The Potential Fate of Local Model Building

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May 06, 2011

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PRD **83**, 086008 [arXiv:1101.3346] & work in progress

Motivation

- F-Theory: Vacua with general branes in type IIB string theory
- Exceptional symmetries available, so interesting for GUT model building (as generalisations of perturbative intersecting brane models)
- Usually, consider local models: Focus on brane stack or points within the stack and decouple bulk of the compactification manifold
- Advantages: Simple, hope for predictivity from genericity
- Some questions cannot be addressed, e.g. moduli stabilisation
- Obvious problem: Existence of global completion
- Consider $SU(5)$ GUT with matter parity to forbid proton decay
- Models very constrained locally – global completion impossible

Contents

String Phenomenology

1. Find realistic particle physics models in string theory:
   - Spontaneously broken $\mathcal{N} = 1$ SUSY in four dimensions
   - Gauge group (standard model or GUT)
   - Matter content
   - Masses and mixings
   - Proton stability
2. Look for imprints of string theory in low-energy physics:
   - Mediation schemes, patterns of soft masses
   - Exotics below GUT/Planck scale
   - Thresholds, gauge coupling unification

GUTs and Strings

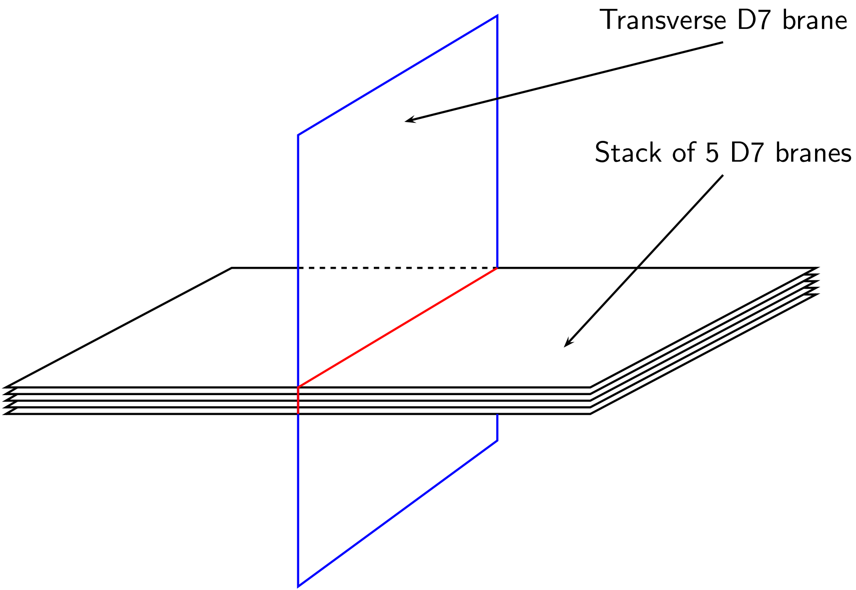
Promising paths:

- $E_8 \times E_8$ heterotic string on orbifolds or smooth Calabi–Yaus
  - Global models, i.e. full compactification space is specified
  - Gauge fields live in bulk, matter in bulk or on lower-dimensional subspaces
- Type II theories with intersecting branes $\rightsquigarrow$ F-theory
  - Mostly local models, i.e. focus on branes and "decouple" bulk
  - Gauge fields on branes, matter on intersections of branes

General features:

- Exceptional symmetry groups (though not as gauge groups in four dimensions)
- Nontrivial pattern of gauge and matter fields localised on different subspaces of compactification space

Cartoon: Intersecting Branes in Perturbative Type IIB



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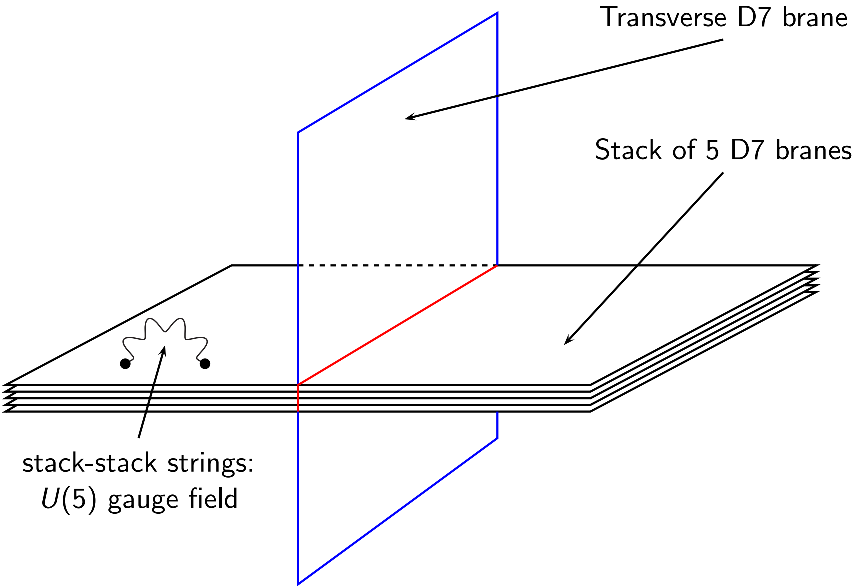

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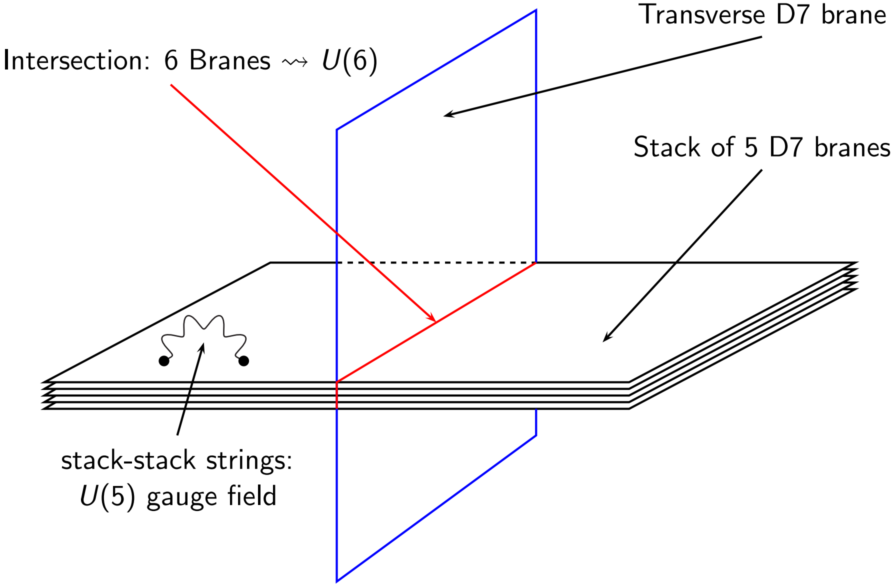

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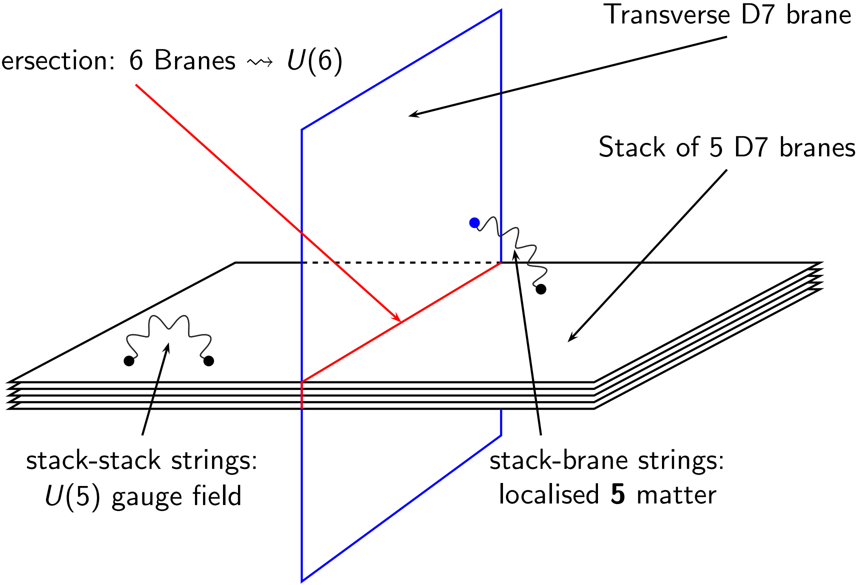

Intersecting Branes

- Stacks of D7 branes and their intersections: $U(N)$ gauge groups, bifundamental matter $\left(N, \bar{M}\right)$
- Include O7 planes: Realise $SO(2N)$ gauge groups and two-index antisymmetric representations, e.g. $\mathbf{10}$ of $SU(5)$
- Matter localised at intersection where symmetry is enhanced, representations can be inferred form decomposition of adjoint of higher group

Intersecting Branes

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- Matter localised at intersection where symmetry is enhanced, representations can be inferred form decomposition of adjoint of higher group
- Problems:
  - For $SU(5)$ GUTs, top quark Yukawa coupling forbidden perturbatively – might be generated by instantons, but then it's small
  - No spinor representations of $SO(10)$
- Both top Yukawa and $SO(10)$ spinors require exceptional local symmetry groups
- Type IIB has more general (p, q) branes, but these cannot be treated perturbatively

F-Theory: Axiodilaton Monodromy

Type IIB contains complex scalar field: axiodilaton

$$\tau = C_0 + \mathrm{i}e^{-\phi}$$

When encircling a 7-brane, τ undergoes $SL(2, \mathbb{Z})$ monodromy transformation

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d}$$

E.g. for a single D7 brane at $z = 0$,

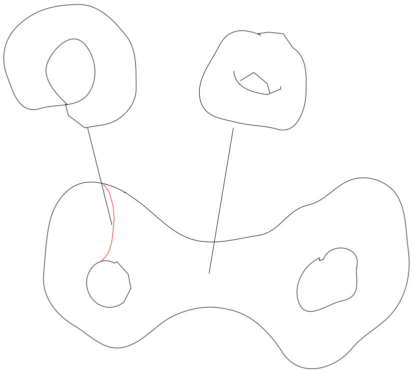
$$\tau \longrightarrow \tau + 1 \quad \Rightarrow \quad \tau \sim \ln z$$

$\Rightarrow$ at brane positions, τ diverges

F-Theory: Extra Torus

[Vafa 96]

Key idea of F-Theory: $SL(2, \mathbb{Z})$ is also symmetry of torus complex structure $\rightsquigarrow$ describe variation of τ by auxiliary torus over every point of compactification space B_6: elliptic fibration



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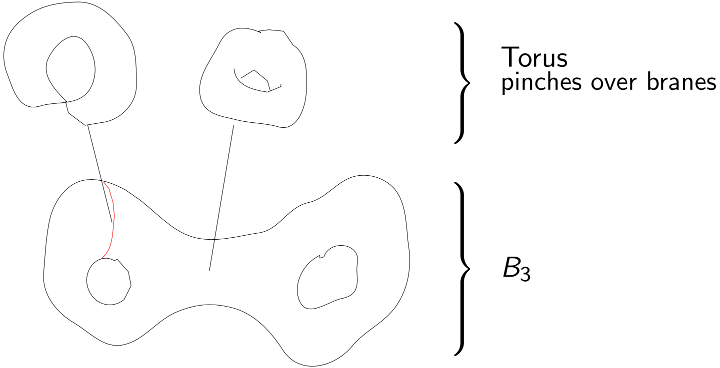

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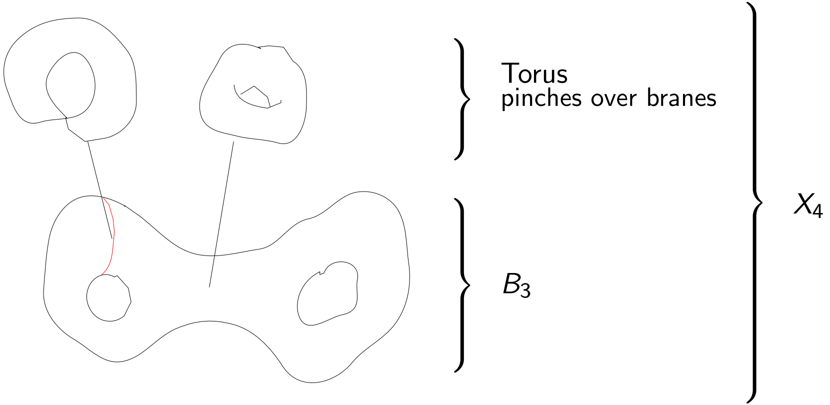

F-Theory: Tate Model

[Bershadsky et al. 96]

Elliptic fibration: Torus over complex three-dimensional base, described by Tate model

$$y^2 = x^3 + a_5 xy + a_4 x^2 + a_3 y + a_2 x + a_0$$

x, $y \in \mathbb{C}$, a_k: functions on the base
Brane positions ⇔ torus degenerates ⇔ Discriminant vanishes:

$$\Delta = \text{polynomial in the } a_k = 0$$

Type of brane (stack), i.e. gauge symmetry, determined by vanishing orders of the a_k and Δ (cf. ADE classification of singularities) [Kodaira '60s]

Intersections with other branes ⇔ Local symmetry enhancement ⇔ Locally Δ and a_k vanish to higher order: Matter curves

Local $SU(5)$ GUT

To engineer e.g. $SU(5)$ GUT, take brane position locally given by coordinate $w = 0$ and choose Tate model appropriately:

$$y^2 = x^3 + a_5\, xy + \ a_4\ \ x^2 + \ \ a_3\ \ y + \ \ a_2\ \ x + \ \ a_0$$

a_k: functions on base

Local $SU(5)$ GUT

To engineer e.g. $SU(5)$ GUT, take brane position locally given by coordinate $w = 0$ and choose Tate model appropriately:

$$y^2 = x^3 + b_5\, xy + b_4 w\, x^2 + b_3 w^2\, y + b_2 w^4\, x + b_0 w^5$$

a_k: functions on base
b_k: functions on brane stack

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a_k: functions on base
b_k: functions on brane stack
Discriminant becomes

$$\Delta = w^5 \left(b_5^4 P + w b_5^2 \left(8 b_4 P + b_5 R\right) + \mathcal{O}\left(w^2\right)\right)$$

P, R: polynomials in the b_k

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P, R: polynomials in the b_k
Locally, $SU(5)$ is enhanced

to $SU(6)$:	$P = 0$	$\Rightarrow$	localised **5**
to $SO(10)$:	$b_5 = 0$	$\Rightarrow$	localised **10**

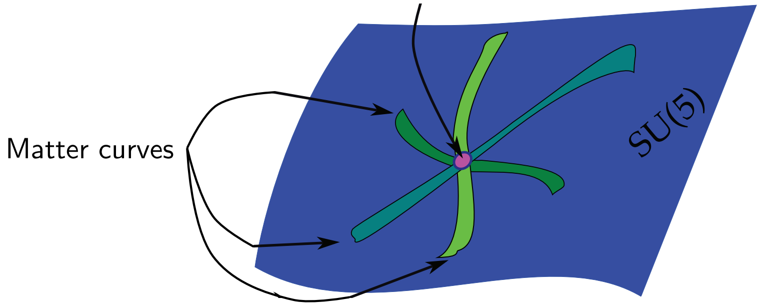
Yukawa Couplings

Matter curves: Curves of symmetry enhancement, representations determined by the adjoint of the higher group

Finally, matter curves meet in points $\rightsquigarrow$ further symmetry enhancement to G_P

Intersection of three matter curves with localised matter representations $R_{1,2,3}$ leads to Yukawa couplings from triple adjoint interaction of G_P:

$$(\text{ad } G_P)^3 = R_1 R_2 R_3 + \cdots$$



[picture from Sakura Schäfer-Nameki]

Global, Semilocal, Local

For F-Theory GUTs, different degrees of locality:

- *Global* model: Specify full compactification space (CY fourfold): Includes all branes, fluxes, obeys consistency conditions, can stabilise moduli etc. [Blumenhagen et al.; Grimm et al.; Marsano et al.;...]
- *Semilocal* model: Focus on the GUT surface (brane stack) S and matter curves within S: Decouples bulk of compactification space, certain consistency conditions included
  [Hayashi et al.; Donagi, Wijnholt; Grimm, Weigand; Marsano et al.; Dudas, Palti; CL, Nilles, Stephan;...]
- *Local* model: Consider only points within S where matter curves intersect and interactions are localised: Simple, and hope for predictivity because any good global model must contain good local model and bulk physics decoupled. Certain questions cannot be answered, and actual existence of global completion is not guaranteed. [Donagi, Wijnholt; Heckman, Vafa et al.; Watari et al.;...]

Contents

Yukawa Couplings

- Tate model for $SU(5)$ GUT localised at $w = 0$:

$$y^2 = x^3 + b_5\, xy + b_4 w\, x^2 + b_3 w^2\, y + b_2 w^4\, x + b_0 w^5$$

- Yukawa couplings require $SO(12)$ enhancement for down-type Yukawas,

$$(\mathbf{66})^3 \supset \overline{\mathbf{5}}_{H_d} \overline{\mathbf{5}}_M \, \mathbf{10}_M$$

and E_6 enhancement for up-type,

$$(\mathbf{78})^3 \supset \mathbf{5}_{H_u} \mathbf{10}_M \, \mathbf{10}_M$$

Point of E_8

[Heckman, Tavanfar, Vafa]

- Need E_6 and $SO(12)$ enhancements for up- and down-type Yukawas
- CKM matrix: Favourably, these points coincide (no geometric suppression of quark mixing) $\rightsquigarrow E_7$
- For PMNS matrix: Further enhancement to E_8 (but we do not consider neutrinos in the following)
- Hence: One single Yukawa "point of E_8", all interactions localised here

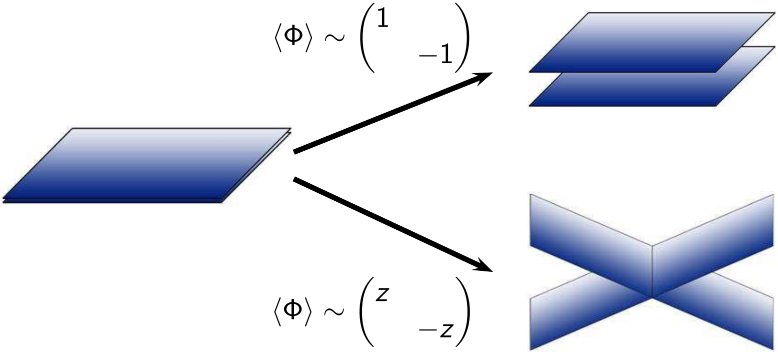
↷ Allows for higher interaction terms – Froggatt–Nielsen type masses using GUT singlets

- Simple and potentially predictive: All (superpotential) interactions determined by group theory, geometric data can be largely ignored

Higgs as Brane Splitter

So far: $SU(5)$, locally enhanced to $SU(6)$, $SO10)$, E_6 etc. up to E_8
Turn this around: E_8 broken to E_6, $SO(10)$ etc., and generically to $SU(5)$

Brane theory contains adjoint Higgs field – parameterises brane motion:



Gauge Theory Description

- On brane stack, super-Yang–Mills theory: contains adjoint scalar field
- VEV for adjoint Higgs: rank-preserving breaking

$$E_8 \longrightarrow (SU(5) \times SU(5)_{\perp}) \longrightarrow SU(5) \times U(1)^4$$

- Extra $U(1)$'s generically massive in F-Theory by geometric Stueckelberg effect, but this cannot be analysed in local model – $U(1)$'s remain as global selection rules [Grimm, Weigand]
- Higgs field varies over S – matter curves now visible as vanishing loci of Higgs eigenvalues

E_8 Higgs

$$E_8 \longrightarrow SU(5) \times SU(5)_\perp$$
$$\mathbf{248} \longrightarrow (\mathbf{24},\mathbf{1}) \oplus (\mathbf{1},\mathbf{24}) \oplus \left[(\mathbf{10},\mathbf{5}) \oplus \left(\mathbf{5},\overline{\mathbf{10}}\right) \oplus \text{c.c.}\right]$$

$$\text{Higgs} \quad \Phi \sim \begin{pmatrix} t_1 &&&& \\ & t_2 &&& \\ && t_3 && \\ &&& t_4 & \\ &&&& t_5 \end{pmatrix} \in (\mathbf{1},\mathbf{24}) \ , \quad \sum_i t_i = 0$$

Connection to Tate model: Deformed E_8 singularity,

$$y^2 = x^3 + b_0 w^5 \quad \longrightarrow \quad y^2 = x^2 + b_0 \prod (w - t_i)$$

$\curvearrowright$ the b_k are symmetric polynomials in the t_i of order k, no b_1 because of tracelessness

Matter Curves

t_i are eigenvalues in the **5** of $SU(5)_\perp$, i.e.

$$\Phi e_i = t_i e_i$$

$\curvearrowright$ **10** of $SU(5)_\perp$ spanned by $e_i \wedge e_j$, $i \neq j$, with eigenvalue $t_i + t_j$

Representations of $SU(5) \times SU(5)_\perp$ appear as $(\mathbf{10}, \mathbf{5}) \oplus \left(\mathbf{5}, \overline{\mathbf{10}}\right)$
$\curvearrowright$ in terms of $SU(5)$ reps, matter curves are given by

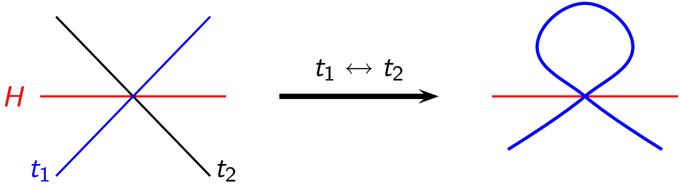
$$\begin{aligned} t_i &= 0 & &\text{localised } \mathbf{10} \\ -t_i - t_j &= 0 & &\text{localised } \mathbf{5} \\ t_i - t_j &= 0 & &\text{localised } \mathbf{1} \end{aligned}$$

t_i double as charges: For gauge-invariant terms, t_i must sum to zero (possibly using $\sum_i t_i = 0$) – realises $U(1)^4 \subset SU(5)_\perp$ selection rules

Monodromy

[Bershadsky et al.]

- The b_k in the Tate model are symmetric polynomials in the t_i
  $\Rightarrow$ Invariant under permutations of the t_i
- Interpretation: Self-intersection, locally distinct-looking branes are the same



- Heavy top requires coupling $\mathbf{5}_{H_u}\mathbf{10}_{\text{top}}\mathbf{10}_{\text{top}}$
  $\rightsquigarrow$ (at least) $\mathbb{Z}_2$ monodromy $t_1 \leftrightarrow t_2$
- Fixes top and up-type Higgs curve: $\mathbf{10}_{\text{top}} \sim \{t_1, t_2\}$, $\mathbf{5}_{H_u} \sim -t_1 - t_2$
- Reduces $SU(5)_\perp$ to lower rank

$SU(5)$ GUT

Decomposition of $SU(5)$ fields into SM representations:

$$\begin{array}{rrcl} \text{Gauge bosons:} & \mathbf{24} & \sim & G \oplus W \oplus B_Y \oplus X \oplus Y \\ \text{Matter:} & \mathbf{10}_M & \sim & Q \oplus u^c \oplus e^c \\ & \overline{\mathbf{5}}_M & \sim & d^c \oplus L \\ \text{Higgs:} & \mathbf{5}_{H_u} & \sim & H_u \oplus T_u \\ & \overline{\mathbf{5}}_{H_d} & \sim & H_d \oplus T_d \end{array}$$

Higgs triplets and X, Y bosons need to be very heavy for proton stability

Break $SU(5)$ by hypercharge flux

$$F_Y \sim \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$$

F_Y must be globally trivial to preserve hypercharge

Yukawa Couplings

Good couplings: Quark and lepton masses, weak-scale μ term

$$W_{\text{good}} = \mu\, \mathbf{5}_{H_u} \overline{\mathbf{5}}_{H_d} + Y_u \mathbf{5}_{H_u} \mathbf{10}_M \mathbf{10}_M + Y_d \overline{\mathbf{5}}_{H_d} \overline{\mathbf{5}}_M \mathbf{10}_M$$

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Bad couplings: Baryon and lepton number violating operators

$$\left.\begin{aligned} W_{\text{bad}} &= \beta\, \mathbf{5}_{H_u}\overline{\mathbf{5}}_M + \lambda\overline{\mathbf{5}}_M\overline{\mathbf{5}}_M\mathbf{10}_M \\ &\quad + W^1\mathbf{10}_M\mathbf{10}_M\mathbf{10}_M\overline{\mathbf{5}}_M + W^2\mathbf{10}_M\mathbf{10}_M\mathbf{10}_M\overline{\mathbf{5}}_{H_d} \\ &\quad + W^3\overline{\mathbf{5}}_M\overline{\mathbf{5}}_M\mathbf{5}_{H_u}\mathbf{5}_{H_u} + W^4\overline{\mathbf{5}}_M\overline{\mathbf{5}}_{H_d}\mathbf{5}_{H_u}\mathbf{5}_{H_u} \\ K_{\text{bad}} &= K^1\mathbf{10}_M\mathbf{10}_M\mathbf{5}_M + K^2\overline{\mathbf{5}}_{H_u}\overline{\mathbf{5}}_{H_u}\mathbf{10}_M \end{aligned}\right\}\ \text{dim-5}$$

dim-3/4 (first line); dim-5 (remaining lines)

Coefficients can contain singlet VEVs, suppressed by M_{GUT} [Conlon, Palti]

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dim-3/4

dim-5

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Some terms related by interchange $\bar{\mathbf{5}}_{H_d} \leftrightarrow \bar{\mathbf{5}}_M$

Matter Parity

[Dimopoulos, Raby, Wilczek; Ibanez, Ross; Dreiner, Luhn, Thormeier]

Various discrete symmetries help for proton stability. Compatibility with $SU(5)$ favours $\mathbb{Z}_2$ which matter parity distinguishes Higgs and matter:

	$\mathbf{5}_{H_u}, \bar{\mathbf{5}}_{H_d}$	$\mathbf{10}_M, \bar{\mathbf{5}}_M$
P_M	$+1$	-1

Forbids all baryon and lepton number violating operators except

$$W^1 \mathbf{10}_M \mathbf{10}_M \mathbf{10}_M \bar{\mathbf{5}}_M \quad \text{and} \quad W^3 \bar{\mathbf{5}}_M \bar{\mathbf{5}}_M \mathbf{5}_{H_u} \mathbf{5}_{H_u}$$

W^3 generates neutrino masses (Weinberg operator), can be tolerated if suppression scale high enough (but will not be generated, so ignore from now on)

W^1 is very strongly constrained ($W^1 \supset QQQL, \bar{u}\bar{u}\bar{d}\bar{e}, \ldots$) – forbid this by clever choice of matter curves (i.e. $U(1)$s)

Contents

Model Requirements

For the local model we require

- P_M defined *at the point of* E_8
- heavy top quark (i.e. rank-one up-type Yukawa matrix at tree level)
- No dim-5 proton decay (the W^1 operator forbidden at all orders)
- Masses for all quarks and leptons after switching on VEVs (down-type Yukawa matrix can be rank-zero or one, but not rank-two)

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Local model building freedom: Freely choose

- Monodromy (at least $\mathbb{Z}_2$)
- Assignment of matter and Higgs zero modes to curves
- Singlet VEVs (for matter parity even singlets)
- Assume: Allowed terms generated with order-one coefficients

Matter Parity

Define $\mathbb{Z}_2$ matter parity in terms of the t_i (i.e. as subgroup of $SU(5)_\perp$):

$$P_M = (-1)^{c_i t_i} \; , \quad c_i = 0, 1 \qquad (\text{defined} \mod 2)$$

- Monodromy $t_1 \leftrightarrow t_2$ requires $c_1 = c_2 = 1$ so $\mathbf{10}_{\text{top}}$ is odd
- Up-type masses always allowed once gauge invariant
- Down-type masses give constraint:

	$\bar{\mathbf{5}}_{H_d}$	$\bar{\mathbf{5}}_M$	$\mathbf{10}_M$
charge	$t_i + t_j$	$t_k + t_l$	t_m

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charge	$t_i + t_j$	$t_k + t_l$	t_m
$c_i t_i$	$0/2$	1	1

  Gauge invariant iff all t_i distinct – can only be matter parity even if even number of $c_i = 1$ (singlets have charge $t_i - t_j$, so don't change the argument)
- Note: W^1 operator has same charge structure

Two Possibilities

Hence, two possible definitions of matter parity:

Case I: $P_M = (-1)^{t_1+t_2+t_3+t_4}$

Case II: $P_M = (-1)^{t_1+t_2}$

Now analyse matter, Higgs and VEV assignment for both cases: $\mathbf{10}_{\text{top}}$ and $\mathbf{5}_{H_u}$ already fixed, need to distribute remaining matter and $\overline{\mathbf{5}}_{H_d}$ according to their matter parity

Main restriction: Forbid W^1, but allow down-type Yukawas

Case I: Matter and VEV Assignment

	Matter **10** Curves		
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	
$\mathbf{10}_3$	t_4	$-$	
	Matter **5** Curves		
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	
	Even Singlet Curves		
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	
$\mathbf{1}_7$	$t_1 - t_2$	$+$	

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$\mathbf{10}_1$	$t_{1,2}$	$-$	top
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$\mathbf{10}_3$	t_4	$-$	
	Matter **5** Curves		
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$\mathbf{5}_5$	$-t_3 - t_5$	$-$	
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	
	Even Singlet Curves		
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	
$\mathbf{1}_7$	$t_1 - t_2$	$+$	

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

Case I: Matter and VEV Assignment

Matter **10** Curves			
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	no matter
$\mathbf{10}_3$	t_4	$-$	matter
Matter **5** Curves			
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	matter
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	no matter
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	matter
Even Singlet Curves			
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	
$\mathbf{1}_7$	$t_1 - t_2$	$+$	

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

$\rightsquigarrow$ no matter on $\mathbf{10}_2$, $\mathbf{5}_5$

Case I: Matter and VEV Assignment

Matter **10** Curves			
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	no matter
$\mathbf{10}_3$	t_4	$-$	matter
Matter 5 Curves			
$\mathbf{5}_3$	$-t_{1,2} - t_5$	$-$	matter
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	no matter
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	matter
Even Singlet Curves			
$\mathbf{1}_1$	$\pm(t_{1,2} - t_3)$	$+$	
$\mathbf{1}_2$	$\pm(t_{1,2} - t_4)$	$+$	
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	
$\mathbf{1}_7$	$t_1 - t_2$	$+$	

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$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

$\rightsquigarrow$ no matter on $\mathbf{10}_2$, $\mathbf{5}_5$

- W^1 with singlets:

e.g. $\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_6\mathbf{1}_4\,,$

$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_3\mathbf{1}_1$

Case I: Matter and VEV Assignment

Matter **10** Curves			
$\mathbf{10}_1$	$t_{1,2}$	$-$	top
$\mathbf{10}_2$	t_3	$-$	no matter
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$\mathbf{5}_3$	$-t_{1,2}-t_5$	$-$	matter
$\mathbf{5}_5$	$-t_3-t_5$	$-$	no matter
$\mathbf{5}_6$	$-t_4-t_5$	$-$	matter
Even Singlet Curves			
$\mathbf{1}_1$	$\pm(t_{1,2}-t_3)$	$+$	no VEV
$\mathbf{1}_2$	$\pm(t_{1,2}-t_4)$	$+$	VEV
$\mathbf{1}_4$	$\pm(t_3-t_4)$	$+$	no VEV
$\mathbf{1}_7$	t_1-t_2	$+$	VEV

- W^1 without singlets:

$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\bar{\mathbf{5}}_6\,,$$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_5\,,$$
$$\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\bar{\mathbf{5}}_3$$

$\rightsquigarrow$ no matter on $\mathbf{10}_2$, $\mathbf{5}_5$

- W^1 with singlets:

e.g. $\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_6\mathbf{1}_4\,,$
$$\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\bar{\mathbf{5}}_3\mathbf{1}_1$$

$\rightsquigarrow$ no VEVs for $\mathbf{1}_1$, $\mathbf{1}_4$ (because of t_3)

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	
$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	
$\bar{\mathbf{5}}_2$	$-t_{1,2} - t_4$	
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	
$\bar{\mathbf{5}}_2$	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	either rank-two Yukawa matrix, or no up-type masses with singlets
$\bar{\mathbf{5}}_2$	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
$\bar{\mathbf{5}}_{H_u}$	$-t_1 - t_2$	No masses at tree level or with singlets μ term
$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	either rank-two Yukawa matrix, or no up-type masses with singlets
$\bar{\mathbf{5}}_2$	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)
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- String-scale μ term for both Higgses on one curve

Case I: Down-Type Higgs

Higgs-like **5** Curves		Down-type Yukawas
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$\bar{\mathbf{5}}_1$	$-t_{1,2} - t_3$	either rank-two Yukawa matrix, or no up-type masses with singlets
$\bar{\mathbf{5}}_2$	$-t_{1,2} - t_4$	No masses at tree level or with singlets
$\bar{\mathbf{5}}_4$	$-t_3 - t_4$	Rank-one Yukawa matrix, bottom quark heavy

- Down-type Higgs needs a factor of t_3 to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two
- String-scale μ term for both Higgses on one curve
- $\bar{\mathbf{5}}_4 = \bar{\mathbf{5}}_{H_d}$ is unique choice, tree-level coupling $\bar{\mathbf{5}}_{H_d} \mathbf{10}_{\text{top}} \bar{\mathbf{5}}_3$

Case I: Yukawas and CKM

- Example Assignment: Third generation on $\mathbf{10}_1$ and $\overline{\mathbf{5}}_3$, light generations on $\mathbf{10}_3$ and $\overline{\mathbf{5}}_6$
- Higgses: $\overline{\mathbf{5}}_{H_u}$ and $\overline{\mathbf{5}}_4$, only $\langle \mathbf{1}_2 \rangle \sim \epsilon$ required at first order
- Ignore $\mathbf{1}_7$ and $\mathcal{O}(1)$ coefficients
- Yukawa matrices (schematically):

$$Y^u \sim Y^d \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}$$

- CKM matrix:

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}$$

- Masses and mixings possible though not a great fit
- Degeneracy because three generations come from two curves

Case II

$$P_M = (-1)^{t_1+t_2}$$

$\rightsquigarrow$ split t's into $t_{\text{odd}} = \{t_1, t_2\}$ and $t_{\text{even}} = \{t_3, t_4, t_5\}$

- Symmetric setup, possible monodromy acting on t_{even}
- $\mathbf{10}_{\text{top}}$ is the unique matter $\mathbf{10}$ curve
- Down-type Higgs unique (up to relabeling)
- Matter-parity even singlets do not mix t_{odd} and t_{even}
- W^1 operator cannot be generated: Charge $4t_{odd} + t_{even}$ cannot be compensated by matter-parity even singlets
- Three possible matter $\overline{\mathbf{5}}$ curves (charges $t_{\text{odd}} - t_{\text{even}}$): model building choice
- Different choices of singlet VEVs possible, achieve masses and mixing

Local Model Summary

- Already locally, rather constrained model: Only two possible definitions of matter parity at the point of E_8
- In both cases, assignments of matter and Higgses is unique or strongly constrained
- Restrictions mainly from forbidding W^1 while allowing for down-type masses
- W^3 operator (neutrino masses) is not generated in any case
- Masses for all matter fields and CKM mixing possible
- Involves choices of zero modes and VEVs by hand – these cannot be calculated in the local framework

Contents

Semilocal Approach

[Friedman, Morgan, Witten; Donagi, Wijnholt]

Now *semilocal* picture: Consider GUT surface S, using spectral cover approach

Main aim: Find homology classes of matter curves which allow to find the flux restrictions and thus the zero mode spectrum.

Two types of fluxes:

- $U(1) \subset SU(5)_{\perp}$ fluxes on matter curves (from the transverse branes): Determines (chiral) 4D zero modes for full GUT multiplets (by index theorem). These are still free parameters up to anomaly cancellation requirements.
- Hypercharge flux on S (globally trivial so hypercharge stays unbroken): Restrictions to matter curves splits $SU(5)$ multiplets, but not independently: homological relations between matter curves lead to relations between the splittings.

Spectral Cover

Spectral cover: Hypersurface in projective threefold

$$\mathbb{P}(K_S \oplus \mathcal{O}_S)$$

with homogeneous coordinates $U : V$ given by spectral equation for Φ. Because of $\mathbb{Z}_2$ monodromy, spectral equation must factorise:

$$\begin{aligned} 0 &= b_0 U^5 + b_2 U^3 V^2 + b_3 U^2 V^3 + b_4 U V^4 + b_5 V^5 \\ &= \left(a_1 V^2 + a_2 UV + a_3 U^2\right)\left(a_4 V + a_7 U\right)\left(a_5 V + a_8 U\right)\left(a_6 V + a_9 U\right) \end{aligned}$$

b_k are sections in line bundles with Chern classes $\eta - kc_1 = (6-k)\, c_1(S) + c_1\left(N_{S/X}\right)$. This determines the bundles for the a_m, and in turn for the matter curves, in terms of three unspecified line bundles $\chi_{7,8,9}$.

Involves particular solution of $b_1 = 0$ constraint – might not be most general one?

Fluxes and Zero Modes

$U(1)$ fluxes: Given by integers $M_{\mathbf{5}}$, $M_{\mathbf{10}}$. Free up to consistency conditions [Dudas, Palti; Marsano]

$$\sum M_{\mathbf{10}} + \sum M_{\mathbf{5}} = 0\,, \qquad M_{\mathbf{10}_1} = -\left(M_{\mathbf{5}_1} + M_{\mathbf{5}_2} + M_{\mathbf{5}_3}\right)$$

Hypercharge flux must be globally trivial, hence

$$0 = F_Y \cdot c_1 = F_Y \cdot \eta \quad \rightsquigarrow \quad \sum_{\mathbf{5}} F_Y = \sum_{\mathbf{10}} F_Y = 0$$

Restrictions to matter curves given by $N_Y = F_Y \cdot$ (homology class). For curve with flux numbers M and N_Y, zero modes given by

$$\mathbf{10} \rightarrow \begin{cases} (\mathbf{3},\mathbf{2}) : M_{\mathbf{10}} \\ (\overline{\mathbf{3}},\mathbf{1}) : M_{\mathbf{10}} - N_Y \\ (\mathbf{1},\mathbf{1}) : M_{\mathbf{10}} + N_Y \end{cases}$$

$$\mathbf{5} \rightarrow \begin{cases} (\mathbf{3},\mathbf{1}) : M_{\mathbf{5}} \\ (\mathbf{1},\mathbf{2}) : M_{\mathbf{5}} + N_Y \end{cases}$$

Upshot

10 Curves

	M	N_Y
$\mathbf{10}_1$	$-(M_{\mathbf{5}_1}+M_{\mathbf{5}_2}+M_{\mathbf{5}_3})$	$-\widetilde{N}$
$\mathbf{10}_2$	$M_{\mathbf{10}_2}$	N_7
$\mathbf{10}_3$	$M_{\mathbf{10}_3}$	N_8
$\mathbf{10}_4$	$M_{\mathbf{10}_4}$	N_9

5 Curves

	M	N_Y
$\mathbf{5}_{H_u}$	$M_{\mathbf{5}_{H_u}}$	$\widetilde{N}$
$\mathbf{5}_1$	$M_{\mathbf{5}_1}$	$-\widetilde{N}$
$\mathbf{5}_2$	$M_{\mathbf{5}_2}$	$-\widetilde{N}$
$\mathbf{5}_3$	$M_{\mathbf{5}_3}$	$-\widetilde{N}$
$\mathbf{5}_4$	$M_{\mathbf{5}_4}$	N_7+N_8
$\mathbf{5}_5$	$M_{\mathbf{5}_5}$	N_7+N_9
$\mathbf{5}_6$	$M_{\mathbf{5}_6}$	N_8+N_9

- Three free parameters $N_{7,8,9}$ for the hypercharge flux, corresponding to three unspecified line bundles
- $\widetilde{N}=N_7+N_8+N_9$
- Split some **5** curves $\Rightarrow$ split some **10** curves [Marsano et al.; Dudas, Palti]

Case I: Matter Sector is Fine

- Doublet-triplet splitting for Higgses ($\widetilde{N} \neq 0$) inevitably splits $\mathbf{10}_{\text{top}}$ and at least one more **10** curve
- However, splitting of matter multiplets is OK as long as there are three generations of zero modes in the end, i.e. other **10** curve must have "opposite" split
- Matter on $\mathbf{10}_1$, $\mathbf{10}_3$, $\mathbf{5}_3$ and $\mathbf{5}_6$, so to have full net generations, we require

$$N_7 = N_9 = 0 \quad \Rightarrow \quad \text{only } N_8 \text{ left free}$$

- No exotics from **10**'s and remaining matter-like **5** curve can be satisfied by choosing appropriate M's

↷ Satisfactory matter sector can be engineered easily

Case I: Higgs Sector is not Fine

- Higgs sector:

	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$
$\mathbf{5}_{H_u}$	$M_{\mathbf{5}_{H_u}}$	$M_{\mathbf{5}_{H_u}} + N_8$
$\mathbf{5}_1$	$M_{\mathbf{5}_1}$	$M_{\mathbf{5}_1} - N_8$
$\mathbf{5}_2$	$M_{\mathbf{5}_2}$	$M_{\mathbf{5}_2} - N_8$
$\mathbf{5}_4$	$M_{\mathbf{5}_4}$	$M_{\mathbf{5}_4} + N_8$

- We can pairwise decouple unwanted triplets from $\mathbf{5}_{H_u}$ and $\mathbf{5}_2$, and from $\mathbf{5}_1$ and $\mathbf{5}_4$ by coupling to VEV for $\mathbf{1}_2$
- However:

$$\#(\text{doublets from } \mathbf{5}_{H_u}, \mathbf{5}_2) = \#(\text{triplets from } \mathbf{5}_{H_u}, \mathbf{5}_2)$$

- Problem persists even when allowing exotics from the matter sector
- Separately, down-type Higgs on $\mathbf{5}_4$ cannot be realised

Case II: Again not Fine

- Only one matter $\mathbf{10}$ curve, split with same parameter as up-type Higgs
- Higgs triplets cannot be decoupled by even matter parity singlets
- No exotic matter $\Rightarrow$ no doublet-triplet splitting
- Ignoring matter sector: Still cannot achieve pair of Higgs doublets

Case II: Again not Fine

- Only one matter **10** curve, split with same parameter as up-type Higgs
- Higgs triplets cannot be decoupled by even matter parity singlets
- No exotic matter $\Rightarrow$ no doublet-triplet splitting
- Ignoring matter sector: Still cannot achieve pair of Higgs doublets

Upshot: In both cases, proper doublet-triplet splitting in Higgs sector does not work, even when allowing for exotics from the matter sector – both models cannot be realised already in semilocal setup!

Contents

Conclusions

- Analysed F-Theory GUT at "point of E_8" and in semilocal approach
- Goal: Find a locally defined matter parity to ensure proton stability
- Local model is already very constrained: Two cases only
- In semilocal embedding, doublet-triplet splitting cannot be realised in either case

$\rightsquigarrow$ Models fail first step towards realisation

Conclusions

- Analysed F-Theory GUT at "point of E_8" and in semilocal approach
- Goal: Find a locally defined matter parity to ensure proton stability
- Local model is already very constrained: Two cases only
- In semilocal embedding, doublet-triplet splitting cannot be realised in either case

$\rightsquigarrow$ Models fail first step towards realisation

- Predictivity of local point in question – Crucial model features required to have nonlocal origin?
- Possible loopholes:
  - Assumed GUT breaking by hypercharge flux – different breaking mechanisms?
  - Non-diagonal Higgs fields ("T-Branes", "Gluing Morphisms") might help to get rid of exotics [Cecotti et al.; Donagi, Wijnholt]