Green–Schwarz Mechanism in Heterotic (2,0) GLSMs

Torsion and NS5 Branes

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Based on: [Blaszczyk,Groot Nibbelink,FR: 1107.0320]

(see also: [Quigley,Sethi: 1107.0714]) and **Sethi's talk**

Motivation

String theory promising candidate for unified description of **fundamental forces**.

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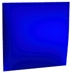
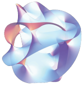
Approaches in $E_8 \times E_8$ heterotic string theory:

- **Orbifold** model building [Blaszczyk, Buchmüller, Groot Nibbelink, Hamaguchi, Kim, Kyae, Lebedev, Nilles, Raby, Ramos–Sanchez, Ratz, FR, Trapletti, Vaudrevange, Wingerter, . . .]
- **Calabi–Yau** model building [Anderson, Bouchard, Braun, Donagi, Gray, He, Lukas, Ovrut, Palti, Pantev, Waldram, . . .]
- **Free fermionic** constructions [Faraggi, Nanopoulos, Yuan, . . .]
- **Gepner Models** [Dijkstra, Gato–Rivera, Huiszoon, Schellekens, . . .]

Motivation – Compactification Geometries

 Orbifold	 Calabi–Yau
singular, non–generic	smooth, generic

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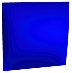
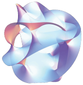
 Orbifold	 Calabi–Yau
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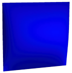
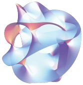
 Orbifold	 Calabi–Yau
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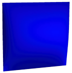
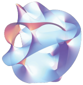
 Orbifold	 Calabi–Yau
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exact CFT calculations possible	only SUGRA approximation
Anomaly drives model away from Orbifold point	Phenomenology requires torsion and/or NS5 branes

Motivation – Problems

Problem 1

Evidence for **purely stringy constraints** that are **only seen** in **exact CFT** caluclation on the **orbifold** and **NOT** on **CY**

[Blaszczyk,Groot Nibbelink,FR,Trapletti,Vaudrevange]

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Need a framework capable of **describing** both **departure** from the **orbifold** and **torsion**

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[Blaszczyk,Groot Nibbelink,FR,Trapletti,Vaudrevange]

Problem 2

Need a framework capable of describing both departure from the orbifold and torsion

Suggestion

Use Gauged Linear Sigma Models

Outline

Definition of GLSM

Consider **2D SUSY** with **Abelian** gauge groups and field content:

superfield type	superfield notation	charge	bosonic DOF	fermionic DOF
chiral	Ψ^a	$(q_I)^a$	z^a	ψ^a
chiral–Fermi	Λ^α	$(Q_I)^\alpha$	h^α	λ^α
gauge	$(V, A)^I$	0	$a^I_\sigma, a^I_{\bar\sigma}$	Φ^I
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Geometry:

D–Term:

$$V_D = \left[(q_I)^a|z^a|^2 + (q_I)^m|x^m|^2 - \xi_I\right]^2, \qquad \xi_I:\ \text{FI–parameter}$$

F–Term:

$$W_{\text{geom}} = \Gamma^\mu P_\mu(\Psi) \Rightarrow P_\mu(\Psi) = 0$$

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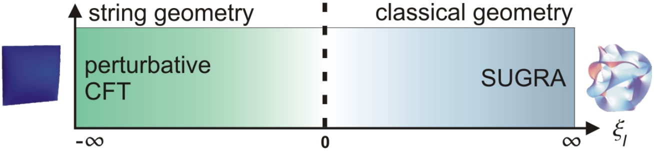

Geometry

Geometry described by polynomial equations in z^a and ξ_I.

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Gauge group:

Fermionic Transformation:

$\delta_\Theta \Lambda^\alpha = M^\alpha{}_i(\Psi)\Theta_i$

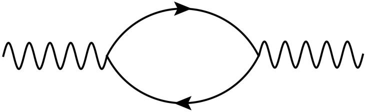
F–Term:

$W_{\text{bundle}} = \Phi^m N_{m\alpha}(\Psi)\Lambda^\alpha \Rightarrow N_{m\alpha}(\Psi)\Lambda^\alpha = 0$

Gauge group

Gauge group and particle content given by (naturally arising) monad construction via $\ker(N)/\mathrm{im}(M)$.

Anomalies



$$\mathcal{A}_{IJ} := q_I \cdot q_J - Q_I \cdot Q_J \, ,$$

$$q_I \cdot q_J := \sum_a (q_I)^a (q_J)^a + \sum_m (q_I)^m (q_J)^m \, ,$$

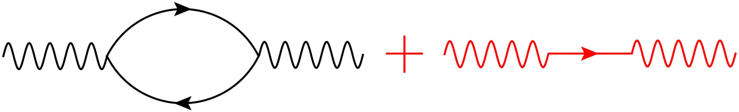
$$Q_I \cdot Q_J := \sum_\alpha (Q_I)^\alpha (Q_J)^\alpha + \sum_\mu (Q_I)^\mu (Q_J)^\mu \, .$$

Problem

In general **many** $U(1)$ gauge groups

$\Rightarrow$ **Huge amount** of stringent **anomaly conditions**.

Anomalies



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Idea

Introduce **new fields** to obtain **Green–Schwarz mechanism** on the world–sheet to **cancel gauge anomalies**. [Adams,Ernebjerg,Lapan]

Green–Schwarz mechanism

Green–Schwarz mechanism needs fields that transform with shifts.

Our approach

$\Rightarrow$ Use **logarithm** of **coordinate fields** Ψ

$$W_{\text{FI}} = \left[\rho_I^0 + T_{XI} \ln \left| R^X(\Psi) \right| \right] F^I \quad \Rightarrow \quad \mathcal{T}_{IJ} = r_I^X T_{XI}$$

$$\mathcal{A}_{IJ} = q_I \cdot q_J - Q_I \cdot Q_J + \mathcal{T}_{IJ}$$

with

- ρ_I^0: constant FI parameter
- $R^X(\Psi)$: homogeneous polynomials w/ charges r_I^X
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Consequences

$$H = dB + \omega_{\mathrm{L}} - \omega_{\mathrm{YM}}$$

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Relationship orbifold ↔ CY

BIs on CY ↔ Modular invariance on orbifold.

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Relationship orbifold $\leftrightarrow$ CY

BIs on **CY** $\leftrightarrow$ **Modular invariance** on **orbifold**.

NS5 and anti–NS5 branes, torsion

- $\mathrm{tr}\mathcal{R}^2 < \mathrm{tr}\mathcal{F}^2 \quad \Rightarrow$ **NS5** and **torsion**
- $\mathrm{tr}\mathcal{R}^2 > \mathrm{tr}\mathcal{F}^2 \quad \Rightarrow$ **anti–NS5** and **torsion**
- $[\mathrm{tr}\mathcal{R}^2] = [\mathrm{tr}\mathcal{F}^2] \quad \Rightarrow$ **torison**

Example

1.) No anomalies

superfield	$\Psi^{a=1,\ldots,8}$	$\Gamma^{\mu=1,\ldots,4}$	$\Lambda^{\alpha=1,\ldots,8}$	$\Phi^{m=1,\ldots,4}$
lowest component	z^a	γ^μ	λ^α	x^m
gauge charge	1	-2	1	-2

$\mathbb{P}^7[2,2,2,2]$ with $SU(3)$ bundle

$$\mathcal{A}_{11} = \frac{1}{2}\left[q_1^2 - Q_1^2 + \mathcal{T}_{11}\right]$$

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$\mathcal{T}_{11} = T = 0$

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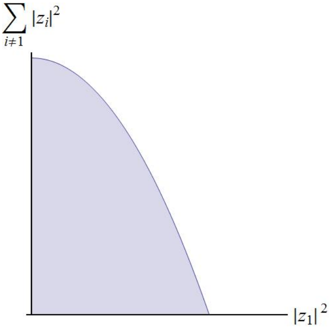

$\xi = 1$

$$\xi > 0 \;\Rightarrow\; |x^a| = 0\,, \quad V_D \stackrel{!}{=} 0 \;\Rightarrow\; \sum_{a=1}^{8} |z^a|^2 = \xi$$

Geometry **compact**, no anomalies.

2.) $T > 0$, compact geometry, effective curve

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$$\mathcal{T}_{11} = T = 2$$

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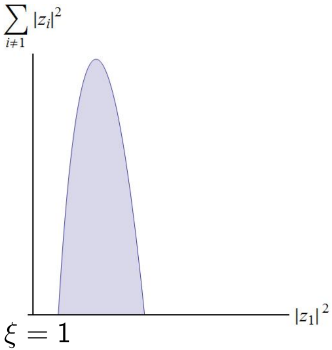

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Geometry still **compact**, anomalies canceled by **NS5 branes**.

3.) $T < 0$, decompactified geometry, non–effective curve

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$$\mathcal{T}_{11} = T = -8$$

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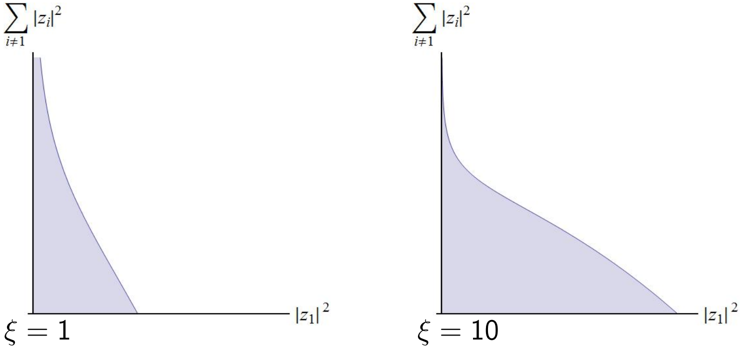

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Geometry **decompactified**, anomalies canceled with **anti–NS5 branes**.

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Thank you for your attention!