GLSM Description of Heterotic Compactification Spaces

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Based on: [Blaszczyk,Groot Nibbelink,FR: 1111.5852]

Motivation

String theory promising candidate for unified description of **fundamental forces**.

- **Orbifold** model building **successful** in producing quasi–realistic models [Blaszczyk, Buchmüller, Groot Nibbelink, Hamaguchi, Kim, Kyae, Lebedev, Nilles, Raby, Ramos–Sanchez, Ratz, FR, Trapletti, Vaudrevange, Wingerter, . . .]
- But: **Target space dynamics** (anomalies) and **phenomenology** (breaking of GG, decoupling of exotics) **requires vevs** for twisted states ⇒ **SUGRA approximation** on resolution space [Blaszczyk, Cabo Bizet, Groot Nibbelink, Ha, Held, Honecker, Klevers, Nilles, Plöger, FR, Trapletti, Vaudrevange, Walter, . . .]
- Effort to match the theories
  - On the level of **anomalies** [Blaszczyk, Cabo Bizet, Groot Nibbelink, Nilles, FR, Trapletti]
  - On the level of **GLSMs** [Groot Nibbelink]

Outline

1. GLSMs and Orbifolds
2. Example $T^6/\mathbb{Z}_3$: GLSM resolution
3. Exploring the moduli space

Toroidal Orbifold

- Underlying compactification spaces are (products of) tori
- **Complex + Kähler structure** s.t. space has **additional symmetries**
- **Dividing out** these **symmetries** produces a space with **singularities** at the **fixed points**

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Gauged linear sigma models

- Here: $\mathcal{N} = (2,2)$ gauge theory with GG $U(1)^N$ on 2D WS
- The fields on the WS correspond to coordinates in TS
- The $U(1)$ charges correspond to weights of toric spaces
- The F–terms cut out the target space manifold (here: CY)
- The D–terms specify the geometric phase of the theory
- The FI–parameters correspond to Kähler moduli

Description of GLSMs and orbifolds

Construction procedure of the two approaches

Free CFT	GLSM
Start with $T^2 \times T^2 \times T^2$	Start with ell. curves $\subset \mathbb{CP} \times \mathbb{CP} \times \mathbb{CP}$

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Divide out $\mathbb{Z}_N$ orbifold action	Introduce extra fields + $U(1)$ gaugings. **Vev** of fields generate $\mathbb{Z}_N$

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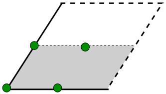
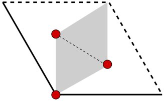
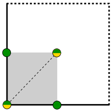
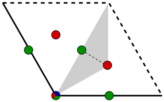
GG and **spectrum** via **shift** embedding and **WLs**	**GG** and **spectrum** from **monad** construction

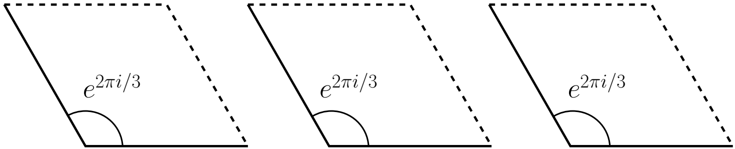
Different 2–tori

Point Group	Geometry	extra *n*–volution	projective embedding
$\mathbb{Z}_2$		$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{CP}^3_{1111}[2,2]$
$\mathbb{Z}_3$		$\mathbb{Z}_3$	$\mathbb{CP}^2_{111}[3]$
$\mathbb{Z}_4$		$\mathbb{Z}_2$	$\mathbb{CP}^2_{112}[4]$
$\mathbb{Z}_6$		—	$\mathbb{CP}^2_{123}[6]$

Part II

Example

1. Start with 3 two–tori with complex structure $\tau = e^{2\pi i/3}$



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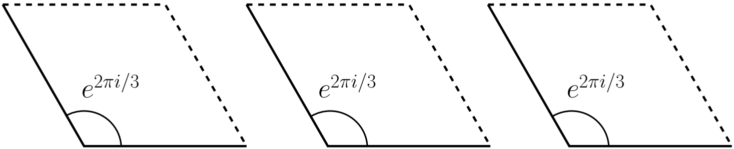
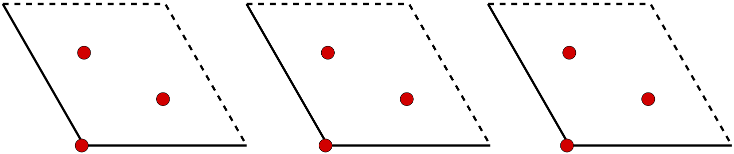


2. Divide out $\mathbb{Z}_N \Rightarrow$ 27 fixed points



$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3
R_1	1	1	1	0	0	0	0	0	0	-3	0	0
R_2	0	0	0	1	1	1	0	0	0	0	-3	0
R_3	0	0	0	0	0	0	1	1	1	0	0	-3

F–terms:

$$\sum_{\rho=1}^{3} {z_{i\rho}}^3 = 0, \quad i = 1, 2, 3\,,$$
$$c_i {z_{i\rho}}^2 = 0, \quad i, \rho = 1, 2, 3$$

D–terms:

$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|c_i|^2 = a_i, \quad i = 1, 2, 3$$

Geometry:

a_i: sizes of tori
$a_i > 0 \Rightarrow c_i = 0$ (assume this case for now)
$z_{i\rho}$: torus coordinate $u \in \mathbb{C}$: $z_{i\rho} \leftrightarrow \wp(u), \wp'(u)$

$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3	x_{111}
R_1	1	1	1	0	0	0	0	0	0	-3	0	0	**0**
R_2	0	0	0	1	1	1	0	0	0	0	-3	0	**0**
R_3	0	0	0	0	0	0	1	1	1	0	0	-3	**0**
$\boldsymbol{E_{111}}$	**1**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**-3**

F–terms:

$$z_{i1}^3 x_{111} + \sum_{\rho=2}^3 z_{i\rho}^3 = 0, \quad i = 1,2,3$$

D–terms:

$$\sum_{\rho=1}^3 |z_{i\rho}|^2 = a_i, \quad i = 1,2,3$$
$$|z_{11}|^2 + |z_{21}|^2 + |z_{31}|^2 - 3|x_{111}|^2 = b_{111}$$

Geometry:

b_{111}: sizes of exceptional cycles

$b_{111} < 0 \Rightarrow x_{111} \neq 0 \Rightarrow \langle x_{111} \rangle$ generates $\mathbb{Z}_3$ with 27 **FP** $z_{11} = z_{21} = z_{31} = 0$

$b_{111} > 0 \Rightarrow x_{111} = 0$ possible, **FP** $z_{11} = z_{21} = z_{31} = 0$ **forbidden** $\Rightarrow$ **smooth**

$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3	x_{111}	x_{211}
R_1	1	1	1	0	0	0	0	0	0	-3	0	0	**0**	**0**
R_2	0	0	0	1	1	1	0	0	0	0	-3	0	**0**	**0**
R_3	0	0	0	0	0	0	1	1	1	0	0	-3	**0**	**0**
E_{111}	**1**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**-3**	**0**
E_{211}	**0**	**1**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**-3**

F–terms:

$$z_{11}^3 x_{111} + z_{12}^3 x_{211} + z_{13}^3 = 0, \quad z_{i\rho}^3 x_{111} x_{211} + \sum_{\rho=2}^3 z_{i\rho}^3 = 0, \quad i = 2,3$$

D–terms:

$$\sum_{\rho=1}^3 |z_{i\rho}|^2 = a_i, \quad i = 1,2,3$$
$$|z_{1\alpha}|^2 + |z_{21}|^2 + |z_{31}|^2 - 3|x_{\alpha 11}|^2 = b_{\alpha 11}\,, \quad \alpha = \mathbf{1}, \mathbf{2}$$

Geometry:

$\langle x_{111}\rangle \neq 0$ generates $\mathbb{Z}_3$ with FP $z_{11}=z_{21}=z_{31}=0$ (**shielded** for $b_{111}>0$)
$\langle x_{211}\rangle \neq 0$ generates $\mathbb{Z}_3$ with FP $z_{12}=z_{21}=z_{31}=0$ (**shielded** for $b_{211}>0$)
$R_1 - E_{111} - E_{211}$ generates $\mathbb{Z}_3$ with FP $z_{13}=z_{21}=z_{31}=0$ (**NOT shielded**!)

$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3	x_{111}	x_{211}	x_{311}
R_1	1	1	1	0	0	0	0	0	0	-3	0	0	**0**	**0**	**0**
R_2	0	0	0	1	1	1	0	0	0	0	-3	0	**0**	**0**	**0**
R_3	0	0	0	0	0	0	1	1	1	0	0	-3	**0**	**0**	**0**
$\boldsymbol{E_{111}}$	**1**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**-3**	**0**	**0**
$\boldsymbol{E_{211}}$	**0**	**1**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**-3**	**0**
$\boldsymbol{E_{311}}$	**0**	**0**	**1**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**-3**

F–terms:

$$z_{11}^3 x_{111} + z_{12}^3 x_{211} + z_{13}^3 x_{311} = 0, \quad z_{i\rho}^3 x_{111} x_{211} x_{311} + \sum_{\rho=2}^3 z_{i\rho}^3 = 0, \ \ i = 2,3$$

D–terms:

$$\sum_{\rho=1}^3 |z_{i\rho}|^2 = a_i, \quad i = 1,2,3$$

$$|z_{1\alpha}|^2 + |z_{21}|^2 + |z_{31}|^2 - 3|x_{111}|^2 = b_{\alpha 11}\,, \quad \alpha = 1,2,3$$

Geometry:

$\langle x_{111} \rangle \neq 0$ generates $\mathbb{Z}_3$ with FP $z_{11} = z_{21} = z_{31} = 0$ (**shielded** for $b_{111} > 0$)
$\langle x_{211} \rangle \neq 0$ generates $\mathbb{Z}_3$ with FP $z_{12} = z_{21} = z_{31} = 0$ (**shielded** for $b_{211} > 0$)
$\langle x_{311} \rangle \neq 0$ generates $\mathbb{Z}_3$ with FP $z_{13} = z_{21} = z_{31} = 0$ (**shielded** for $b_{311} > 0$)

Summary of the procedure:

To build an orbifold/resolution **GLSM** model

1. Choose tori description appropriate for orbifold action
2. Introduce exceptional divisors to smoothen the singularities
3. Set FI parameters $a \gg 0 > b$ to study orbifold or $a \gg b > 0$ to study blowup
4. Construct inherited divisors and linear equivalences
5. Read off intersection numbers

In this way, the resolution phase can be studied using a GLSM which can be smoothly connected to the orbifold.

The procedure confirms the results of previous approaches which proceeded via polytopes and gluings. [Lust,Reffert,Scheidegger,Stieberger]

But we can **do more**! Having a **GLSM** realization, we can **probe** whole **moduli space** by simply varying **FI** parameter.

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Can answer question

What happens to a swiss cheese when the holes are bigger than the cheese?

But we can **do more**! Having a **GLSM** realization, we can **probe** whole **moduli space** by simply varying **FI** parameter.

$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3	x_{111}	x_{222}	x_{333}
R_1	1	1	1	0	0	0	0	0	0	-3	0	0	0	0	0
R_2	0	0	0	1	1	1	0	0	0	0	-3	0	0	0	0
R_3	0	0	0	0	0	0	1	1	1	0	0	-3	0	0	0
E_{111}	1	0	0	1	0	0	1	0	0	0	0	0	-3	0	0
E_{222}	0	1	0	0	1	0	0	1	0	0	0	0	0	-3	0
E_{333}	0	0	1	0	0	1	0	0	1	0	0	0	0	0	-3

Superpotential:

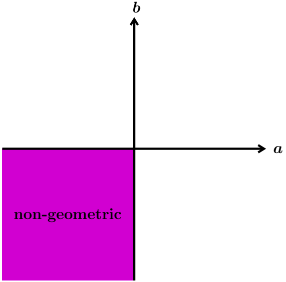
$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho} x_{\rho\rho\rho}$$

D–terms:

$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|c_i|^2 = a_i \,, \qquad i = 1,2,3$$

$$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_{\rho\rho\rho}|^2 = b_{\rho\rho\rho} \,, \qquad \rho = 1,2,3$$

For simplicity: Set $a_i = a$ and $b_{\rho\rho\rho} = b$.



Superpotential:

$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_{\rho\rho\rho}$$

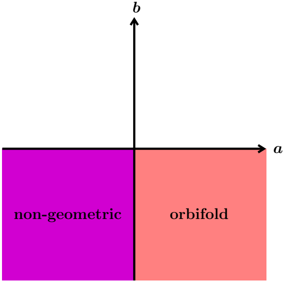
D–terms:

$$\sum_{\rho=1}^3 |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^3 |z_{i\rho}|^2 - 3|x_{\rho\rho\rho}|^2 = b$$

$a < 0,\ b < 0$:

$$\langle c_i \rangle = \frac{\sqrt{a}}{3}, \qquad \langle x_{\rho\rho\rho} \rangle = \frac{\sqrt{b}}{3}, \qquad z_{i\rho} = 0$$

Target space is a **point**.



Superpotential:

$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_{\rho\rho\rho}$$

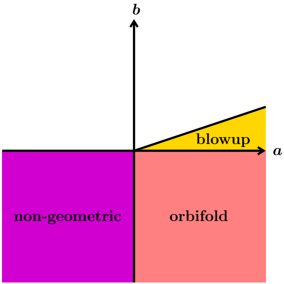
D–terms:

$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_{\rho\rho\rho}|^2 = b$$

$a > 0,\ b < 0$:

$$c_i = 0, \qquad \langle x_{\rho\rho\rho}\rangle > 0, \qquad \langle z_{i\rho}\rangle \geq 0$$

Target space is the $T^6/\mathbb{Z}_3$ orbifold.



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$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_{\rho\rho\rho}$$

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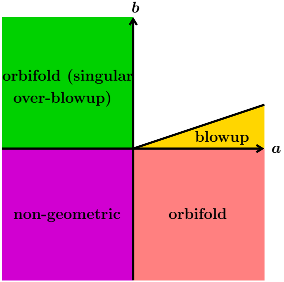
$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_{\rho\rho\rho}|^2 = b$$

$a > 3b > 0$:

$$c_i = 0, \qquad \langle x_{\rho\rho\rho} \rangle \geq 0, \qquad \langle z_{i\rho} \rangle \geq 0$$

Target space is the **resolution CY** of the $T^6/\mathbb{Z}_3$ orbifold.



Superpotential:

$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_{\rho\rho\rho}$$

D–terms:

$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_{\rho\rho\rho}|^2 = b$$

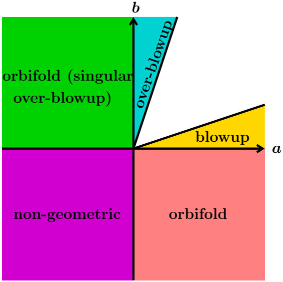
$a < 0,\ b > 0$:

Note complete symmetry of the model under

$x_{\rho\rho\rho} \leftrightarrow c_i,\ z_{i\rho} \leftrightarrow z_{\rho i},\ a_i \leftrightarrow b_{\rho\rho\rho}$

$\langle c_i \rangle > 0, \qquad x_{\rho\rho\rho} = 0, \qquad \langle z_{i\rho} \rangle \geq 0$

Target space again $T^6/\mathbb{Z}_3$ orbifold, with roles of x and c exchanged.



Superpotential:

$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_{\rho\rho\rho}$$

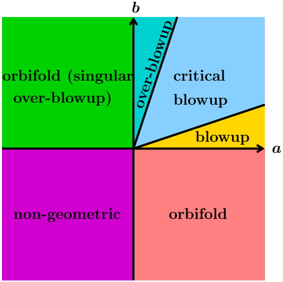
D–terms:

$$\sum_{\rho=1}^3 |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^3 |z_{i\rho}|^2 - 3|x_{\rho\rho\rho}|^2 = b$$

$b > 3a > 0$:

$\langle c_i \rangle \geq 0, \qquad x_{\rho\rho\rho} = 0, \qquad \langle z_{i\rho} \rangle \geq 0$

Target space is the resolution CY of the "other" $T^6/\mathbb{Z}_3$ orbifold.



Superpotential:

$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_{\rho\rho\rho}$$

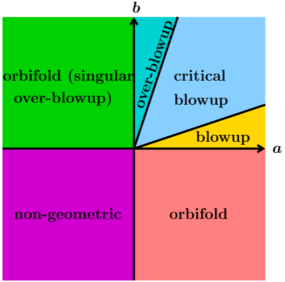
D–terms:

$$\sum_{\rho=1}^3 |z_{i\rho}|^2 - 3|c_i|^2 \quad = a$$
$$\sum_{i=1}^3 |z_{i\rho}|^2 - 3|x_{\rho\rho\rho}|^2 = b$$

$a > 0,\ b \in \left[\frac{a}{3}, 3a\right]$:

$\langle c_{i\neq\rho}\rangle \geq 0, \qquad \langle x_{\rho\rho\rho}\rangle \geq 0, \qquad \langle z_{i\rho}\rangle \geq 0$

Target space is hybrid phase with blowup limits for $b \downarrow \frac{a}{3}$ and $b \uparrow 3a$.



Superpotential:

$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_{\rho\rho\rho}$$

D–terms:

$$\sum_{\rho=1}^3 |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^3 |z_{i\rho}|^2 - 3|x_{\rho\rho\rho}|^2 = b$$

Note that in general

- The **dimension** of the **TS** **can jump** between the phases
- There can be **flop-transitions** also "outside" the **CY**
- There can be several **distinct singular phases** [Aspinwall, Greene, Morrison, Plesser, . . .]

There is a vast (moduli) space to be explored!

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Can **probe** entire **moduli space**. Access to **new phases** which exhibit **interesting phenomena**.

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Thank you for your attention!