g. by non-R $\mathbb{Z}_6^X$ with

$$q_{\mathbf{10}} = 1\,, \qquad q_{\bar{\mathbf{5}}} = 5\,, \qquad q_{H_u} = 4\,, \qquad q_{H_d} = 0$$

$\mathbb{Z}_2$ matter parity as subgroup

Requiring $SO(10)$ relations for matter, we need $R = 1$

Contents

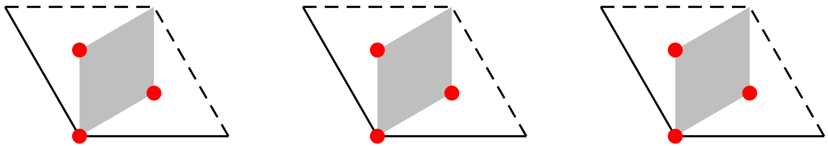
Overview of Heterotic GS Mechanisms

In heterotic models, all axions arise from B_2 in 10D with transformation
Distinguish Orbifolds and smooth Calabi–Yaus with vector bundles:

Orbifolds	Calabi–Yau X with Gauge Bundle
single two-form $b_2 \leftrightarrow$ universal axion a_0	$B_2 = b_2 + \beta_r E_r$, $E_r \in H^2(X) \rightsquigarrow$ additional axions β_r
Universality: a_0 couples to reduction of X_4^{uni}	Couplings $\beta_r X_4^r$ depend on gauge background and curvature – some remnants of universality
(at most) one anomalous $U(1)$ (determined by shift of a_0)	Number of anomalous $U(1)$s given by rank of bundle

$T^6/\mathbb{Z}_3$ Orbifold

For illustration, consider simple $T^6/\mathbb{Z}_3$ orbifold model



$V = \frac{1}{3}\left(1,1,-2,0^5\right)\left(0^8\right)$, no Wilson lines
$\Rightarrow$ standard embedding, 27 equivalent fixed points

Gauge group	$E_6 \times SU(3)\,[\times E_8]$
spectrum	$3\left(\mathbf{27},\overline{\mathbf{3}}\right) + 27\left[\left(\mathbf{27},\mathbf{1}\right) + 3\left(\mathbf{1},\mathbf{3}\right)\right]$

In particular, no anomalous $U(1)$, hence universal axion does not shift under gauge transformations, and no FI term has to be cancelled

Blowup

[Groot Nibbelink et al. 07-09]

Idea: VEVs for twisted states (blow-up modes) corresponds to smoothing out singularities – connection to smooth Calabi–Yau with bundles

In particular: VEVs for twisted non-oscillator states $(\mathbf{27}, \mathbf{1})$
$\leftrightarrow$ line bundles (i.e. Abelian fluxes)

Blowup

[Groot Nibbelink et al. 07-09]

Idea: VEVs for twisted states (blow-up modes) corresponds to smoothing out singularities – connection to smooth Calabi–Yau with bundles
In particular: VEVs for twisted non-oscillator states $(\mathbf{27}, \mathbf{1})$
$\leftrightarrow$ line bundles (i.e. Abelian fluxes)

Procedure:

1. Replace fixed points by exceptional divisors E_r ($\mathbb{P}^2$s)
2. Turn on Abelian gauge flux along the exceptional divisors,

$$\mathcal{F} = V_r^I H_I E_r \,, \quad H_I\text{: Cartan generators of } E_8$$

(automatically $(1,1)$-form)

Note: Line bundles don't reduce rank, axion shifts do:

$$B_2 = b_2 - \beta_r E_r \,, \quad \delta B_2 = -\operatorname{tr} \lambda \mathcal{F} \quad \Rightarrow \quad \delta \beta_r = \operatorname{tr} \lambda V_r$$

Bundle Vectors

Bundle has to satisfy DUY eqns. (analogue of D-term equation)

$$0 = \frac{1}{2}\int_X J \wedge J \wedge \mathcal{F} = \sum_r \mathrm{vol}(E_r) V_r \,, \quad \text{with all } \mathrm{vol}(E_r) > 0$$

Take three bundle vectors from p_{sh} of twisted **27** which sum to zero, distribute among exceptional divisors

$\rightsquigarrow$ flux quantisation and Bianchi Identity fulfilled automatically

DUY eqns. easily satisfied for arbitrary large volumes

Bundle Vectors

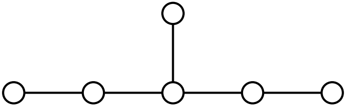
Bundle has to satisfy DUY eqns. (analogue of D-term equation)

$$0 = \frac{1}{2}\int_X J \wedge J \wedge \mathcal{F} = \sum_r \mathrm{vol}(E_r) V_r \,, \quad \text{with all } \mathrm{vol}(E_r) > 0$$

Take three bundle vectors from p_{sh} of twisted **27** which sum to zero, distribute among exceptional divisors

$\rightsquigarrow$ flux quantisation and Bianchi Identity fulfilled automatically
DUY eqns. easily satisfied for arbitrary large volumes

Each bundle vector breaks $E_6 \rightarrow SO(10) \times U(1)$, only two are independent.



Bundle Vectors

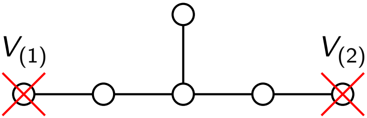
Bundle has to satisfy DUY eqns. (analogue of D-term equation)

$$0 = \frac{1}{2}\int_X J \wedge J \wedge \mathcal{F} = \sum_r \mathrm{vol}(E_r) V_r \,, \quad \text{with all } \mathrm{vol}(E_r) > 0$$

Take three bundle vectors from p_{sh} of twisted **27** which sum to zero, distribute among exceptional divisors

$\rightsquigarrow$ flux quantisation and Bianchi Identity fulfilled automatically
DUY eqns. easily satisfied for arbitrary large volumes

Each bundle vector breaks $E_6 \;\to\; SO(10) \times U(1)$, only two are independent.



$\Rightarrow$ gauge group $SO(8) \times U(1)_A \times U(1)_B \times SU(3)$

Massless spectrum depends on distribution (given by (k, p, q) with $k + p + q = 27$)

Blowup Anomalies

Blowup: two $U(1)$'s, different spectrum
$\Rightarrow$ anomaly polynomial $I_6 = \int_X I_{12}$ with backgrounds inserted, or from triangle diagrams

$$\Rightarrow I_6 \sim {F_A}^3 \cdot \left(\frac{k-6}{12}\right) + F_A F_B^2 \cdot \left(\frac{k-18}{4}\right)$$
$$+ F_A \left[\operatorname{tr} F_{SU(3)}^2 + \operatorname{tr} F_{SO(8)}^2 + \frac{7}{48} \operatorname{tr} R^2\right] \cdot \left(\frac{k-9}{2}\right)$$
$$+ F_B \left[\frac{1}{8} F_B^2 + \frac{1}{48} F_A^2 + \operatorname{tr} F_{SU(3)}^2 + \operatorname{tr} F_{SO(8)}^2 + \frac{7}{48} \operatorname{tr} R^2\right] \cdot \left(\frac{p-q}{2}\right)$$

Blowup Anomalies

Blowup: two $U(1)$'s, different spectrum
$\Rightarrow$ anomaly polynomial $I_6 = \int_X I_{12}$ with backgrounds inserted, or from triangle diagrams

$$\Rightarrow I_6 \sim F_A{}^3 \cdot \left(\frac{k-6}{12}\right) + F_A F_B^2 \cdot \left(\frac{k-18}{4}\right)$$
$$+ F_A \left[\operatorname{tr} F_{SU(3)}^2 + \operatorname{tr} F_{SO(8)}^2 + \frac{7}{48} \operatorname{tr} R^2\right] \cdot \left(\frac{k-9}{2}\right)$$
$$+ F_B \left[\frac{1}{8} F_B^2 + \frac{1}{48} F_A^2 + \operatorname{tr} F_{SU(3)}^2 + \operatorname{tr} F_{SO(8)}^2 + \frac{7}{48} \operatorname{tr} R^2\right] \cdot \left(\frac{p-q}{2}\right)$$

- For $p = q$, $U(1)_B$ is omalous, while $U(1)_A$ is always anomalous

Blowup Anomalies

Blowup: two $U(1)$'s, different spectrum
$\Rightarrow$ anomaly polynomial $I_6 = \int_X I_{12}$ with backgrounds inserted, or from triangle diagrams

$$\Rightarrow I_6 \sim F_A{}^3 \cdot \left(\frac{k-6}{12}\right) + F_A F_B^2 \cdot \left(\frac{k-18}{4}\right)$$
$$+ F_A \left[\operatorname{tr} F^2_{SU(3)} + \operatorname{tr} F^2_{SO(8)} + \frac{7}{48} \operatorname{tr} R^2\right] \cdot \left(\frac{k-9}{2}\right)$$
$$+ F_B \left[\frac{1}{8} F_B^2 + \frac{1}{48} F_A^2 + \operatorname{tr} F^2_{SU(3)} + \operatorname{tr} F^2_{SO(8)} + \frac{7}{48} \operatorname{tr} R^2\right] \cdot \left(\frac{p-q}{2}\right)$$

- For $p = q$, $U(1)_B$ is omalous, while $U(1)_A$ is always anomalous
- Remnant universality: Coefficients of non-Abelian groups from one E_8 are equal, and proportional to gravitational anomaly (only true if one E_8 unbroken)

Axion Shifts and massive $U(1)_B$

Axions β_r shift under $U(1)_{A,B}$ – universal axion does not!
$\Rightarrow$ $U(1)_A$ and $U(1)_B$ always massive, even if one of them is omalous:

$$\int_X H_3 \wedge *H_3 = A^I_\mu A^{\mu\,J} M^2_{IJ} + \cdots\,, \qquad M^2_{IJ} = V^I_r V^J_s \cdot \int_X E_r \wedge *_6\, E_s$$

Mass matrix is positive definite and always rank-two (and depends on the Kähler parameters)

Axion Shifts and massive $U(1)_B$

Axions β_r shift under $U(1)_{A,B}$ – universal axion does not!
$\Rightarrow$ $U(1)_A$ and $U(1)_B$ always massive, even if one of them is omalous:

$$\int_X H_3 \wedge *H_3 = A_\mu^I A^{\mu\, J} M_{IJ}^2 + \cdots\,, \qquad M_{IJ}^2 = V_r^I V_s^J \cdot \int_X E_r \wedge *_6\, E_s$$

Mass matrix is positive definite and always rank-two (and depends on the Kähler parameters)

$\rightarrow$ Stueckelberg mass possible without anomaly (but not *vice versa*)

Note: Still a coupling of the universal axion to X_4, as required by supersymmetry

Contents

Gauge Symmetry

Remnant non-R symmetries: discrete subgroup of $U(1)_A \times U(1)_B$ which leaves VEVs invariant
Blow-up modes:

$$\mathbf{1}_{4,0}\,, \quad \mathbf{1}_{-2,-2}\,, \quad \mathbf{1}_{-2,2}$$

$\Rightarrow$ discrete remnant $\mathbb{Z}_4 \times \mathbb{Z}_4$, generated by

$$T_\pm : \phi_{(q_A,q_B)} \longrightarrow \exp\left\{ \frac{2\pi \mathrm{i}}{4} \left(q_A \pm q_B\right) \right\} \phi_{(q_A,q_B)}$$

However: Charges of all massless fields are even under both $\mathbb{Z}_4$s
$\rightsquigarrow$ only $\mathbb{Z}_2 \times \mathbb{Z}_2$ realised on massless spectrum

Both $\mathbb{Z}_2$ factors are omalous

R Symmetries

- R symmetries do not commute with SUSY $\leftrightarrow \theta$ transforms, and different components of SUSY multiplets have different charges
- Only defined up to mixing with non-R symmetries
- For $\mathcal{N} = 1$ SUSY, only one $\theta \rightsquigarrow$ only one $U(1)$ or $\mathbb{Z}_N$ R symmetry – otherwise can redefine generators such that only one acts on θ
- Usual convention: θ has charge 1 $\Rightarrow$ Superpotential W has charge 2 ($\curvearrowright \mathbb{Z}_2$ doesn't really count as an R symmetry)

R Symmetries

- R symmetries do not commute with SUSY $\leftrightarrow$ θ transforms, and different components of SUSY multiplets have different charges
- Only defined up to mixing with non-R symmetries
- For $\mathcal{N} = 1$ SUSY, only one $\theta \rightsquigarrow$ only one $U(1)$ or $\mathbb{Z}_N$ R symmetry – otherwise can redefine generators such that only one acts on θ
- Usual convention: θ has charge 1 $\Rightarrow$ Superpotential W has charge 2 ($\curvearrowright \mathbb{Z}_2$ doesn't really count as an R symmetry)
- In compactifications, internal Lorentz transformations treat spinors and scalars differently $\rightsquigarrow$ can lead to R symmetries in 4D
- Orbifolds are special points in moduli space, so expect more symmetries – in particular, for general smooth spaces, expect no R symmetry in general

R Symmetries from Orbifolds

R transformations from sublattice rotations act as

$$\mathcal{R} : \Phi \longrightarrow e^{2\pi \mathrm{i} v R} \Phi$$

where (for Z_3 orbifolds) $v = \left(\underline{\frac{1}{3}, 0, 0}\right)$, $R = q_{\mathrm{sh}} - \Delta N$

R Symmetries from Orbifolds

R transformations from sublattice rotations act as

$$\mathcal{R} : \Phi \longrightarrow e^{2\pi i v R} \Phi$$

where (for Z_3 orbifolds) $v = \left(\underline{\tfrac{1}{3}, 0, 0}\right)$, $R = q_{\mathrm{sh}} - \Delta N$
Symmetry conventions somewhat tricky:

- For bosons, both v and R quantised in units of $\frac{1}{3}$, so $\mathbb{Z}_9$ symmetry (i.e. $\mathcal{R}^9 = \mathbb{1}$)
- For fermions, $R^{\mathrm{f}} = R^{\mathrm{b}} - \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$, so θ has charge $\frac{1}{6}$ (i.e. $\mathbb{Z}_6$ "R symmetry")
- Hence: $\mathbb{Z}_{18}$ symmetry with charges for

$$(\text{bosons}, \text{fermions}, \theta) = \frac{1}{18} (2n, 2n - 3, 3)$$

- Can redefine charges such that θ has charge 1 and superpotential has charge $2 \mod 6$, but then fields have non-integer charges

Model: VEV picture

Our blow-up modes have

$$R = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

Seek unbroken combinations of the three sublattice rotations and $U(1)_{A,B}$:

$$\begin{aligned}
\mathbf{1}_{4,0} &\longrightarrow \mathcal{R}_1^p\,\mathcal{R}_2^q\,\mathcal{R}_3^r\;T_A\;T_B\mathbf{1}_{4,0} = \mathbf{1}_{4,0}\,,\\
\mathbf{1}_{-2,-2} &\longrightarrow \mathcal{R}_1^p\,\mathcal{R}_2^q\,\mathcal{R}_3^r\;T_A\;T_B\mathbf{1}_{-2,-2} = \mathbf{1}_{-2,-2}\,,\\
\mathbf{1}_{-2,2} &\longrightarrow \mathcal{R}_1^p\,\mathcal{R}_2^q\,\mathcal{R}_3^r\;T_A\;T_B\mathbf{1}_{-2,2} = \mathbf{1}_{-2,2}
\end{aligned}$$

However, this implies $p + q + r = 3 \Rightarrow$ only a $\mathbb{Z}_2$ R symmetry survives in blow-up

Model: GLSM Description

[Witten '93;Groot Nibbelink '10; Blaszczyk et al. '11]

Algebraically, describe the orbifold by ($\mathbb{P}^2[3]$ is a T^2)

$$\frac{\mathbb{P}^2[3] \times \mathbb{P}^2[3] \times \mathbb{P}^2[3]}{\mathbb{Z}_3}$$

Blowup (crepant resolution) in $(0,2)$ GLSM description:

- 2D supersymmetric field theory with $U(1)$ gauge symmetries, fields$\sim$coordinates
- Geometry given by F and D term equations, GLSM FI terms become CY Kähler parameters
- In IR, flows to worldsheet description
- To resolve singularities, introduce extra coordinates (exceptional divisors) and $U(1)$s
- Gauge bundle given by "chiral-Fermi" superfields Λ_I with charges determined by the bundle vectors

R Symmetries in the GLSM

Set of F and D terms fixes geometry.
$\exists$ discrete transformations of the fields which leave F and D terms invariant
$\rightsquigarrow$ R symmetries if holomorphic three-form Ω transforms [Witten 85]

$$\Omega \sim \eta^T \Gamma_{ijk} \eta \mathrm{d}z^i \mathrm{d}z^j \mathrm{d}z^k \quad \Rightarrow \quad Q_R(\Omega) = Q_R(W) = 2$$

Different types of R symmetries:

- Phases $z \to e^{2\pi \mathrm{i}/3} z$: always possible (but see next slide)
  $\rightsquigarrow$ $\mathbb{Z}_6$ R symmetries
- Permutations of fields: Only possible for special values of Kähler parameters – corresponds to groupwise exchange of exceptional divisors

R Symmetry breaking in GLSM

$\mathbb{P}^2$ coordinates $z_{i\alpha}$ only appear as $z_{i\alpha}^3$ or $|z_{i\alpha}|^2$
$\Rightarrow$ unbreakable $\mathbb{Z}_3$ rotations?
(Presumably) broken by marginal deformations of Kähler potential terms in presence of gauge bundles (correspond to massless charged matter, $\phi_{4\mathrm{d}}$ 4D modes)

$$\int \mathrm{d}^2\theta^+ \phi_{4\mathrm{d}}(x^\mu) N(z) \qquad \underbrace{\Lambda\,\bar{\Lambda}}_{\text{gauge bundle fields}}$$

Fits with orbifold: Bundle corresponds to blowup
Presence of deformations controlled by Kähler (FI) parameters
$\Rightarrow$ Generically, no R symmetry in blow-up (all FI terms large), but enhanced at certain loci of parameter space

Contents

Conclusions

- Discussed (discrete) symmetries in 4D low-energy theories from the heterotic string
- Anomalies are generically not universal: Not required for anomaly cancellation, not generic from unification
- orbifold anomalous $U(1)$ is the exception because it is cancelled by the universal axion
- For blowups of heterotic orbifolds, many axions possible

Conclusions

- Discussed (discrete) symmetries in 4D low-energy theories from the heterotic string
- Anomalies are generically not universal: Not required for anomaly cancellation, not generic from unification
- orbifold anomalous $U(1)$ is the exception because it is cancelled by the universal axion
- For blowups of heterotic orbifolds, many axions possible
- Line bundles do reduce the rank via the axion shift – also omalous $U(1)$s can become massive
- Blow-ups can leave gauged discrete subgroups unbroken – important for phenomenology
- On orbifold, R symmetries exist but are broken by the blow-up

Outlook

- Found nice agreement between orbifold and blow-up picture, up to some subtleties
- "Geometry part" of GLSM generically has many "unbreakable" R-like symmetries – seem to be broken by the gauge bundle, but better understanding of their breaking required
- Linked to determination of charged massless spectrum
- Non-generic type of R symmetries: Exchange symmetries, appearing for certain loci in Kähler moduli space, e.g. exchange of exceptional divisors if their volumes are equal
- Study these symmetries for more realistic models, including Wilson lines etc.