Local SU(5) Unification from the Heterotic String

Christoph Lüdeling
ITP, Universität Heidelberg


1 Introduction
2 The Model
3 Anomaly Cancellation
4 Local GUT
5 Outlook
Introduction

- **GUT:** Attractive features:
  - \(SU(3) \times SU(2) \times U(1) \subset SU(5), SO_{10} \ldots\), gauge couplings unify
  - Unification matter into larger multiplets

- **Drawbacks in 4d GUTS**
  - Large Higgs representations required
  - Doublet–triplet–splitting
  - Yukawa couplings do not unify

- **Drawbacks can be addressed in higher-dimensional orbifold GUTs**

- **Nice possibility:** Heterotic String:
  - \(E_8 \times E_8\) gauge symmetry included
  - Simple orbifold compactifications with realistic four-dimensional matter content and gauge group possible
  - UV completion

[Kobayashi, Raby, Zhang; Buchmüller, Hamaguchi, Lebedev, Ratz; Kim, Kim, Kyae; Förste, Nilles, Vaudrevange, Wingerter, Ramos-Sanchez, . . . ]
Choose a torus with discrete isometry ("twist") with fixed points
- Mod out by this isometry, fixed points become singularities
- Fixing boundary conditions at fixed points requires embedding the twist into gauge group and choosing Wilson lines
- Gauge symmetry reduced at fixed points (but rank usually preserved)
- Twisted sectors: States localised at fixed points
The Model: Geometry

- Torus: $G_2 \times SU(3) \times SO(4)$ root lattice, $\mathbb{Z}_{6-II} = \mathbb{Z}_3 \times \mathbb{Z}_2$ twist:

  - Obtain effective 6D Theory on $T^2/\mathbb{Z}_2$ orbifold
  - Internal zero modes and $\mathbb{Z}_3$ twisted states show up as bulk states, $\mathbb{Z}_2$ twisted states are localised at orbifold fixed points
The Model: Effective $T^2/\mathbb{Z}_2$ Orbifold

2 MSSM generations

SU(5)

SU(2) × SU(4)

SU(6)

$\bar{5} + 10$

$4 \times 2$

$35$

$9 \times (6 + \bar{6})$

$20$

exotics
Anomalies

• Orbifold have bulk and brane anomalies
• Anomaly cancellation by Green–Schwarz mechanism requires factorisation of anomaly polynomials, $I_8 = X_4 Y_4$ and $I_6^f = X_4^f Y_2$
• $O(500)$ conditions, but guaranteed by string theory (and modular invariance conditions on twist vectors and Wilson lines): Check of spectrum
• Anomalous U(1)'s induce localised FI terms

\[
\xi_0 = 148 \left( \frac{g M_P^2}{384 \pi^2} \right) \delta^{(2)}(z - z_0)
\]

\[
\xi_1 = 80 \left( \frac{g M_P^2}{384 \pi^2} \right) \delta^{(2)}(z - z_1)
\]

• These lead to localisation of bulk fields, break the U(1) and need to be cancelled to obtain SUSY vacuum

[Lee, Nilles, Zucker]
Local SU(5) GUT

• Local GUT: At fixed points, boundary conditions break bulk gauge group to smaller groups,

\[
\text{in our case: } \quad SU(6) \rightarrow \begin{cases} 
SU(5) \\
SU(2) \times SU(4)
\end{cases}
\]

• In zero mode spectrum, only the intersection of local groups survives, which is \(G_{\text{SM}} = SU(3) \times SU(2) \times U(1)\)
• Localised fields come in complete multiplets of local GUT group
• Due to other branes, bulk fields form split multiplets
• Due to higher symmetry, decoupling of exotics much more transparent that in four-dimensional limit
On branes, SUSY is broken to $\mathcal{N} = 1$

Bulk Matter:
1. Hypermultiplets, split as $H = (H_L, H_R)$ into chiral multiplet
2. Bulk vector multiplets split as $V = (A, \phi)$ into vector and chiral multiplets

Only one $\mathcal{N} = 1$ multiplet survives projection
Decoupling

- Several pairs of $5 + \bar{5}$ and most exotics decoupled easily
- Remaining $5$'s and $\bar{5}$'s:

<table>
<thead>
<tr>
<th>Bulk:</th>
<th>$5$</th>
<th>$5_1$</th>
<th>$\bar{5}_0$</th>
<th>$\bar{5}$</th>
<th>$\bar{5}_1$</th>
<th>$\bar{5}_2$</th>
<th>$5_0$</th>
<th>$5_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero modes:</td>
<td>$\text{SU}(3) \times \text{SU}(2)$</td>
<td>(1, 2)</td>
<td>(1, 2)</td>
<td>(3, 1)</td>
<td>(1, 2)</td>
<td>(1, 2)</td>
<td>(3, 1)</td>
<td>(3, 1)</td>
</tr>
<tr>
<td>$\text{U}(1)_{B-L}$</td>
<td>0</td>
<td>0</td>
<td>$-\frac{2}{3}$</td>
<td>0</td>
<td>0</td>
<td>$-\frac{1}{3}$</td>
<td>$\frac{2}{3}$</td>
<td>$-1$</td>
</tr>
<tr>
<td>MSSM content</td>
<td>$H_u$</td>
<td></td>
<td></td>
<td></td>
<td>$H_d$</td>
<td>$d_3$</td>
<td></td>
<td>$l_3$</td>
</tr>
</tbody>
</table>

$2 \times (\bar{5} + 10)$ generations on the branes
$2 \times (\bar{5} + 10)$ generations in the bulk
$5 + \bar{5}$ Higgses in the bulk
Split Multiplets

- Bulk generations:

\[
\begin{align*}
\bar{5}_{(3)} &= (\bar{3}, 1) + (1, 2) \\
\bar{5}_{(4)} &= (\bar{3}, 1) + (1, 2)
\end{align*}
\]

\[
\begin{align*}
10_{(3)} &= (3, 2) + (\bar{3}, 1) + (1, 1) \\
10_{(4)} &= (3, 2) + (\bar{3}, 1) + (1, 1)
\end{align*}
\]

One generation remains, avoiding SU(5) mass relations

- Higgses:

\[
\begin{align*}
5_u &= (3, 1) + (1, 2) \\
\bar{5}_d &= (\bar{3}, 1) + (1, 2)
\end{align*}
\]

Orbifold projection solves doublet–triplet–splitting
Yukawa Couplings

\[ W = C_{(ij)}^{(u)} 5_u \cdot 10(i) \cdot 10(j) + C_{(ij)}^{(d)} 5_d \cdot \bar{5}(i) \cdot 10(j) \]

\[ C_{(ij)}^{(u)} = \begin{pmatrix} a_1 & 0 & a_2 & a_3 \\ 0 & a_1 & a_2 & a_3 \\ a_2 & a_2 & 0 & g \\ a_3 & a_3 & g & a_4 \end{pmatrix}, \quad C_{ij}^{(d)} = \begin{pmatrix} 0 & 0 & b_1 & b_2 \\ 0 & 0 & b_1 & b_2 \\ b_3 & b_3 & b_4 & 0 \\ b_5 & b_5 & b_6 & b_5^2 \end{pmatrix} \]

\[ W = Y_{ij}^u h_u u_i^c q_j + Y_{ij}^d h_d d_i^c q_j + Y_{ij}^l h_d l_i^c e_j^c \]

\[ Y_{ij}^u = \begin{pmatrix} a_1 & 0 & a_3 \\ 0 & a_1 & a_3 \\ a_2 & a_2 & g \end{pmatrix}, \quad Y_{ij}^d = \begin{pmatrix} 0 & 0 & b_2 \\ 0 & 0 & b_2 \\ b_5 & b_5 & b_7 \end{pmatrix}, \quad Y_{ij}^l = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_1 \\ b_3 & b_3 & b_4 \end{pmatrix} \]
Outlook

- Constructed local 6D GUT from the heterotic string
- Doublet–triplet splitting achieved easily, SU(5) mass relations avoided due to split bulk multiplets
- More symmetry in 6D $\Rightarrow$ simple decoupling of unwanted states
- Supersymmetric vacuum: four-dimensional $D$-term vanishes
- Open Questions:
  - Phenomenology needs to be improved (CKM mixing, $R$-parity)
  - Stabilisation of moduli, in particular, size of two-dimensional torus
  - Profiles of bulk fields due to localised FI terms
  - Blowup/resolution of singularities, generalisation to K3 internal space