GLSM Description of Heterotic Compactification Spaces

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Based on: [Blaszczyk,Groot Nibbelink,FR: 1111.5852]

Motivation

Popular heterotic compactification spaces for model building:

- Orbifolds [Blaszczyk, Buchmüller, Groot Nibbelink, Hamaguchi, Kim, Kyae, Lebedev, Nilles, Quevedo, Raby, Ramos–Sanchez, Ratz, FR, Trapletti, Vaudrevange, Wingerter, . . .]
- Calabi–Yaus with vector/line bundles [Anderson, Blaszczyk, Bouchard, Braun, Cabo Bizet, Donagi, Groot Nibbelink, Gray, Ha, Held, Honecker, Klevers, Lukas, Nilles, Ovrut, Palti, Pantev, Plöger, FR, Trapletti, Vaudrevange, Waldram, Walter, . . .]
- Free fermionic constructions [Faraggi, Nanopoulos, Yuan, . . .]
- Gepner Models [Dijkstra, GatoRivera, Huiszoon, Schellekens, . . .]

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Our approach:

- Start with **orbifold** where theory is **well** under **control** (**CFT** description)
- Try to **map** the theory to a **smooth CY** space via **blowup** procedure

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Effort to match the theories:

- On the level of **anomalies**
  [Blaszczyk, Cabo Bizet, Groot Nibbelink, Nilles, FR, Trapletti]
- On the level of **GLSMs** [Groot Nibbelink]

Motivated by this, we worked out in [1111.5852] the **GLSM** description of **orbifold** resolutions. To our surprise, we found **more** than just the **orbifold** and the **blowup** phase $\rightsquigarrow$ this talk

Outline

1. Orbifolds and GLSMs
2. Example $T^6/\mathbb{Z}_3$: GLSM resolution
3. Exploring the moduli space

Toroidal Orbifold [Dixon,Harvey,Vafa,Witten,. . .]

- Underlying compactification spaces are (products of) tori
- **Complex structure** s.t. space has **additional symmetries**
- **Dividing out** these **symmetries** produces a space with **singularities** at the **fixed points**

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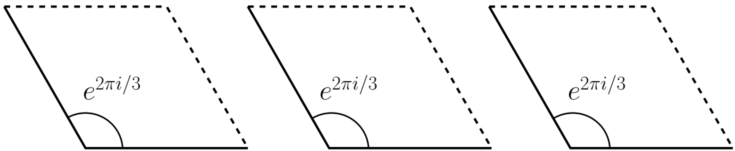
Gauged linear sigma models [Witten]

- Here: $\mathcal{N} = (2,2)$ **gauge theory** with GG $U(1)^N$ **on** 2D **WS**
- The **fields** on the **WS** correspond to **coordinates** in **TS**
- The $U(1)$ charges correspond to weights of toric spaces
- The **F–terms** cut out the **target space** manifold (here: CY)
- The **D–terms** specify the **geometric phase** of the theory
- The **FI–parameters** correspond to **Kähler moduli**

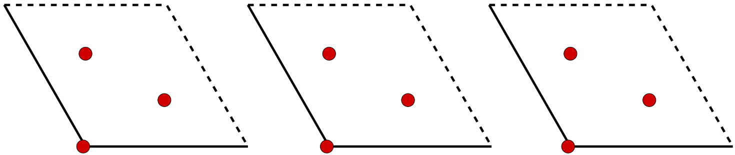
Part II

Example

1. Start with 3 two–tori with complex structure $\tau = e^{2\pi i/3}$



2. Divide out $\mathbb{Z}_3$ $\Rightarrow$ 27 fixed points



$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3
R_1	1	1	1	0	0	0	0	0	0	-3	0	0
R_2	0	0	0	1	1	1	0	0	0	0	-3	0
R_3	0	0	0	0	0	0	1	1	1	0	0	-3

F–terms for c_i and $z_{i\rho}$:

$$z_{i1}{}^3 + z_{i2}{}^3 + z_{i3}{}^3 = 0\,, \qquad i = 1,2,3\,,$$
$$c_i z_{i\rho}{}^2 = 0\,, \qquad i,\rho = 1,2,3$$

D–terms:

$$|z_{i1}|^2 + |z_{i2}|^2 + |z_{i3}|^2 - 3|c_i|^2 = a_i\,, \qquad i = 1,2,3$$

Geometry:

a_i: sizes of tori

$a_i > 0 \Rightarrow$ at least 3 $z_{i,\rho} \neq 0 \Rightarrow c_i = 0$ (assume this case for now)

$z_{i1},\ z_{i2},\ z_{i3}$: coordinate of i^{th} torus

$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3	x_1
R_1	1	1	1	0	0	0	0	0	0	-3	0	0	**0**
R_2	0	0	0	1	1	1	0	0	0	0	-3	0	**0**
R_3	0	0	0	0	0	0	1	1	1	0	0	-3	**0**
E_1	**1**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**-3**

F–terms for c_i:

$$z_{i1}^3 x_1 + z_{i2}^3 + z_{i3}^3 = 0\,, \qquad i = 1, 2, 3$$

D–terms:

$$|z_{i1}|^2 + |z_{i2}|^2 + |z_{i3}|^2 = a_i\,, \qquad i = 1, 2, 3$$

$$|z_{11}|^2 + |z_{21}|^2 + |z_{31}|^2 - 3|x_1|^2 = b_1$$

Geometry: [Aspinwall, Plesser]

b_1: sizes of exceptional cycles

$b_1 < 0 \Rightarrow x_1 \neq 0 \Rightarrow \langle x_1 \rangle$ breaks E_1 to $\mathbb{Z}_3$ with 27 **FP** $z_{11} = z_{21} = z_{31} = 0$

$b_1 > 0 \Rightarrow$ **FP** $z_{11} = z_{21} = z_{31} = 0$ **forbidden** $\Rightarrow$ **smooth**

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$$|z_{1\rho}|^2 + |z_{2\rho}|^2 + |z_{3\rho}|^2 - 3|x_\rho|^2 = b_\rho\,, \qquad \rho = 1,2,3$$

Geometry:

$$\left.\begin{array}{l} \langle x_1\rangle \neq 0 \text{ generates } \mathbb{Z}_3 \text{ with FP } z_{11}=z_{21}=z_{31}=0 \\ \langle x_2\rangle \neq 0 \text{ generates } \mathbb{Z}_3 \text{ with FP } z_{12}=z_{22}=z_{32}=0 \\ \langle x_3\rangle \neq 0 \text{ generates } \mathbb{Z}_3 \text{ with FP } z_{13}=z_{23}=z_{33}=0 \end{array}\right\} 3 \times 9 \text{ fixed points}$$

General procedure:

To build an orbifold/resolution **GLSM** model

1. Choose toric description appropriate for orbifold action
2. Introduce exceptional divisors to smoothen the singularities
3. Set FI parameters $a \gg 0 > b$ to study orbifold or $a \gg b > 0$ to study blowup
4. Construct inherited divisors and linear equivalences
5. Read off intersection numbers

In this way, the resolution phase can be studied using a GLSM which can be smoothly connected to the orbifold.

The procedure confirms the results of previous approaches which proceeded via polytopes and gluings. [Lust,Reffert,Scheidegger,Stieberger]

Probing the Moduli Space

But we can **do more**! Having a **GLSM** realization, we can **probe** whole **moduli space** by simply varying **FI** parameter.

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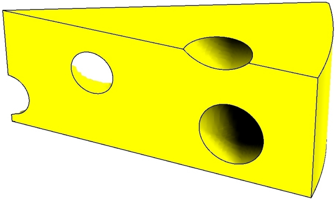

Can answer question

What happens to a **swiss cheese** when the **holes** are **bigger** than the **cheese**?

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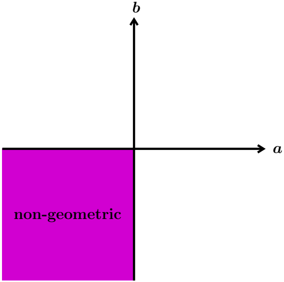
Superpotential:
$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$

D–terms:
$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|c_i|^2 = a_i\,, \qquad i = 1, 2, 3$
$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_\rho|^2 = b_\rho\,, \qquad \rho = 1, 2, 3$

For simplicity: Set $a_i = a$ and $b_\rho = b$.

Phase I: Non-Geometric Regime



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$a < 0, \ b < 0$:

$$\langle c_i \rangle = \frac{\sqrt{a}}{3}, \qquad \langle x_\rho \rangle = \frac{\sqrt{b}}{3}, \qquad z_{i\rho} = 0$$

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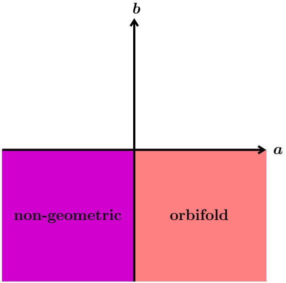
Target space is a **point**.

Phase II: Orbifold



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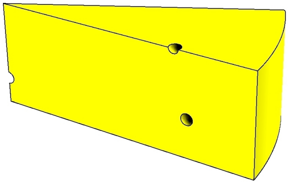

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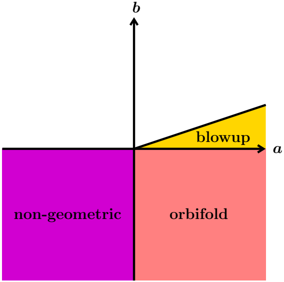
Target space is the $T^6/\mathbb{Z}_3$ **orbifold**.

Phase III: Blowup



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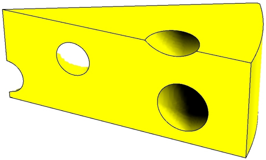

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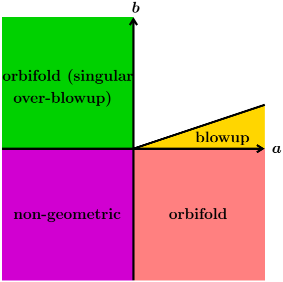
Target space is the **resolution CY** of the $T^6/\mathbb{Z}_3$ orbifold.

Phase IV: Singular Over-Blowup



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$a < 0, \; b > 0$:

Note complete symmetry of the model under

$x_\rho \leftrightarrow c_i, \;\; z_{i\rho} \leftrightarrow z_{\rho i}, \;\; a \leftrightarrow b$

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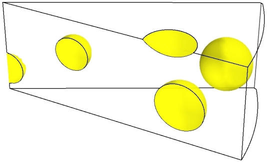

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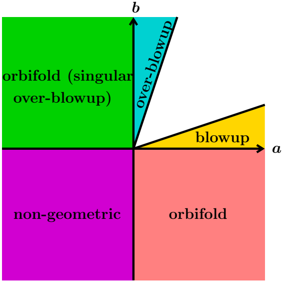
Target space again $T^6/\mathbb{Z}_3$ **orbifold**, with x and c exchanged.

Phase V: Over-Blowup



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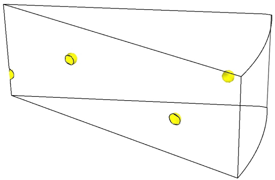

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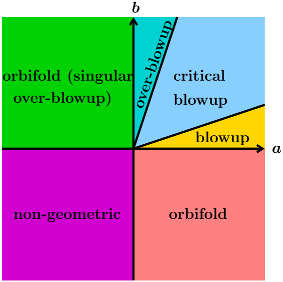
Target space is the **resolution CY** of the “other” $T^6/\mathbb{Z}_3$ orbifold.

Phase VI: Critical Blowup



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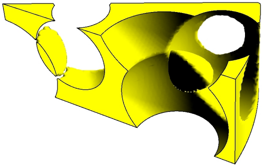

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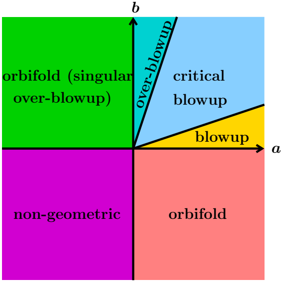
Target space is **hybrid phase** with blowup limits for $b \downarrow \frac{a}{3}$ & $b \uparrow 3a$.

Summary



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Note that in general

- The **dimension** of the **TS can jump** between the phases
- There can be **flop-transitions** also "outside" the **CY**
- There can be several **distinct singular phases** [Aspinwall, Greene, Morrison, Plesser, . . .]

There is a vast (moduli) space to be explored!

Conclusion

GLSMs can be used to describe different compactification spaces for string **model building**.

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Thank you for your attention!