Anomalies and Discrete Symmetries in Heterotic String Constructions

Fabian Rühle

Bethe Center for Theoretical Physics
Bonn University

Northeastern University – 07/16/2012

Based on:

[Lüdeling,FR,Wieck: 1203.5789], [Blaszczyk,Groot Nibbelink,FR: 1111.5852],

[Blaszczyk,Cabo Bizet,Nilles,FR: 1108.0667]

Motivation

The minimal supersymmetric extension of the Standard Model (**MSSM**) is phenomenologically well **motivated**

- Alleviates the **hierarchy problem**
- Allows for **gauge coupling unification**
- Provides a natural **Dark Matter** candidate
- ...

Motivation

The minimal supersymmetric extension of the Standard Model (**MSSM**) is phenomenologically well **motivated**

- Alleviates the **hierarchy problem**
- Allows for **gauge coupling unification**
- Provides a natural **Dark Matter** candidate
- ...

But it also has **problems**

- Introduces dimension four and five **proton decay** operators
- Needs explanation for smallness of **μ-term**

Motivation

The minimal supersymmetric extension of the Standard Model (**MSSM**) is phenomenologically well **motivated**

- Alleviates the **hierarchy problem**
- Allows for **gauge coupling unification**
- Provides a natural **Dark Matter** candidate
- ...

But it also has **problems**

- Introduces dimension four and five **proton decay** operators
- Needs explanation for smallness of **μ-term**

$\Rightarrow$ Introduce symmetries that suppress/forbid the terms

Motivation

The minimal supersymmetric extension of the Standard Model (**MSSM**) is phenomenologically well **motivated**

- Alleviates the **hierarchy problem**
- Allows for **gauge coupling unification**
- Provides a natural **Dark Matter** candidate
- ...

But it also has **problems**

- Introduces dimension four and five **proton decay** operators
- Needs explanation for smallness of **μ-term**

$\Rightarrow$ Introduce symmetries that suppress/forbid the terms

- Lacks **UV completion**

Motivation

The minimal supersymmetric extension of the Standard Model (**MSSM**) is phenomenologically well **motivated**

- Alleviates the **hierarchy problem**
- Allows for **gauge coupling unification**
- Provides a natural **Dark Matter** candidate
- ...

But it also has **problems**

- Introduces dimension four and five **proton decay** operators
- Needs explanation for smallness of **μ-term**

$\Rightarrow$ Introduce symmetries that suppress/forbid the terms

- Lacks **UV completion**

$\Rightarrow$ Embed in UV complete theory like String Theory

Motivation - Discrete Symmetries

In the past, many **discrete symmetries** proposed to forbid bad terms

$$\begin{aligned} W_{\text{bad}} \supset \quad & \mu H_u H_d + LH \\ & + LQd^c + Qd^c d^c + LLe^c \\ & + QQQL + u^c u^c d^c e^c + \dots \end{aligned}$$

In the past, many **discrete symmetries** proposed to forbid bad terms

$$W_{\text{bad}} \supset \mu H_u H_d + \cancel{LH} + \cancel{LQd^c} + \cancel{Qd^c d^c} + \cancel{LLe^c} + QQQL + u^c u^c d^c e^c + \ldots$$

- **$\mathbb{Z}_2$ Matter Parity** [Farrar,Fayet; Dimopoulos,Raby,Wilczek]

In the past, many **discrete symmetries** proposed to forbid bad terms

$$W_{\text{bad}} \supset \mu H_u H_d + LH + LQd^c + Qd^c d^c + LLe^c + \cancel{QQQL} + u^c \cancel{u^c d^c} e^c + \dots$$

- **$\mathbb{Z}_2$ Matter Parity** [Farrar,Fayet; Dimopoulos,Raby,Wilczek]
- **$\mathbb{Z}_3$ Baryon triality** [Ibanez,Ross]

In the past, many **discrete symmetries** proposed to forbid bad terms

$$W_{\text{bad}} \supset \mu H_u H_d + \cancel{LH} + \cancel{LQd^c} + \cancel{Qd^c d^c} + \cancel{LLe^c} + \cancel{QQQL} + u^c \cancel{u^c d^c} e^c + \dots$$

- **$\mathbb{Z}_2$ Matter Parity** [Farrar,Fayet; Dimopoulos,Raby,Wilczek]
- **$\mathbb{Z}_3$ Baryon triality** [Ibanez,Ross]
- **$\mathbb{Z}_6$ anomaly-free proton hexality**
  [Babu,Gogoladze,Wang; Dreiner,Luhn,Thormeier]

Motivation - Discrete Symmetries

In the past, many **discrete symmetries** proposed to forbid bad terms

$$\begin{aligned} W_{\text{bad}} \supset\ & \cancel{\mu H_u H_d} + \cancel{LH} \\ & + \cancel{LQd^c} + \cancel{Qd^c d^c} + \cancel{LLe^c} \\ & + \cancel{QQQL} + u^c \cancel{u^c d^c} e^c + \ldots \end{aligned}$$

- **$\mathbb{Z}_2$ Matter Parity** [Farrar,Fayet; Dimopoulos,Raby,Wilczek]
- **$\mathbb{Z}_3$ Baryon triality** [Ibanez,Ross]
- **$\mathbb{Z}_6$ anomaly-free proton hexality** [Babu,Gogoladze,Wang; Dreiner,Luhn,Thormeier]
- **$\mathbb{Z}_4^R$ anomaly-universal R-symmetry** [Lee,Raby,Ratz,Ross,Schieren,Schmidt-Hoberg,Vaudrevange]

Motivation - Discrete Symmetries

In the past, many **discrete symmetries** proposed to forbid bad terms

$$
\begin{aligned}
W_{\text{bad}} \supset \quad & \cancel{\mu H_u H_d} + \cancel{LH} \\
& + \cancel{LQd^c} + \cancel{Qd^c d^c} + \cancel{LLe^c} \\
& + \cancel{QQQL} + u^c \cancel{u^c d^c} e^c + \ldots
\end{aligned}
$$

- **$\mathbb{Z}_2$ Matter Parity** [Farrar,Fayet; Dimopoulos,Raby,Wilczek]
- **$\mathbb{Z}_3$ Baryon triality** [Ibanez,Ross]
- **$\mathbb{Z}_6$ anomaly-free proton hexality** [Babu,Gogoladze,Wang; Dreiner,Luhn,Thormeier]
- **$\mathbb{Z}_4^R$ anomaly-universal R-symmetry** [Lee,Raby,Ratz,Ross,Schieren,Schmidt-Hoberg,Vaudrevange]
- ...

Motivation - String Embedding

Much effort spent on construction of **MSSM**–like models in last decade.

Motivation - String Embedding

Much effort spent on construction of **MSSM**–like models in last decade.

Approaches in $E_8 \times E_8$ heterotic string theory:

- **Orbifold** model building [Blaszczyk, Buchmüller, Groot Nibbelink, Hamaguchi, Kim, Kyae, Lebedev, Nilles, Raby, Ramos–Sanchez, Ratz, FR, Trapletti, Vaudrevange, Wingerter, . . .]
- **Calabi–Yau** model building [Anderson, Bouchard, Braun, Donagi, Gray, He, Lukas, Ovrut, Palti, Pantev, Waldram, . . .]
- **Free fermionic** constructions [Faraggi, Nanopoulos, Yuan, . . .]
- **Gepner Models** [Dijkstra, Gato–Rivera, Huiszoon, Schellekens, . . .]

I will focus on the first two approaches.

Outline

1. Anomalies
   - Definition and Description
   - Green–Schwarz mechanism
2. Green–Schwarz mechanism in String Theory
   - Structure of anomaly polynomial
   - Introduction of Orbifolds/CYs
   - GS in orbifold compactifications
   - GS in smooth CY compactifications
3. Example: The $T^6/\mathbb{Z}_3$ orbifold and its resolution
   - Orbifold construction
   - Toric resolution of orbifold
4. Remnant discrete symmetries
   - Remnant non-R symmetries
   - Remnant R symmetries
   - Calculation of (non-) R symmetry charges
5. Conclusion

Part I

Anomalies

Definition of Anomalies

Definition of Anomaly

An anomaly is a symmetry of the classical theory which is broken by quantum effects. Gauge anomalies render theory inconsistent and have to be absent!

Properties of Anomalies

- Anomalies arise at 1-loop
- They are determined by the chiral spectrum
- Can be determined from variation of path integral measure [Fujikawa]
- Can be described in terms of anomaly polynomial [Wess,Zumino;Stora;Alvarez-Gaume,Ginsparg]

From **path integral**:

- Look at trafo $\Psi \to \Psi'$ parameterized by trafo parameter λ:

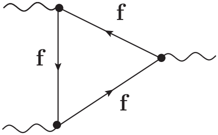
$$\int \mathcal{D}\Psi e^{iS} \to \int \mathcal{D}\Psi' J(\lambda) e^{iS}\,, \quad J(\lambda) = e^{i\mathcal{A}} = e^{i\int d^D x\, I_D}$$

- It is more convenient to work with **anomaly polynomial** I_{D+2}

$$\mathrm{d}I_D = \delta_\lambda I_{D+1} \quad \mathrm{d}I_{D+1} = I_{D+2} \qquad \text{“Descent equations”}$$

- **Anomaly form** I_D: linear in trafo parameter λ, polynomial in gauge connections and field strengths
- **Chern Simons form** I_{D+1}: poynomial in gauge connections and field strengths
- **Anomaly polynomial** I_{D+2}: closed and gauge invariant polynomial in the field strengths

From **Feynman diagram** (here $D = 4$):



- **Internal** legs: Chiral fermions f
- **External** legs: Gauge bosons / Gravity

- $\mathcal{A} = \mathcal{A}_{G-G-U(1)} + \mathcal{A}_{U(1)_A - U(1)_B - U(1)_C} + \mathcal{A}_{\text{grav}-\text{grav}-U(1)}$

$$\mathcal{A}_{G-G-U(1)} \propto \sum_{\text{f}} q_f \, \ell(r(\text{f}))$$

$$\mathcal{A}_{U(1)_A - U(1)_B - U(1)_C} \propto \sum_{\text{f}} q_f^A q_f^B q_f^C$$

$$\mathcal{A}_{\text{grav}-\text{grav}-U(1)} \propto \sum_{\text{f}} q_f$$

Calculation of **discrete** $\mathbb{Z}_N$ anomalies:

- Useful to think of $\mathbb{Z}_N \subset U(1)$
- Quadratic/cubic/mixed $U(1)$-$\mathbb{Z}_N$ anomalies ill-defined [Banks,Seiberg]
- $\Rightarrow$ Anomalies $\mathcal{A}_{G-G-\mathbb{Z}_N}$, $\mathcal{A}_{U(1)_A-U(1)_B-\mathbb{Z}_N}$, $\mathcal{A}_{\text{grav}-\text{grav}-\mathbb{Z}_N}$

$$\mathcal{A}_{G-G-\mathbb{Z}_N} \propto \left[\sum_f q_f(\mathbb{Z}_N)\,\ell(r(f))\right] \mod \eta$$

$$\mathcal{A}_{U(1)_A-U(1)_B-\mathbb{Z}_N} \propto \left[\sum_f q_f^A q_f^B q_f(\mathbb{Z}_N)\right] \mod \eta$$

$$\mathcal{A}_{\text{grav}-\text{grav}-\mathbb{Z}_N} \propto \left[\sum_f q_f(\mathbb{Z}_N)\right] \mod \eta$$

$$\eta = \begin{cases} \frac{N}{2} & \text{if } N \text{ is even} \\ N & \text{if } N \text{ is odd} \end{cases}$$

Green–Schwarz mechanism

Anomalies can be cancelled via the **Green–Schwarz** mechanism. This requires

1. **Factorization** of anomaly polynomial:
   $I_{D+2} = X_k Y_{D+2-k}$
2. **$(k-2)$-form field B_{k-2}** with gauge trafo:
   $\delta B_{k-2} = -X_{k-2}$ (descent of X_k)
3. **Coupling** to Y_{D+2-k}:
   $S_{\text{GS}} = \int \frac{1}{2} |\mathrm{d}B_{k-2} + X_{k-1}|^2 + B_{k-2} Y_{D+2-k}$

Note

Exchanging $Y_{D+2-k} \leftrightarrow X_{k-2}$ corresponds to $B_{k-2} \leftrightarrow \widetilde{B}_{D-k}$

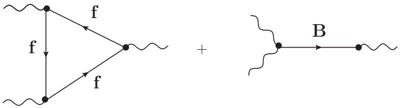
Anomalies can be cancelled via the **Green–Schwarz** mechanism. This requires

1. **Factorization** of anomaly polynomial:
   $I_{D+2} = X_k Y_{D+2-k}$
2. **$(k-2)$-form field B_{k-2}** with gauge trafo:
   $\delta B_{k-2} = -X_{k-2}$ (descent of X_k)
3. **Coupling** to Y_{D+2-k}:
   $S_{\text{GS}} = \int \frac{1}{2}|\mathrm{d}B_{k-2}+X_{k-1}|^2 + B_{k-2}Y_{D+2-k}$

Note

Exchanging $Y_{D+2-k} \leftrightarrow X_{k-2}$ corresponds to $B_{k-2} \leftrightarrow \widetilde{B}_{D-k}$

Additional contribution from B field



Green–Schwarz mechanism in 4D

GS mechanism in 4D (i.e. $D = 4, k = 2$) with 1 anomalous $U(1)_A$:

1. **Factorization**: $I_6 = X_4 Y_2$ where
   - $Y_2 = \mathrm{d}A_A$ is the field strength of $U(1)_A$
   - $X_4 = \mathcal{A}_{\text{grav}-\text{grav}-U(1)_A} \mathrm{tr} R^2 + \sum_i \mathcal{A}_{G_i - G_i - U(1)_A} \mathrm{tr} F_i^2$
2. **0-form field** (axion) a with gauge trafo:
   $\delta_\lambda a = -\lambda$
3. **Coupling** to X_4:
   $S_{\mathrm{GS}} = \int \frac{1}{2} |\mathrm{d}a + Y_1|^2 + a X_4 = \int \frac{1}{2} |\mathrm{d}a + A_A|^2 + a X_4$

Consequences

- Anomalies are **cancelled**
- Axionic coupling gives **Stückelberg mass** to $U(1)_A$
- **Axion** in 4D is dual to 2-form field B_2 (in the sense that $* H_3 = H_1$) → use later

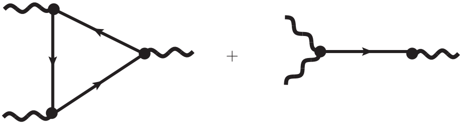
Part II

GS mechanism in String Theory

The introduction of GS axions might seem ***ad hoc***, but is automatically **implemented** in **string theory**.

- Start with 10D (heterotic) SUGRA
- Factorize **anomaly polynomial** $I_{12} = X_4^{10D}\ X_8^{10D}$
  $X_4^{10D} = \mathrm{tr}\mathfrak{R}^2 - \mathrm{tr}\mathfrak{F}_1^2 - \mathrm{tr}\mathfrak{F}_2^2$
- Cancel **anomaly** with $\delta\mathfrak{B}_2$ which is the descent of X_4^{10D}
- For dimensional reduction, decompose 10D 2-forms
  - $\mathfrak{B} \to B + \mathcal{B}$
  - $\mathfrak{R} \to R + \mathcal{R}$
  - $\mathfrak{F}_i \to F_i + \mathcal{F}_i$
- Integrate out internal space (Orbifold/smooth CY) to obtain I_6 in 4D SUGRA
- 4D Anomalies cancelled by B_2 and $\mathcal{B}_2$

$$I_6 = \frac{1}{(2\pi)^6} \int_X \Big\{ \tfrac{1}{6} \left(\mathrm{tr}[\mathcal{F}_1 F_1]\right)^2 + \tfrac{1}{4} \left(\mathrm{tr}\mathcal{F}_1{}^2 - \tfrac{1}{2}\mathrm{tr}\mathcal{R}^2\right) \mathrm{tr}F_1{}^2 - \tfrac{1}{16} \left(\mathrm{tr}\mathcal{F}_1{}^2 - \tfrac{5}{12}\mathrm{tr}\mathcal{R}^2\right) \mathrm{tr}R^2 \Big\} \mathrm{tr}[\mathcal{F}_1 F_1] + (1 \to 2)$$

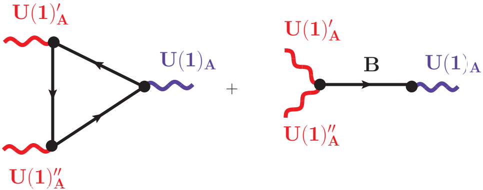


General remarks:

- $\mathcal{F}$,$\mathcal{R}$: internal (6D), F,R: external (4D)
- $\mathcal{F}$ Abelian
- $\mathcal{F}$=$\mathcal{F}_1 \oplus \mathcal{F}_2 \in E_8 \otimes E_8$

Structure of 4D anomaly polynomial

$$I_6 = \frac{1}{(2\pi)^6}\int_X \Big\{ \tfrac{1}{6}(\mathrm{tr}[\mathcal{F}_1 F_1])^2 + \tfrac{1}{4}\left(\mathrm{tr}\mathcal{F}_1^2 - \tfrac{1}{2}\mathrm{tr}\mathcal{R}^2\right)\mathrm{tr}F_1^2$$
$$-\tfrac{1}{16}\left(\mathrm{tr}\mathcal{F}_1^2 - \tfrac{5}{12}\mathrm{tr}\mathcal{R}^2\right)\mathrm{tr}R^2\Big\}\mathrm{tr}[\mathcal{F}_1 F_1] + (1 \to 2)$$

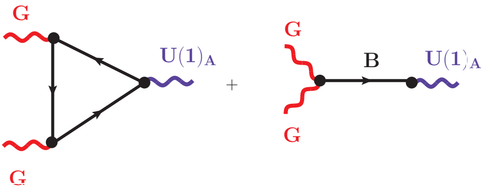


1^{st} term:

- $\mathrm{tr}[\mathcal{F}F]$ projects onto Abelian part of F with $\mathcal{F} \not\perp F$
- Generically $\mathrm{tr}[\mathcal{F}F] = 0 \Leftrightarrow \mathcal{F} \perp F \Leftrightarrow I_6 = 0$
- Anomalies: $U(1)_A \times U(1)'_A \times U(1)''_A$, $U(1)_A^2 \times U(1)'_A$, $U(1)_A^3$

Structure of 4D anomaly polynomial

$$I_6 = \frac{1}{(2\pi)^6}\int_X \Big\{ \tfrac{1}{6}\left(\mathrm{tr}[\mathcal{F}_1 F_1]\right)^2 + \tfrac{1}{4}\left(\mathrm{tr}\mathcal{F}_1^2 - \tfrac{1}{2}\mathrm{tr}\mathcal{R}^2\right) \mathrm{tr}F_1^2 - \tfrac{1}{16}\left(\mathrm{tr}\mathcal{F}_1^2 - \tfrac{5}{12}\mathrm{tr}\mathcal{R}^2\right)\mathrm{tr}R^2 \Big\} \mathrm{tr}[\mathcal{F}_1 F_1] + (1 \to 2)$$

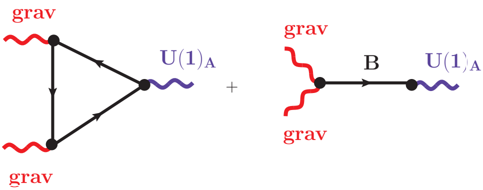


2nd term:

- $\mathrm{tr}[\mathcal{F}F]$ projects onto Abelian part of F with $\mathcal{F} \not\perp F$
- From $\mathrm{tr}F^2$, we get Abelian and non-Abelian anomalies
- Anomalies: $U(1)_A \times \mathrm{G} \times \mathrm{G}$, $\mathrm{G} = U(1), U(1)_A, SU(N), SO(N), \ldots$

Structure of 4D anomaly polynomial

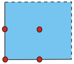
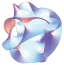
$$I_6 = \frac{1}{(2\pi)^6}\int_X \Big\{ \tfrac{1}{6}\left(\mathrm{tr}[\mathcal{F}_1 F_1]\right)^2 + \tfrac{1}{4}\left(\mathrm{tr}\mathcal{F}_1^2 - \tfrac{1}{2}\mathrm{tr}\mathcal{R}^2\right)\mathrm{tr}F_1^2 - \tfrac{1}{16}\left(\mathrm{tr}\mathcal{F}_1^2 - \tfrac{5}{12}\mathrm{tr}\mathcal{R}^2\right)\mathrm{tr}R^2 \Big\}\mathrm{tr}[\mathcal{F}_1 F_1] + (1 \to 2)$$



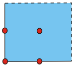
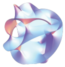
3rd term:

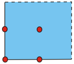
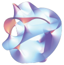
- $\mathrm{tr}[\mathcal{F}F]$ projects onto Abelian part of F with $\mathcal{F} \not\perp F$
- From $\mathrm{tr}R^2$, we get gravity anomalies
- Anomalies: $U(1)_A \times \mathrm{grav} \times \mathrm{grav}$

Heterotic Compactification Spaces

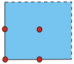
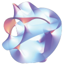
 Orbifold	 Calabi–Yau
Geometry given by **quotient** of T^6 by discrete $\mathbb{Z}_N$ group	**Geometry** given by **topological data** like divisors, Intersection numbers,. . .

Heterotic Compactification Spaces

 Orbifold	 Calabi–Yau
Geometry given by **quotient** of T^6 by discrete $\mathbb{Z}_N$ group	**Geometry** given by **topological data** like divisors, Intersection numbers,. . .
Gauge sector described by Orbifold **shift** and **Wilson lines**	**Gauge sector** describe by stable **vector bundle**

 Orbifold	 Calabi–Yau
Geometry given by **quotient** of T^6 by discrete $\mathbb{Z}_N$ group	**Geometry** given by **topological data** like divisors, Intersection numbers,. . .
Gauge sector described by Orbifold **shift** and **Wilson lines**	**Gauge sector** describe by stable **vector bundle**
Impose modular invariance for consistency	**Impose Bianchi Identities** and **DUY** equations for consistency

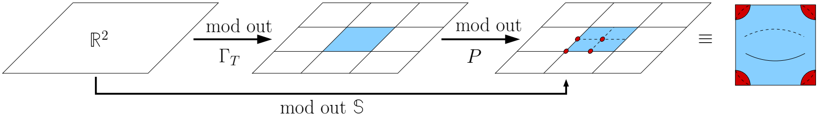
Heterotic Compactification Spaces

 Orbifold	 Calabi–Yau
Geometry given by **quotient** of T^6 by discrete $\mathbb{Z}_N$ group	**Geometry** given by **topological data** like divisors, Intersection numbers,...
Gauge sector described by Orbifold **shift** and **Wilson lines**	**Gauge sector** describe by stable **vector bundle**
Impose modular invariance for consistency	**Impose Bianchi Identities** and **DUY** equations for consistency
exact CFT calculations possible	only **SUGRA approximation**

Procedure

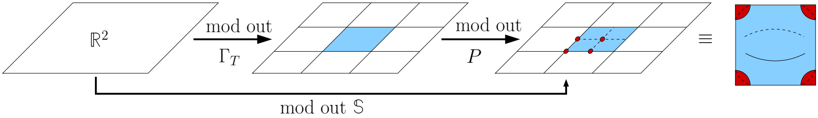
Start at **Orbifold** and extrapolate to **CY** regime

Construction mechanism:



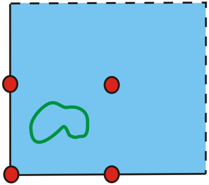
- Start with $\mathbb{R}^2$ plane
- Divide out torus lattice Γ_T
- Divide out orbifold action

Construction mechanism:



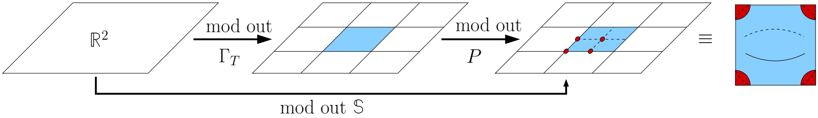
- Start with $\mathbb{R}^2$ plane
- Divide out torus lattice Γ_T
- Divide out orbifold action

Kinds of Strings:



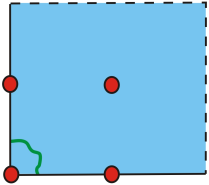
Untwisted strings

Construction mechanism:



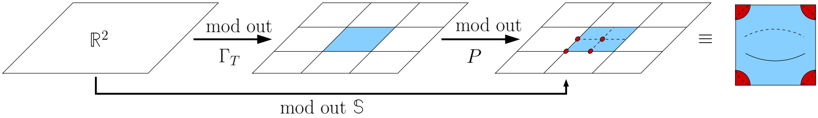
- Start with $\mathbb{R}^2$ plane
- Divide out torus lattice Γ_T
- Divide out orbifold action

Kinds of Strings:



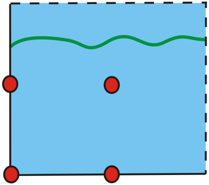
Twisted strings

Construction mechanism:



- Start with $\mathbb{R}^2$ plane
- Divide out torus lattice Γ_T
- Divide out orbifold action

Kinds of Strings:



Massive strings

Anomaly cancellation on Orbifolds:

On **orbifolds**, there is a unique Kalb-Ramond field B_2 (with dual axion a), thus

- all **anomalies** are **proportional** such that the same **axionic** coupling can cancel all at once
- at most 1 **anomalous** $U(1)$ for suitable choice of $U(1)$ basis
- **GS anomaly** cancellation ensured by **modular invariance** conditions

Anomaly universality

Coupling $a\,X_4 \to \mathcal{A}_{\text{grav-grav-}U(1)} \sim \mathcal{A}_{\text{G-G-}U(1)} \sim \mathcal{A}_{U(1)_A\text{-}U(1)_B\text{-}U(1)_C}$

Heterotic Compactification Spaces - Orbifolds

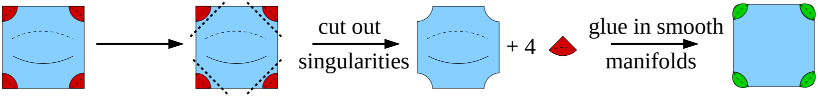
Consequences: [Lee,Raby,Ratz,Ross,Schieren,Schmidt-Hoberg,Vaudrevange]

- **Anomalies** ok as long as all are **proportional**
- There is a unique $\mathbb{Z}_4^R$ symmetry that
  - assumes family universality
  - works after doublet-triplet splitting
  - is compatible with $SO(10)$ GUT
  - forbids dim 4 and 5 proton decay operators
  - forbids the μ term
  - Realized in string theory in a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold construction
    [Blaszczyk,Groot Nibbelink,Ratz,FR,Trapletti,Vaudrevange]

Summary

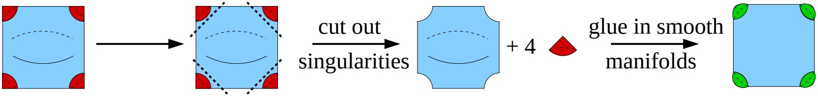
- On **orbifolds** all **anomalies** are **universal**
- very constrained choice for discrete symmetries

Construction mechanism:



- Start with **Orbifold**
- Cut out **singularitites**
- **Glue** in compact **smooth surfaces**
- Describe via **Gauged Linear Sigma Models** (GLSMs) [Witten]

Construction mechanism:



- Start with **Orbifold**
- Cut out **singularitites**
- **Glue** in compact **smooth surfaces**
- Describe via **Gauged Linear Sigma Models** (GLSMs) [Witten]

Terminology:

- **Divisors** $\hat{=}$ Codimension 1 **hypersurfaces** $\Leftrightarrow$ $(1,1)$ forms
- **Inherited Divisors** $\hat{=}$ **Torus** away from singularitites
- **Exceptional Divisors** $\hat{=}$ Smooth **surfaces glued** into the orbifold **singularitites**

Anomaly cancellation in Blowup:

In **blowup**, there are 4D **axions** arising from both B_2 and $\mathcal{B}_2$, thus:

- **anomalies** are non-universal
- as many **anomalous** $U(1)$'s as **rank of line bundle**
- **Absence** of non-Abelian **anomalies** ensured by **Bianchi identities** [Witten]

Anomaly cancellation in Blowup:

In **blowup**, there are 4D **axions** arising from both B_2 and $\mathcal{B}_2$, thus:

- **anomalies** are non-universal
- as many **anomalous** $U(1)$'s as **rank of line bundle**
- **Absence** of non-Abelian **anomalies** ensured by **Bianchi identities** [Witten]

Connection between **blowup** with line bundles and **orbifold**:

- **Blowup modes** $\leftrightarrow$ **twisted orbifold states**
- **Kähler parameters** $\leftrightarrow$ **vev** (real part) of **blowup modes**
- **Axions** in $\mathcal{B}_2$ $\leftrightarrow$ **phases** of **blowup modes**
- $E_8 \times E_8$ weights $\leftrightarrow$ **Orbifold matter** states

Using this **correspondence** and the non-triviality of the **anomaly polynomial**, the **orbifold** spectrum can be **matched** completely with the **blowup** spectrum [Nibbelink,Nilles,Trapletti;Blaszczyk,Cabo Bizet,Nilles,FR]

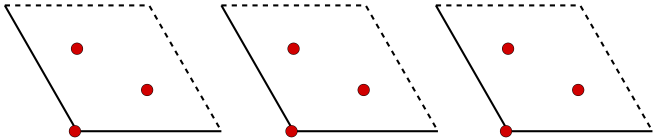
Part III

Example: The $T^6/\mathbb{Z}_3$ Orbifold and its resolution

Orbifold

Compactify on 6D Lie root lattice $SU(3)^3$ and divide out **orbifold** $\mathbb{Z}_3$ action θ:

$$\theta : (z_1, z_2, z_3) \mapsto (e^{2\pi i/3} z_1, e^{2\pi i/3} z_2, e^{-2\pi i\, 2/3} z_3)$$



- **Orbifold action** given by **twist** vector $v = \frac{1}{3}(1, 1, -2)$
- **Modular invariance** requires a **shift** V in the $E_8 \times E_8$ gauge sector s.t. $3(V^2 - v^2) = 0 \bmod 2$
- Choose **Standard embedding** $V = \frac{1}{3}(1, 1, -2, 0^5)(0^8)$ "$= v$"

Gauge group: $[E_6 \times SU(3)]_{\text{vis}} \times [E_8]_{\text{hid}}$

Matter: $3(\mathbf{27}, \overline{\mathbf{3}}; \mathbf{1}) + 27[(\mathbf{27}, \mathbf{1}; \mathbf{1}) + 3(\mathbf{1}, \mathbf{3}; \mathbf{1})]$

Consistency requirements

Want to construct **smooth CY** with line bundles from orbifold using **toric** (algebraic) **geometry**. Impose

- **Bianchi identity** (ensures absence of purely non-Abelian anomalies [Witten]):

$$H = dB + \omega_{YM} - \omega_L \to \int_{\mathcal{C}_4} dH = \int_{\mathcal{C}_4} \mathrm{tr}\mathcal{F}^2 - \mathrm{tr}\mathcal{R}^2 \stackrel{!}{=} 0$$

- **Donaldson–Uhlenbeck–Yau** (ensures 4d $\mathcal{N} = 1$ SUSY)

$$\int_X J \wedge J \wedge \mathcal{F} = 0$$

In order to solve these **equations**, need **divisors** $\mathcal{C}_4$ and their **intersection numbers**

Analytic Description of T^2

- Introduce **complex coordinate** $u \in \mathbb{C}/\Gamma_T$
- Torus described by **double-periodic function** $\wp(u)$ with $\wp(u+1) = \wp(u+\tau) = \wp(u)$

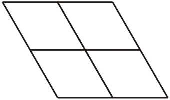
Toric description of $T^6/\mathbb{Z}^3$

Analytic Description of T^2

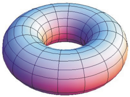
- Introduce **complex coordinate** $u \in \mathbb{C}/\Gamma_T$
- Torus described by **double-periodic function** $\wp(u)$ with $\wp(u+1) = \wp(u+\tau) = \wp(u)$

Algebraic Description of T^2

- Introduce 3 **homogeneous coordinates** z_1, z_2, z_3 in $\mathbb{P}^2$
- Impose **cubic equation** $z_1^3 + z_2^3 + z_3^3 + t\, z_1 z_2 z_3 = 0$
- Impose **condition** on **absolute values** $|z_1|^2 + |z_2|^2 + |z_3|^2 = a$
- t corresponds to CS, a to Kähler parameter of the torus



$u \in \mathbb{C}/\Gamma_T$



$z_1, z_2, z_3 \in \mathbb{C}$

One finds that for $\tau = e^{2\pi i/3} \Rightarrow t = 0$

$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}
R_1	1	1	1	0	0	0	0	0	0
R_2	0	0	0	1	1	1	0	0	0
R_3	0	0	0	0	0	0	1	1	1

$$\sum_{\rho=1}^{3} z_{i\rho}^3 = 0\,, \quad i = 1,2,3$$

$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 = a_i\,, \quad i = 1,2,3$$

- Introduce 3 times 3 z's to descibe the three T^2
- Divide by orbifold $\mathbb{Z}_3$
- Resolve fixed points by introducing 27 x's that resolve the FPs by gluing in 27 $\mathbb{P}^2$ at the singularities

[Blaszczyk,Groot Nibbelink, FR]

Resolution of $T^6/\mathbb{Z}^3$ with line bundles

$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	x_{111}	$\ldots$	x_{333}
R_1	1	1	1	0	0	0	0	0	0	0	0	0
R_2	0	0	0	1	1	1	0	0	0	0	0	0
R_3	0	0	0	0	0	0	1	1	1	0	0	0
E_{111}	1	0	0	1	0	0	1	0	0	-3	0	0
$\vdots$		$\ddots$			$\ddots$			$\ddots$			$\ddots$	
E_{333}	0	0	1	0	0	1	0	0	1	0	0	-3

$$\sum_{\rho=1}^{3} z_{1\rho}^3 \prod_{\beta,\gamma=1}^{3} x_{\rho\beta\gamma} = 0\,, \quad \sum_{\rho=1}^{3} z_{2\rho}^3 \prod_{\alpha,\gamma=1}^{3} x_{\alpha\rho\gamma} = 0\,, \quad \sum_{\rho=1}^{3} z_{3\rho}^3 \prod_{\alpha,\beta=1}^{3} x_{\alpha\beta\rho} = 0\,,$$

$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 = a_i\,, \qquad i = 1,2,3$$

$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|x_{\alpha\beta\gamma}| = b_{\alpha\beta\gamma}\,, \qquad \alpha,\beta,\gamma = 1,2,3$$

Resolution of $T^6/\mathbb{Z}^3$ with line bundles

- Introduce **exceptional divisors** $E_{\alpha\beta\gamma}$ at $x_{\alpha\beta\gamma} = 0$
- Introduce **gauge flux** $\mathcal{F} = E_{\alpha\beta\gamma} V^I_{\alpha\beta\gamma} H_I$
  - The H_I are the 16 Cartan generators of $E_8 \times E_8$
  - The 16×27 matrix $V^I_{\alpha\beta\gamma}$ describes the gauge line bundle at the 27 fixed points
- Note that in the **orbifold limit** the $E_{\alpha\beta\gamma}$ are shrunk to a point $\Rightarrow$ **flux** is located at fixed points

Resolution of $T^6/\mathbb{Z}^3$ with line bundles

- Introduce **exceptional divisors** $E_{\alpha\beta\gamma}$ at $x_{\alpha\beta\gamma} = 0$
- Introduce **gauge flux** $\mathcal{F} = E_{\alpha\beta\gamma} V^I_{\alpha\beta\gamma} H_I$
  - The H_I are the 16 Cartan generators of $E_8 \times E_8$
  - The 16×27 matrix $V^I_{\alpha\beta\gamma}$ describes the gauge line bundle at the 27 fixed points
- Note that in the **orbifold limit** the $E_{\alpha\beta\gamma}$ are shrunk to a point $\Rightarrow$ **flux** is located at fixed points

To make contact with the **orbifold** description:

- Choose the $V_{\alpha\beta\gamma}$ to coincide with the internal $E_8 \times E_8$ **momentum** of some **twisted orbifold state** located at (α, β, γ)
- **Vev** of **orbifold state** generates the **blowup** of the $E_{\alpha\beta\gamma}$

Field redefinitions:

$$\Phi_{\alpha\beta\gamma}^{\text{BU-Mode}} = e^{b_{\alpha\beta\gamma}+i\beta_{\alpha\beta\gamma}}$$

$$\Phi_{\alpha\beta\gamma}^{\text{CY}} = e^{-b_{\alpha\beta\gamma}-i\beta_{\alpha\beta\gamma}}\ \Phi_{\alpha\beta\gamma}^{\text{Orb}} \quad \Rightarrow \quad Q^{\text{CY}} = Q^{\text{Orb}} + V_{\alpha\beta\gamma}$$

Note:

- Kähler parameters $b_{\alpha\beta\gamma} \propto \text{vol}(E_{\alpha\beta\gamma})$
  - $b_{\alpha\beta\gamma} \to \infty$: **Blowup limit**
  - $b_{\alpha\beta\gamma} \ll 0$: **Orbifold limit**

  [Aspinwall,Greene,Morrison]
- **Kalb-Ramond** 2-form $\mathfrak{B}_2 = B_2 + \beta_{\alpha\beta\gamma} E_{\alpha\beta\gamma}$
- **Axions** $\beta_{\alpha\beta\gamma} \to \beta_{\alpha\beta\gamma} + \lambda_I V_{\alpha\beta\gamma}^I$
- **Gauge bundle** is sum of **line bundles**
  - Gauge group rank not reduced by **bundle**
  - $U(1)$'s in direction of **line bundle anomalous**
  - **Anomaly cancelled** by **axions** β, but $U(1)$'s massive

Choose 3 different **bundle vectors** from $(\mathbf{27},\mathbf{1})$ of $E_6 \times SU(3)$

- $V_1 = \frac{1}{3}(2,2,2,0^5)(0^8)$ at k fixed points
- $V_2 = \frac{1}{3}(-1,-1,-1,3,0^4)(0^8)$ at p fixed points
- $V_3 = -(V_1 + V_2)$ at $q \equiv 27 - p - q$ fixed points

$$\Rightarrow \mathcal{F} = \sum_{i=1}^{k} E_i V_1^I H_I + \sum_{j=k+1}^{k+p} E_j V_2^I H_I + \sum_{n=k+p+1}^{27} E_n V_3^I H_I$$

Resolution of $T^6/\mathbb{Z}^3$ with line bundles

Choose 3 different **bundle vectors** from $(\mathbf{27},\mathbf{1})$ of $E_6 \times SU(3)$

- $V_1 = \frac{1}{3}(2,2,2,0^5)(0^8)$ at k fixed points
- $V_2 = \frac{1}{3}(-1,-1,-1,3,0^4)(0^8)$ at p fixed points
- $V_3 = -(V_1 + V_2)$ at $q \equiv 27 - p - q$ fixed points

$$\Rightarrow \mathcal{F} = \sum_{i=1}^{k} E_i V_1^I H_I + \sum_{j=k+1}^{k+p} E_j V_2^I H_I + \sum_{n=k+p+1}^{27} E_n V_3^I H_I$$

Check consistency conditions:

Bianchi Identities

$$\int_{E_{\alpha\beta\gamma}} \mathrm{tr}\mathcal{F}^2 = \int_{E_{\alpha\beta\gamma}} \mathrm{tr}\mathcal{R}^2 \quad \Rightarrow \quad V_1^2 = V_2^2 = V_3^2 = \frac{4}{3}$$

Choose 3 different **bundle vectors** from $(\mathbf{27},\mathbf{1})$ of $E_6 \times SU(3)$

- $V_1 = \frac{1}{3}(2,2,2,0^5)(0^8)$ at k fixed points
- $V_2 = \frac{1}{3}(-1,-1,-1,3,0^4)(0^8)$ at p fixed points
- $V_3 = -(V_1 + V_2)$ at $q \equiv 27 - p - q$ fixed points

$$\Rightarrow \mathcal{F} = \sum_{i=1}^{k} E_i V_1^I H_I + \sum_{j=k+1}^{k+p} E_j V_2^I H_I + \sum_{n=k+p+1}^{27} E_n V_3^I H_I$$

Check consistency conditions:

Bianchi Identities

$$\int_{E_{\alpha\beta\gamma}} \mathrm{tr}\mathcal{F}^2 = \int_{E_{\alpha\beta\gamma}} \mathrm{tr}\mathcal{R}^2 \quad \Rightarrow \quad V_1^2 = V_2^2 = V_3^2 = \frac{4}{3}$$

DUY equations

$$\int J \wedge J \wedge \mathcal{F} = 0 \quad \Rightarrow \quad \sum_{\alpha\beta\gamma} V^I_{\alpha\beta\gamma} \mathrm{vol}(E_{\alpha\beta\gamma}) = 0 \ \ \forall I$$

$$\sum_{i=1}^{k} V_1 \ \mathrm{vol}(E_i) + \sum_{j=k+1}^{k+p} V_2 \ \mathrm{vol}(E_j) + \sum_{n=k+p+1}^{27} V_3 \ \mathrm{vol}(E_n) = 0$$

Resolution of $T^6/\mathbb{Z}^3$ with line bundles

The **gauge bundle** breaks $E_6 \to SO(8) \times U(1)_A \times U(1)_B$:

$\mathbf{27} \to \mathbf{8}_{s(1,-1)} + \mathbf{8}_{c(1,1)} + \mathbf{8}_{v(-2,0)} + \mathbf{1}_{(-2,-2)} + \mathbf{1}_{(-2,2)} + \mathbf{1}_{(4,0)}$

The **gauge bundle** breaks $E_6 \to SO(8) \times U(1)_A \times U(1)_B$:
$\mathbf{27} \to \mathbf{8}_{s(1,-1)} + \mathbf{8}_{c(1,1)} + \mathbf{8}_{v(-2,0)} + \mathbf{1}_{(-2,-2)} + \mathbf{1}_{(-2,2)} + \mathbf{1}_{(4,0)}$

Calculate **anomaly polynomial** $I_6 = \int_X I_{12}$ in background:

$$I_6 \sim F_A^3 \left(\frac{k-6}{12}\right) + F_A F_B^2 \left(\frac{k-18}{4}\right) + F_A \left[\mathrm{tr} F^2_{SU(3)} + \mathrm{tr} F^2_{SO(8)} + \frac{7}{48}\mathrm{tr} R^2\right] \left(\frac{k-9}{2}\right) + F_B \left[\mathrm{tr} F^2_{SU(3)} + \mathrm{tr} F^2_{SO(8)} + \frac{7}{48}\mathrm{tr} R^2 + \frac{1}{48} F_A^2 + \frac{1}{8} F_B^2\right] \left(\frac{p-q}{2}\right)$$

- $U(1)_A$ always anomalous, $U(1)_B$ non-anomalous iff $p = q$
- **Remnant anomaly universality** from **orbifold**:
  - Coefficients of **non-Abelian anomaly** from same E_8 **prop.**
  - Coefficients of of **non-Abelian** and of **grav. anomaly prop.**

Axionic shifts and massive $U(1)$'s

Axions $\beta_{\alpha\beta\gamma}$ **shift** under $U(1)_A$ and $U(1)_B$
$\Rightarrow$ In general both $U(1)$'s **massive**, even if **not anomalous**:

$$S \subset \int_X H_3 \wedge *H_3 = A^I_\mu A^\mu_I M_{IJ} + \ldots\,, \quad M_{IJ} = V^I_r V^J_s \int_X E_r *_6 E_s$$

Mass matrix M_{IJ} is positive definite, of rank 2, and depends on the Kähler parameters.

Note

Stückelberg mass possible **without** an **anomalous** $U(1)$
$\rightarrow$ rank reduction from line bundles

Part IV

Remnant discrete symmetries

Non-R symmetries arise as **discrete subgroups** of $U(1)_A$ and $U(1)_B$ which **leave vevs** of blowup modes **invariant**

$\mathbf{27} \to \mathbf{8}_{s(1,-1)} + \mathbf{8}_{c(1,1)} + \mathbf{8}_{v(-2,0)} + \mathbf{1}_{(-2,-2)} + \mathbf{1}_{(-2,2)} + \mathbf{1}_{(4,0)}$

Blowup modes:

$\mathbf{1}_{(-2,-2)}$, $\mathbf{1}_{(-2,2)}$, $\mathbf{1}_{(4,0)}$ corresponding to V_1, V_2, V_3

Leave **discrete $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry** generated by

$$T_\pm : \quad \phi_{(q_a,q_b)} \to e^{\frac{2\pi i}{2}(q_A \pm q_B)}\, \phi_{(q_A,q_B)}$$

Both **symmetries** are **non-anomalous**

Properties of R **symmetries**

- R **symmetries** do **not commute** with **SUSY**
- Grassmann coordinate θ **transforms** under R-**symmetries**
- R **symmetries** only defined up to **mixing** with **non**-R **symmetries**
- Usual choice of normalization: θ has charge 1 $\rightarrow$ Superpotential has charge 2

Remnant R symmetries

Properties of R **symmetries**

- R **symmetries** do **not commute** with **SUSY**
- Grassmann coordinate θ **transforms** under R-**symmetries**
- R **symmetries** only defined up to **mixing** with **non-R symmetries**
- Usual choice of normalization: θ has charge 1 $\rightarrow$ Superpotential has charge 2

Origin of R-symmetries

- **Lorentz symmetry** of **internal compactification space** treat bosons and fermions differently $\rightarrow$ can give rise to R **symmetries** in 4D
- **Orbifolds** are special points in moduli space of **enhanced symmetry** $\rightarrow$ expect **more R symmetries** than on generic **CY**

Remnant R symmetries – Orbifold

R-**charge** on the **orbifold** defined via a combination of **right-moving momenta** q and **oscillator numbers** ΔN:
$R = q - \Delta N$ with $q = \frac{1}{3}(1,1,1)$ [Kobayashi,Raby,Zhang]

Remnant **symmetry** of internal space: Sublattice rotations by $2\pi/3$ in each T^2:

$$T_k^R : \phi \to e^{2\pi i/3 R_k} \phi$$

Order of the **symmetry**:

- For bosons, $R_k \in \frac{1}{3}\mathbb{Z} \Rightarrow \mathbb{Z}_9$ R-**symmetry**
- For fermions, $R^{\text{f}} = R - (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$, i.e. θ has charge $\frac{1}{6} \Rightarrow \mathbb{Z}_6^R$ symmetry

Summary of conventions

Choose lcm(9,6)=18 $\Rightarrow \mathbb{Z}_{18}$ R-**symmetry** where all fields have integer charges: (bosons,fermions,θ)=$\frac{1}{18}$(2n,2n-3,3)

Remnant R symmetries – Orbifold

Our orbifold **blowup modes** have

$$R = q - \Delta N = \frac{1}{3}(1,1,1)$$

To identify remnant **R-symmetries**, search for **invariant combinations** of T_k^R with $T_{U(1)_A}$ and $T_{U(1)_B}$:

$$\mathbf{1}_{(-2,-2)} \to (T_1^R)^a (T_2^R)^b (T_3^R)^c T_{U(1)_A} T_{U(1)_B} \mathbf{1}_{(-2,-2)} \stackrel{!}{=} \mathbf{1}_{(-2,-2)}$$
$$\mathbf{1}_{(-2,2)} \to (T_1^R)^a (T_2^R)^b (T_3^R)^c T_{U(1)_A} T_{U(1)_B} \mathbf{1}_{(-2,2)} \stackrel{!}{=} \mathbf{1}_{(-2,2)}$$
$$\mathbf{1}_{(4,0)} \to (T_1^R)^a (T_2^R)^b (T_3^R)^c T_{U(1)_A} T_{U(1)_B} \mathbf{1}_{(4,0)} \stackrel{!}{=} \mathbf{1}_{(4,0)}$$

Result

One finds that $a + b + c = 3 \Rightarrow$ **only** a (trivial) $\mathbb{Z}_2$ **R-symmetry** remains in **blowup**.

Remnant R symmetries – GLSM

Look at simplified model with 3 exceptional divisors:

$$
\begin{aligned}
0 &= z_{11}^3 x_1 + z_{12}^3 x_2 + z_{13}^3 x_3 \\
0 &= z_{21}^3 x_1 x_2 x_3 + z_{22}^3 + z_{23}^3 \\
0 &= z_{31}^3 x_1 x_2 x_3 + z_{32}^3 + z_{33}^3 \\
a_i &= |z_{i1}|^2 + |z_{i2}|^2 + |z_{i3}|^2 \\
b_\alpha &= |z_{1\alpha}|^2 + |z_{21}|^2 + |z_{31}|^2 - 3|x_\alpha|^2
\end{aligned}
$$

Symmetries:

- $z_{i\alpha} \to e^{2\pi i/3} z_{i\alpha}$
- $(x_1, x_2, x_3) \to e^{2\pi i/3}(x_1, x_2, x_3)$
- ...

Origin of Symmetries

Note that the symmetries are inherited from the special choice of complex structure on the orbifold (absence of $t\, z_{i1} z_{i2} z_{i3}$ term)

Remnant R symmetries – GLSM

How to check which of these symmetries are R-symmetries?

R-**symmetries** will **transform** the holomorphic $(3,0)$ **form** Ω:

$\Omega \sim \eta\Gamma\eta dz^i dz^j dz^k \quad \Rightarrow \quad Q_R(\Omega) = Q_R(W)$ [Witten]

Remnant R symmetries – GLSM

How to check which of these symmetries are R-symmetries?

R-**symmetries** will **transform** the holomorphic $(3,0)$ **form** Ω:
$\Omega \sim \eta\Gamma\eta dz^i dz^j dz^k \quad \Rightarrow \quad Q_R(\Omega) = Q_R(W)$ [Witten]

How are the R-symmetries broken in blowup?

(Presumably) via marginal deformations in Kähler potential under the presence of the gauge bundle:

$$\int d^2\theta^+ \phi_{4D}(x^\mu) N(z,x) \Lambda\overline{\Lambda}$$

- ϕ_{4D}: 4D modes
- $N(z,x)$: Polynomial in the geometry fields $z_{i\alpha}, x_\alpha$
- Λ: WS fermions describing the gauge bundle

$N(z,x)$ might not be compatible with rotational symmetries
$\Rightarrow$ R-**symmetry broken**

Remnant R symmetries – GLSM

To **check transformation** of bundle under **discrete symmetries**:

- **Find discrete transformations** of coordinate **fields** z, x under **symmetry** in question
- Write down **gauge bundle** in ambient space
- **Restrict bundle** to toric hypersurface via Koszul sequence
- Find **contributing monomials**
- **Check transformation** of **monomials** under discrete symmetry

Tools

The last three steps should be automatized using cohomcalg [Jurke] and the Koszul extension [Rahn].

Conclusion

Discrete Symmetries extremely important for model building

- Forbid μ term
- Suppress proton decay operators

Conclusion

Discrete Symmetries extremely important for **model building**

- Forbid μ **term**
- Suppress **proton decay operators**

Discussion of **anomalies** and **GS cancellation mechanism**

- in 4D with axions arising from factorized I_6
- in 10D with factorized I_{12}

Conclusion

Discrete Symmetries extremely important for **model building**

- Forbid **μ term**
- Suppress **proton decay operators**

Discussion of **anomalies** and **GS cancellation mechanism**

- in 4D with axions arising from factorized I_6
- in 10D with factorized I_{12}

Embedding in **String Theory**:

- **Orbifold**: One universal axion $\Rightarrow$ **Anomalies universal**
- **Blowup CY**: Several axions $\Rightarrow$ **Anomalies not universal**

Conclusion

Origin of **discrete symmetries**:

- **Non-R symmetries** are discrete remnants of **higgsed $U(1)$'s**
- **R symmetries** are discrete remnants of **internal Lorentz trafos**

Calculation of **discrete symmetries**:

- **Non-R symmetries** can be calculated from **spectrum**
- **R symmetries** can be calculated from **GLSM**

Thank you for your attention!