String Geometries for Heterotic MSSM models

Fabian Rühle

Bethe Center for Theoretical Physics
Universität Bonn

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Outline

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2. $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold
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4. Conclusion
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Motivation

- String theory requires 10 spacetime dimensions
- We only perceive 4 dimensions $\Rightarrow$ compactify 6 dimensions
- **Compactification geometry** influences 4d **particle content**
- **Orbifold** compacitifications of heterotic string have proven to yield quasi-realistic models
- MSSM-like models constructed for
  - $\mathbb{Z}_{6-II}$ orbifold

    [Lebedev,Nilles,Raby,Ramos-Sanchez,Ratz,Vaudrevange,Wingerter,2006-2008]
  - $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_{2,\text{free}}$

    [Blaszczyk,Groot Nibbelink,Ratz,FR,Trapletti,Vaudrevange,2010]
- **Advantage**: **Exact CFT** calculations possible

Motivation

- Other possibility: Compactification on **CY** (more generic)
- **Disadvantage**: Calculations cumbersome, rely on SUGRA approximation

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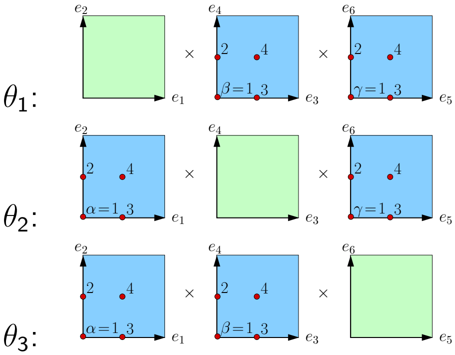
- Transition from Orbifolds to CYs via blowup:
  - CY **more generic** compactification
  - CY is **non-singular**
  - Blowup can **decouple exotics**, **break** down **gauge group**, ...
  - Blowup **needed** to **cancel FI from anomalous** $U(1)$ on orbifold

Question

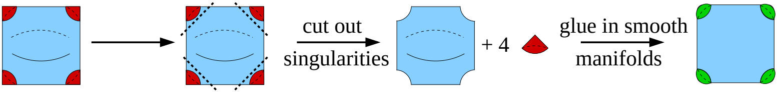
How are physical quantities on the Orbifold and the CY related?

$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_{2,\text{free}}$

- Compactify 6 dimensions on 3 tori
- Orbifold acts as reflection



Blowing up orbifolds



- **Blowup** generated by **giving vevs** to orbifold fields
- Blowup breaks all GGs (including $U(1)$s) under which blowup modes are charged
- Singularieties replaced by **smooth hypersurfaces** (exceptional cycles)
- Additional **Kähler moduli b_r parameterize the geometry** of these cycles
- Imaginary part of complexified Kähler parameter give **axions** β_r
- These model-dependent axions **cancel** $U(1)$ anomalies in blowup [Groot Nibbelink,Held,FR,Trapletti,Vaudrevange, 2009]

Blowing up $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_{2,\text{free}}$ orbifolds

- On the **Orbifold**, we have **local GUTs** ($SU(5), SO(10)$)
- Breaking GUTs by **$U(1)_Y$ flux** $\Rightarrow$ **Stückelberg mass** for $U(1)_Y$ **via GS**
- Break GUT differently $\Rightarrow$ **Freely acting involution $\mathbb{Z}_{2,\text{free}}$**
- Blowup of $\mathbb{Z}_2 \times \mathbb{Z}_2$ with **6 SM generations** constructed

Heterotic model with Abelian fluxes

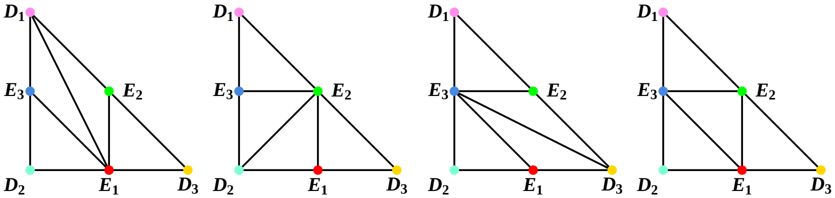
CY models with 3 (6 before $\mathbb{Z}_{2,\text{free}}$) models constructed!

[Blaszczyk,Groot Nibbelink,FR,Trapletti,Vaudrevange,2010]

Toric diagram

To describe **blowup**:

- **Replace** 48 fixed **tori** by exceptional **divisors**
- **Choose triangulation** at 64 fixed tori intersections
- Problem: $\mathcal{O}(10^{33})$ choices

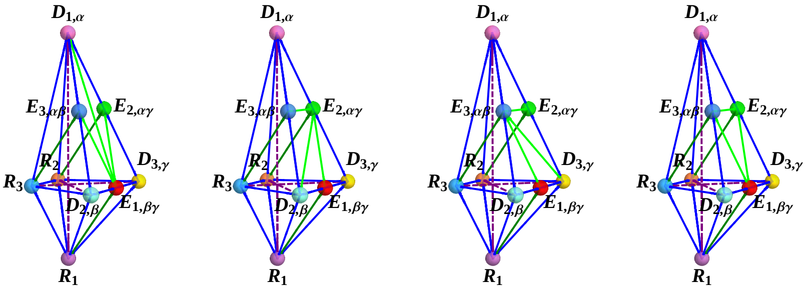


Auxiliary Polyhedra

Add additional divisors to glue together patches

Expand **Kähler form**: $J = a_i R_i - b_r E_r$

$$Vol(\mathcal{C}) = \int_{\mathcal{C}} J, \quad Vol(E_r) = \int_{E_r} J \wedge J, \quad Vol(X) = \int_X J \wedge J \wedge J$$



$b_1 > b_2 + b_3$ $\quad$ $b_2 > b_1 + b_3$ $\quad$ $b_3 > b_1 + b_2$ $\quad$ $b_i < b_j + b_k$

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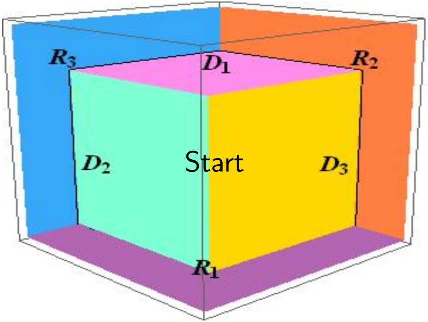

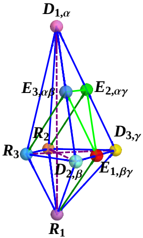

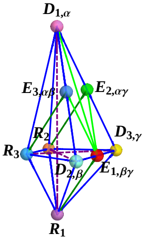

Blowup procedure

1. **Gauge flux** on CY given by **Abelian** line bundles:
   $\mathcal{F} = V_r E_r^I H_I$
2. **Line bundle** vectors V_r correspond to **orbifold shifted momenta** P_r
3. **Gauge flux** has to satisfy the **Bianchi identities**
   $\int \text{tr}\mathcal{F}^2 - \text{tr}\mathcal{R}^2 = 0$
4. Use **index theorem** to compute CY **spectrum**

Comparing Orbifold and Blowup spectrum

Blowup changes massless spectrum by higgsing states massive:
$\Phi_1 \Phi_2 s_{\text{BU}} \to \Phi_1 \Phi_2 \langle s_{\text{BU}} \rangle$

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$$\Phi^r_{\text{BU}} = e^{2\pi(b_r+i\beta_r)}\Phi^r_{\text{Orb}}, \quad \Phi^r_{\text{BU}} \rightarrow e^{2\pi i\lambda_I(V^I_r+P^I_r)}\Phi^r_{\text{Orb}}$$

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Consider simple model:

$$V_1 = (0, \frac{1}{2}, -\frac{1}{2}, 0^5)(0^8), \quad V_2 = (-\frac{1}{2}, 0, \frac{1}{2}, 0^5)(0^8)$$

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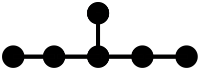

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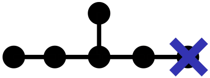

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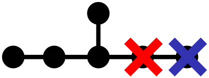

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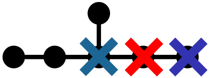

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States in Blowup

Name	Orb. mult.	Resolution multiplicity			
		E_1	E_2	E_3	S
Φ_1	(16, 0, 0)	16	-48	16	16
Φ_2	(0, 0,16)	16	-48	16	16
Φ_3	(16, 0, 0)	16	16	-48	16
Φ_4	(0,16, 0)	16	16	-48	16
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$$W_{\text{tot}} \supset \Phi_1 \Phi_2 e^{-2\pi(b_1 - b_2 + b_3)} + \bar{\Phi}_1 \bar{\Phi}_2 e^{-2\pi(-b_1 + b_2 - b_3)}$$

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Non-perturbative corrections

Mass terms arise from non-perturbative contributions

Conclusion

Phases of the CY Theory

Flop transitions via change of Kähler parameters

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Extra states

Higgsing $\Rightarrow$ states rendered massive in Blowup
Non-perturbative corrections $\Rightarrow$ states massive or massless in Blowup