Matching of Heterotic Orbifold and Blowup Theories via Anomalies

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bctp

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bcgs
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Based on [Blaszczyk, Cabo Bizet, Nilles, FR: 1108.0667]

Outline

1. Orbifold and CY Model Building
2. Blowup procedure
   - Blowup procedure
   - $\mathbb{Z}_7$ Orbifold + resolution
3. Spectrum matching
4. Anomaly matching

Part I

Recap: Orbifold and CY Model Building

Motivation – Heterotic Model Building

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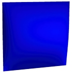
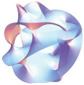
Approaches in $E_8 \times E_8$ heterotic string theory:

- **Orbifold** model building [Blaszczyk, Buchmüller, Groot Nibbelink, Hamaguchi, Kim, Kyae, Lebedev, Nilles, Raby, Ramos–Sanchez, Ratz, FR, Trapletti, Vaudrevange, Wingerter, . . .]
- **Calabi–Yau** model building [Anderson, Bouchard, Braun, Donagi, Gray, He, Lukas, Ovrut, Palti, Pantev, Waldram, . . .]
- **Free fermionic** constructions [Faraggi, Nanopoulos, Yuan, . . .]
- **Gepner Models** [Dijkstra, Gato–Rivera, Huiszoon, Schellekens, . . .]

Motivation – Compactification Geometries

 Orbifold	 Calabi–Yau
singular, non–generic	smooth, generic

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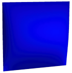
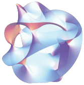
	Orbifold		Calabi–Yau
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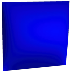
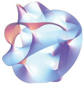
 Orbifold	 Calabi–Yau
singular, non–generic	smooth, generic
simple	complicated
exact CFT calculations possible	only SUGRA approximation

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Berechenbarkeit

Evidence for purely stringy constraints only seen in exact CFT calculation, NOT on CY [Blaszczyk,Groot Nibbelink,FR,Trapletti,Vaudrevange]

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Enhanced symmetry

Orbifold point of **enhanced symmetry**: **good** for **pheno**, but might **miss** generic **features** [Blaszczyk,Groot Nibbelink,FR,Trapletti,Vaudrevange]

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Anomaly on orbifold drives you away from orbifold point to CY

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Model building approach

Start on Orbifold (Berechenbarkeit) and carry it over to CY (generality) via blowup.
BUT: Ensure that the Orbifold and CY model match!

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Anomalies can be canceled by axions. Axions provide a Stückelberg mass for the gauge boson [Green,Schwarz]

$\Rightarrow$ Anomaly canceled, gauge group broken

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- One calls such a symmetry **anomalous symmetry**

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Definition:

- Definition **Orbifold**: $\mathbb{O} = T^6/\mathbb{Z}_N, \quad T^6 = \mathbb{C}^3/\Lambda$
- Model specified via
  - **Twist** vector v: $\mathbb{Z}_N$ Orbifold action on $\mathbb{C}^3$
  - **Shift** vector V: Embedding of Orbifold action in gauge sector
  - **Wilson Lines** W: Constant gauge background

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Properties:

- Central **consistency** requirements: **Modular Invariance** conditions (ensure absence of anomalies)
- At most **one anomalous** $U(1)_A$
- **Green–Schwarz mechanism** ensures cancelation of anomaly
- Conditions for unbroken SUSY: D– and F–terms

CY Model building

Definition:

- Definition **CY**: Ricci–flat Kähler manifold
- Model specified via
  - **Geometry**: Usually given in terms of (intersection of) **hypersurfaces/divisors** in weighted projective spaces
  - **Gauge Group**: Stable **vector bundle**

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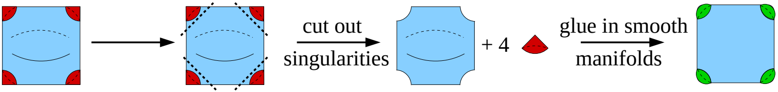
Properties:

- Central **consistency requirements**: **Bianchi Identities** (ensure absence of anomalies)
- **Several anomalous $U(1)$s** possible
- **Green–Schwarz mechanism** ensures cancelation of anomalies
- Conditions for unbroken SUSY: Donaldson–Uhlenbeck–Yau equations

Part II

Blowup procedure

Blowing up orbifolds



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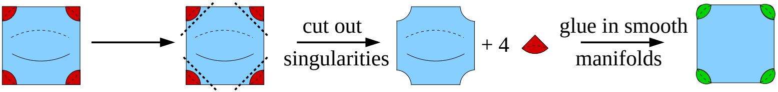

- **Blowup** generated by giving **VEVs** to **orbifold fields**
- Blowup breaks all GGs (including $U(1)$s) under which blowup modes are charged
- **Singularities** replaced by smooth **hypersurfaces** (exceptional divisors)
- Additional **Kähler moduli** parameterize the **size** of these cycles
- **Imaginary part** of complexified Kähler parameter give **axions**
- These model-dependent **axions cancel** $U(1)$ **anomalies** in blowup [Blumenhagen,Honecker,Weigand] [Groot Nibbelink,Trapletti,Nilles] [Blaszczyk,Cabo Bizet,Nilles,FR]

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Theories can be **ambiguous**

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Nevertheless, **complications** can be **avoided**:

- **Orbifold**:
  - No fixed tori $\Rightarrow$ **No brother** models
- **CY**:
  - Unique triangulation $\Rightarrow$ **No flop** transitions
  - Only compact divisors $\Rightarrow$ BIs decouple + solved locally

$\mathbb{Z}_7$ Orbifold

$\mathbb{O} = T^6/\mathbb{Z}_7, \quad T^6 = \mathbb{C}^3/\Lambda_{SU(7)}$

Model specified via [Casas,de la Maccora,Mondragon,Munoz]

- **Twist** vector $v = \frac{1}{7}(1, 2, -3)$: 3 twisted sectors, 7 FP
- **Shift** vector
  $V = \frac{1}{7}(0, 0, -1, -1, -1, 5, -2, 6)(-1, -1, 0, 0, 0, 0, 0, 0)$
- **Wilson Line**
  $W = \frac{1}{7}(-1, -1, -1, -1, -1, -10, 2, -9)(4, 3, -3, 0, 0, 0, 0, 0)$

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Spectrum

GG: $[SU(3) \times SU(2)]_{\text{vis}} \times [SO(10)]_{\text{hidden}} \times U(1)^8$

(3,2,1)	(3,1,1)	($\bar{3}$,1,1)	(1,2,1)	(1,1,10)	(1,1,1)
3	12	18	21	1	133

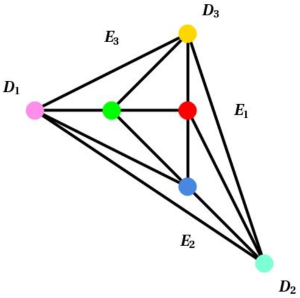
$\mathbb{Z}_7$ Blowup Procedure

Resolve $T^6/\mathbb{Z}_7$ FP by gluing in local $\mathbb{C}^3/\mathbb{Z}_7$ **resolutions**

[Lüst,Reffert,Scheidegger,Stieberger]

Relevant **divisors**: R_a, $a = 1, 2, 3$, $E_{k,\sigma}$, $k = 1, 2, 4$, $\sigma = 1, \ldots, 7$

Get topological data (intersection numbers, Chern classes,. . .) from toric diagram



$\mathbb{Z}_7$ Calabi–Yau

Gauge bundle: choose $U(1)$ line bundle

$$\mathcal{F} = E_{k,\sigma} V^I_{k,\sigma} H_I$$

Properties:

- $U(1)$ **bundles** automatically **stable**
- $V_{k,\sigma}$ only charged under $U(1)^8$, not under non–Abelian groups
- Expand in $E_{k,\sigma}$ only $\Rightarrow$ **gauge flux** vanishes in blowdown
- Spectrum calculable via **Index Theorem** (much easier than bundle **cohomology**)
- $\mathcal{F}$ solves all BIs $\Rightarrow$ anomaly free
- $\mathcal{F}$ solves all DUY equations $\Rightarrow$ SUSY intact for arbitrarily large volumes

Part III

Spectrum Comparison

Spectra on Orbifold and CY

Calculation of **Spectrum**

- **Orbifold**: Construct all $P_{\mathbf{Sh}}$ fulfilling masslessness condition, projection, level matching
- **CY**: Apply Atiyah–Singer index theorem to 480 root vectors of $\Lambda_{E_8 \times E_8}$

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BU	3	10	16	17	1	86

Origin of **differences**:

- Particles **massive** in **blowup**: $\mathcal{W}\supset\Phi^{\text{BU–Mode}}_{k,\sigma}\,\Phi^{\text{Orb}}_{k,\sigma\gamma}\,\Phi^{\text{Orb}}_{k,\sigma\gamma'}$
- $\text{rk}(\mathcal{F})=8\Rightarrow U(1)^8$ broken completely, rest unbroken

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- VEV of Orbifold states $\Leftrightarrow$ Size of blowup cycles

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Blowup modes on **Orbifold** $\mapsto$ **Kähler modulus** + **local axion**

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Twisted states redefined as

$$\Phi_{\sigma,\gamma}^{\text{BU–State}} = e^{-\sum_k \kappa_{k,\sigma}(b_{k,\sigma} + i\beta_{k,\sigma})} \, \Phi^{\text{Orb–State}}$$

$$Q_{\sigma,\gamma}^{\text{BU}} = Q_{k,\sigma}^{\text{Orb}} + \sum_k \kappa_{k,\sigma} V_{k,\sigma}, \quad Q_{\sigma,\gamma}^{\text{BU}} \in \Lambda_{E_8 \times E_8}, \quad Q_{k,\sigma}^{\text{Orb}} = P_{\text{Sh}}$$

Local Multiplicities

Global Multiplicity Operator: $N = \frac{1}{6}\int_X \mathcal{F}^3 - \frac{1}{4}\mathrm{tr}\mathcal{R}^2\mathcal{F}$

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Example:

Name	N	$N(1)$	$N(2)$	$N(3)$	$N(4)$	$N(5)$	$N(6)$	$N(7)$
$\lambda_{(3,2,1)}$	1	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
Q_1	1	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

- only 1 **Orbifold** state redefined to $E_8 \times E_8$ vector $\lambda_{(3,2,1)}$
- state distributed over all FPs $\Rightarrow$ **untwisted** state

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Q_2	1	1	0	0	0	0	0	0
$\lambda'_{(3,2,1)}$	1	1	$-\frac{1}{7}$	$-\frac{1}{7}$	$\frac{1}{7}$	$-\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

- only 1 **Orbifold** state redefined to $E_8 \times E_8$ vector $\lambda'_{(3,2,1)}$
- state located at **FP 1**
- other contributions of $\pm\frac{1}{7}$ correspond to untwisted states that are projected out

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h_{14}	1	0	0	0	1	0	0	0
h_{20}	1	0	0	0	0	1	0	0
$\lambda_{(1,2,1)}$	1	$\frac{1}{7}$	$\frac{1}{7}$	-1	$\frac{6}{7}$	$\frac{6}{7}$	$\frac{1}{7}$	$-\frac{1}{7}$

- several **Orbifold** states (with both chiralities) redefined to same $E_8 \times E_8$ vector $\lambda_{(1,2,1)}$
- states located at **FP 3, 4, 5**

Mass Terms

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$$h_4^{\text{BU}} = e^{-b_{1,1}+b_{4,1}}\ h_4^{\text{Orb}}\ , \qquad h_{17}^{\text{BU}} = e^{b_{1,1}+b_{2,1}-b_{4,1}}\ h_{17}^{\text{Orb}}$$

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Mass term in blowup

$$h_4^{\text{BU}} h_{17}^{\text{BU}} = e^{-b_{1,1}+b_{4,1}+b_{1,1}-b_{4,1}+b_{2,1}} h_4^{\text{Orb}} h_{17}^{\text{Orb}} = \Phi_{2,1}^{\text{BU-Mode}}\ h_4^{\text{Orb}} h_{17}^{\text{Orb}}$$

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- Local mass term generated via VEV of blowup mode $\langle\Phi_{2,1}^{\mathrm{BU-Mode}}\rangle \neq 0$
- $b_{2,1} \to \infty$: term massive, $\quad b_{2,1} \to -\infty$: zero mass
- Non–local (instantonic) mass terms NOT seen in blowup

Local R–Symmetry

Observation

- Mass terms via redefinition at work in most cases
- **BUT**: Redefinition sometimes not unique

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$\mathbb{C}^3/\mathbb{Z}_7$ **Orbifold** has locally $U(1)^3_R$ **R–symmetry** $z_i \to e^{i\alpha} z_i$

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Local R–Symmetry

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without R–symmetry:

$$\mathcal{W} \supset \begin{pmatrix} s_{111} & s_{112} & s_{113} \end{pmatrix} \begin{pmatrix} a_{11}\Phi^{\mathrm{BM}}_{2,4} & a_{12}\Phi^{\mathrm{BM}}_{2,4} & a_{13}\Phi^{\mathrm{BM}}_{4,4} \\ a_{21}\Phi^{\mathrm{BM}}_{2,4} & a_{22}\Phi^{\mathrm{BM}}_{2,4} & a_{23}\Phi^{\mathrm{BM}}_{4,4} \\ a_{31}\Phi^{\mathrm{BM}}_{2,4} & a_{32}\Phi^{\mathrm{BM}}_{2,4} & a_{33}\Phi^{\mathrm{BM}}_{4,4} \end{pmatrix} \begin{pmatrix} s_{25} \\ s_{26} \\ s_{70} \end{pmatrix}$$

$\Rightarrow$ **6** singlets massive

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with R–symmetry:

$$\mathcal{W} \supset (s_{111}\;\; s_{112}\;\; s_{113}) \begin{pmatrix} 0 & 0 & 0 \\ a_{21}\Phi^{\text{BM}}_{2,4} & 0 & a_{23}\Phi^{\text{BM}}_{4,4} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} s_{25} \\ s_{26} \\ s_{70} \end{pmatrix}$$

$\Rightarrow$ 2 singlets massive, 4 singlets massless

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Result

We see 4 singlets $\Rightarrow$ **R–symmetry** violation **suppressed** by **volume**
R–symmetry+**consistency** with spectrum $\Rightarrow$ Redefinition **unique**

Mass Terms – Summary

Mass Terms I – Local mass terms

- Local mass terms between states at same FP induced via VEVs
- Kähler parameters govern size of mass

Mass Terms II – Non–local mass terms

- Non–local, instantonic mass terms between states at different FPs not seen in blowup
- Mass term suppressed as $e^{-\text{vol}(\mathcal{C})}$

Mass Terms III – R–parity protected states

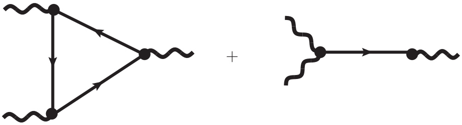
- States protected by local R–Symmetry massless in blowup
- R–symmetry broken non–locally by lattice
- Effect again suppressed by volume

Part IV

Anomalies

Anomaly Polynomial

$$I_6 = \frac{1}{(2\pi)^6}\int_X \Big\{ \tfrac{1}{6}\left(\mathrm{tr}[\mathcal{F}_1 F_1]\right)^2 + \tfrac{1}{4}\left(\mathrm{tr}\mathcal{F}_1^2 - \tfrac{1}{2}\mathrm{tr}\mathcal{R}^2\right)\mathrm{tr}F_1^2 - \tfrac{1}{16}\left(\mathrm{tr}\mathcal{F}_1^2 - \tfrac{5}{12}\mathrm{tr}\mathcal{R}^2\right)\mathrm{tr}R^2 \Big\}\mathrm{tr}[\mathcal{F}_1 F_1] + (1 \to 2)$$



General remarks:

- $\mathcal{F}, \mathcal{R}$: internal (6D), F, R : external (4D)
- $\mathcal{F}$ Abelian: $\mathcal{F} = E_r V_r^I H_I$
- $\mathcal{F} = \mathcal{F}_1 \oplus \mathcal{F}_2 \in E_8 \otimes E_8$

Anomaly Polynomial

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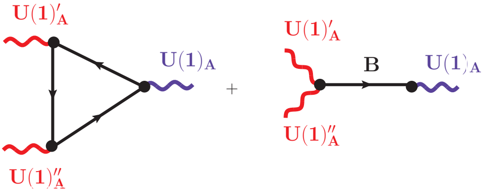

1st term:

- $\mathrm{tr}[\mathcal{F}F]$ projects onto Abelian part of F
- Generically $\mathrm{tr}[\mathcal{F}F] = 0 \Leftrightarrow \mathcal{F} \perp F \Leftrightarrow I_6 = 0$
- Anomalies: $U(1)_A \times U(1)'_A \times U(1)''_A,\ \ U(1)_A^2 \times U(1)'_A,\ \ U(1)_A^3$

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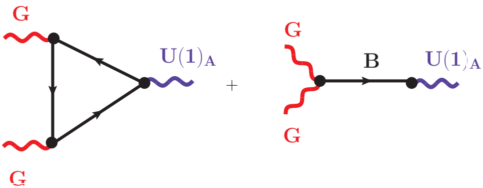

2nd term:

- $\mathrm{tr}[\mathcal{F}F]$ projects onto Abelian part of F with $\mathcal{F} \not\perp F$
- From $\mathrm{tr}F^2$, we get Abelian and non-Abelian anomalies
- Anomalies: $U(1)_A \times \mathrm{G} \times \mathrm{G}$, $\mathrm{G} = U(1), U(1)_A, SU(N), SO(N), \ldots$

Anomaly Polynomial

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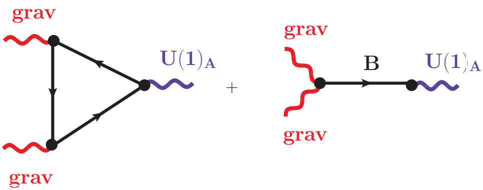

3rd term:

- $\mathrm{tr}[\mathcal{F}F]$ projects onto Abelian part of F with $\mathcal{F} \not\perp F$
- From $\mathrm{tr}R^2$, we get gravity anomalies
- Anomalies: $U(1)_A \times \text{grav} \times \text{grav}$

Calculation of Anomalies

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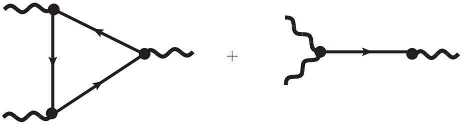

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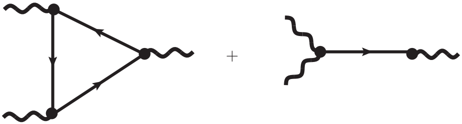

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Result

Both results agree ⇒ **strong** consistency check on the **spectrum**

Anomaly Matching

Orbifold perspective

One anomalous $U(1)$ + one axion a^{Orb} to cancel it via $a^{\text{Orb}} X_4^{\text{Orb}}$

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Contribution to anomaly from redefinition $I_6^{\text{BU}} = I_6^{\text{Orb}} + I_6^{\text{red}}$
I_6^{red} accounted for by blowup modes: $\sum_{k,\sigma} \tau_{k,\sigma} X_4^{\text{red}}$

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Calabi–Yau perspective

Start with 10D anomaly polynomial + integrate out internal space

$$I_{12} = X_4 X_8 = X_{2,6} X_{4,0} + X_{2,2} X_{4,4}$$
$$I_6^{\text{BU}} = \int_X I_{12} = X_2^{\text{uni}} X_4^{\text{uni}} + \sum_{k,\sigma} X_2^{k,\sigma} X_4^{k,\sigma}$$

Anomaly canceled by universal and local contributions

$$a^{\text{uni}} X_4^{\text{uni}} + \sum_{k,\sigma} \beta^{k,\sigma} X_4^{k,\sigma}$$

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Relate orbifold blowup to CY anomaly

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- The **blowup modes** indeed provide the local **axions** to cancel the anomalies in **blowup**
- The universal **axion** on the **orbifold** receives contribution from the **blowup modes**

Conclusion

- **Blowup** procedure
  - VEV of **blowup** field $\hat{=}$ size of **blowup** cycle
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Thank you for your attention!