Local SU(5) Unification from the Heterotic String

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W. Buchmüller, CL, J. Schmidt, arXiv:0707.1651

Introduction

- GUT: Attractive features:
  - $SU(3) \times SU(2) \times U(1) \subset SU(5), SO_{10} \ldots$, gauge couplings unify
  - Unification matter into larger multiplets
- Drawbacks in 4d GUTS
  - Large Higgs representations required
  - Doublet–triplet–splitting
  - Yukawa couplings do not unify
- Drawbacks can be addressed in higher-dimensional orbifold GUTs
- Nice possibility: Heterotic String:
  - $E_8 \times E_8$ gauge symmetry included
  - Simple orbifold compactifications with realistic four-dimensional matter content and gauge group possible
  - UV completion

[Kobayashi, Raby, Zhang; Buchmüller, Hamaguchi, Lebedev, Ratz; Kim, Kim, Kyae; Förste, Nilles, Vaudrevange, Wingerter, Ramos-Sanchez,. . .]

Heterotic Orbifold Compactification

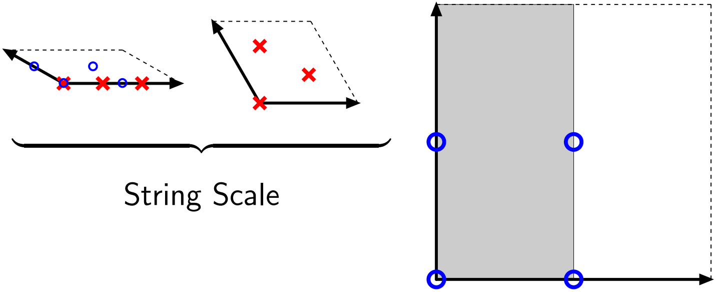
- Choose a torus with discrete isometry ("twist") with fixed points
- Mod out by this isometry, fixed points become singularities
- Fixing boundary conditions at fixed points requires embedding the twist into gauge group and choosing Wilson lines
- Gauge symmetry reduced at fixed points (but rank usually preserved)
- Twisted sectors: States localised at fixed points

The Model: Geometry

[Buchmüller, Hamaguchi, Lebedev, Ratz]

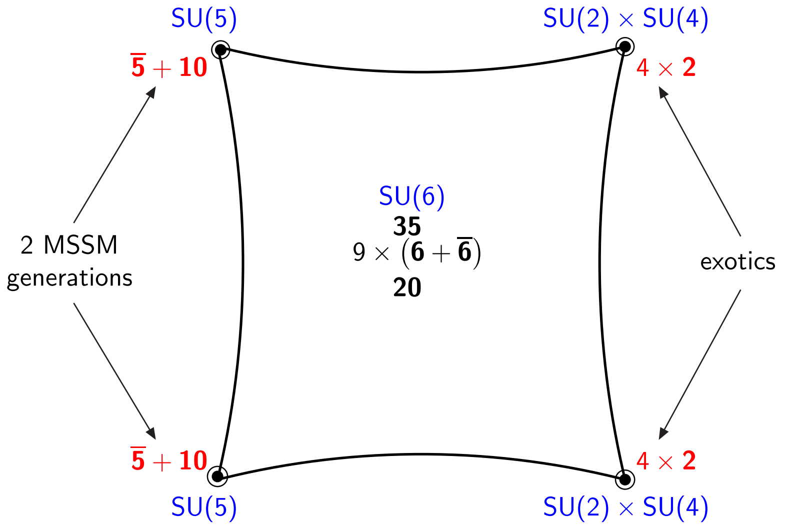
- Torus: $G_2 \times SU(3) \times SO(4)$ root lattice, $\mathbb{Z}_{6-II} = \mathbb{Z}_3 \times \mathbb{Z}_2$ twist:

[Kobayashi,Raby,Zhang]



- Obtain effective 6D Theory on $T^2/\mathbb{Z}_2$ orbifold
- Internal zero modes and $\mathbb{Z}_3$ twisted states show up as bulk states, $\mathbb{Z}_2$ twisted states are localised at orbifold fixed points

The Model: Effective $T^2/\mathbb{Z}_2$ Orbifold



Anomalies

- Orbifold have bulk and brane anomalies
- Anomaly cancellation by Green–Schwarz mechanism requires factorisation of anomaly polynomials, $I_8 = X_4 Y_4$ and $I_6^f = X_4^f Y_2$
- $\mathcal{O}(500)$ conditions, but guaranteed by string theory (and modular invariance conditions on twist vectors and Wilson lines): Check of spectrum
- Anomalous U(1)'s induce localised FI terms

$$\xi_0 = 148 \left(\frac{g M_\mathrm{P}^2}{384\pi^2} \right) \delta^{(2)}(z - z_0)$$
$$\xi_1 = 80 \left(\frac{g M_\mathrm{P}^2}{384\pi^2} \right) \delta^{(2)}(z - z_1)$$

- These lead to localisation of bulk fields, break the U(1) and need to be cancelled to obtain SUSY vacuum [Lee, Nilles, Zucker]

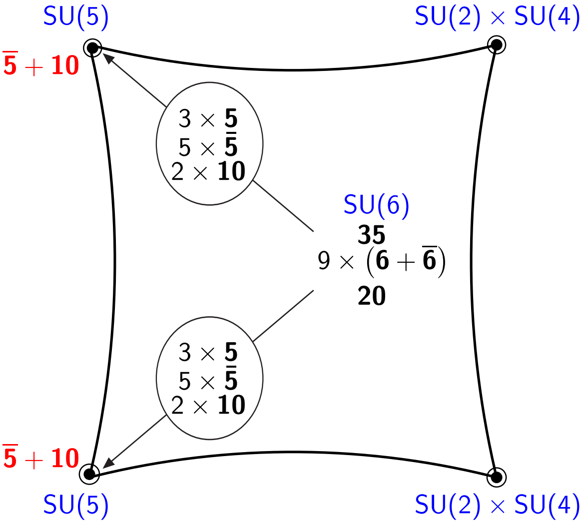
Local SU(5) GUT

- Local GUT: At fixed points, boundary conditions break bulk gauge group to smaller groups,

$$\text{in our case:} \quad SU(6) \longrightarrow \begin{cases} SU(5) \\ SU(2) \times SU(4) \end{cases}$$

- In zero mode spectrum, only the intersection of local groups survives, which is $G_{SM} = SU(3) \times SU(2) \times U(1)$
- Localised fields come in complete multiplets of local GUT group
- Due to other branes, bulk fields form split multiplets
- Due to higher symmetry, decoupling of exotics much more transparent that in four-dimensional limit

Projection



- On branes, SUSY is broken to $\mathcal{N} = 1$
- Bulk Matter: Hypermultiplets, split as $H = (H_L, H_R)$ into chiral multiplet
- Bulk vector multiplets split as $V = (A, \phi)$ into vector and chiral multiplets
- Only one $\mathcal{N} = 1$ multiplet survives projection

Decoupling

- Several pairs of $\mathbf{5}+\bar{\mathbf{5}}$ and most exotics decoupled easily
- Remaining $\mathbf{5}$'s and $\bar{\mathbf{5}}$'s:

Bulk: / Zero modes:	$\mathbf{5}$	$\mathbf{5}_1$	$\bar{\mathbf{5}}_0^c$	$\bar{\mathbf{5}}$	$\bar{\mathbf{5}}_1$	$\bar{\mathbf{5}}_2$	$\mathbf{5}_0^c$	$\mathbf{5}_2^c$
$SU(3)\times SU(2)$	$(1,\mathbf{2})$	$(1,\mathbf{2})$	$(\mathbf{3},1)$	$(1,\mathbf{2})$	$(1,\mathbf{2})$	$(\bar{\mathbf{3}},1)$	$(\bar{\mathbf{3}},1)$	$(1,\mathbf{2})$
$U(1)_{B-L}$	0	0	$-\frac{2}{3}$	0	0	$-\frac{1}{3}$	$\frac{2}{3}$	-1
MSSM content	H_u				H_d	d_3		l_3

$2\times(\bar{\mathbf{5}}+\mathbf{10})$ generations on the branes
$2\times(\bar{\mathbf{5}}+\mathbf{10})$ generations in the bulk
$\mathbf{5}+\bar{\mathbf{5}}$ Higgses in the bulk

Split Multiplets

- Bulk generations:

$$\bar{\mathbf{5}}_{(3)} = \cancel{(\bar{\mathbf{3}}, 1)} + (1, \mathbf{2}) \qquad \mathbf{10}_{(3)} = \cancel{(\mathbf{3}, \mathbf{2})} + (\bar{\mathbf{3}}, 1) + (1, 1)$$
$$\bar{\mathbf{5}}_{(4)} = (\bar{\mathbf{3}}, 1) + \cancel{(1, \mathbf{2})} \qquad \mathbf{10}_{(4)} = (\mathbf{3}, \mathbf{2}) + \cancel{(\bar{\mathbf{3}}, 1)} + \cancel{(1, 1)}$$

One generation remains, avoiding SU(5) mass relations

- Higgses:

$$\mathbf{5}_u = \cancel{(\mathbf{3}, 1)} + (1, \mathbf{2})$$
$$\bar{\mathbf{5}}_d = \cancel{(\bar{\mathbf{3}}, 1)} + (1, \mathbf{2})$$

Orbifold projection solves doublet–triplet–splitting

Yukawa Couplings

$$W = C^{(u)}_{(ij)} \mathbf{5}_u \mathbf{10}_{(i)} \mathbf{10}_{(j)} + C^{(d)}_{(ij)} \mathbf{5}_d \bar{\mathbf{5}}_{(i)} \mathbf{10}_{(j)}$$

$$C^{(u)}_{(ij)} = \begin{pmatrix} a_1 & 0 & a_2 & a_3 \\ 0 & a_1 & a_2 & a_3 \\ a_2 & a_2 & 0 & g \\ a_3 & a_3 & g & a_4 \end{pmatrix}, \qquad C^{(d)}_{ij} = \begin{pmatrix} 0 & 0 & b_1 & b_2 \\ 0 & 0 & b_1 & b_2 \\ b_3 & b_3 & b_4 & 0 \\ b_5 & b_5 & b_6 & b_5^2 \end{pmatrix}$$

$$W = Y^u_{ij} h_u u^c_i q_j + Y^d_{ij} h_d d^c_i q_j + Y^l_{ij} h_d l_i e^c_j$$

$$Y^u_{ij} = \begin{pmatrix} a_1 & 0 & a_3 \\ 0 & a_1 & a_3 \\ a_2 & a_2 & g \end{pmatrix}, \quad Y^d_{ij} = \begin{pmatrix} 0 & 0 & b_2 \\ 0 & 0 & b_2 \\ b_5 & b_5 & b_7 \end{pmatrix}, \quad Y^l_{ij} = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_1 \\ b_3 & b_3 & b_4 \end{pmatrix}$$

Outlook

- Constructed local 6D GUT from the heterotic string
- Doublet–triplet splitting achieved easily, SU(5) mass relations avoided dur to split bulk multiplets
- More symmetry in 6D $\rightsquigarrow$ simple decoupling of unwanted states
- Supersymmetric vacuum: four-dimensional D-term vanishes
- Open Questions:
  - Phenomenology needs to be improved (CKM mixing, R-parity)
  - Stabilisation of moduli, in particular, size of two-dimensional torus
  - Profiles of bulk fields due to localised FI terms
  - Blowup/resolution of singularities, generalisation to K3 internal space