Heterotic String Compactification with Gauged Linear Sigma Models

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LMU Fields & Strings – 04/25/2013

Based on:

[Lüdeling,FR,Wieck: 1203.5789], [Blaszczyk,Groot Nibbelink,FR: 1111.5852],

[Blaszczyk,Groot Nibbelink,FR: 1107.0320]

Motivation

- Start with **heterotic string** on $\mathcal{M}_{1,3} \times X$
  $\Rightarrow$ 4D **physics** is a **function** of **compactification manifold** X
- Want $\mathcal{N} = 1$ **SUSY** in 4D
  $\Rightarrow$ X is **CY threefold** [Candelas,Horowitz,Strominger,Witten]
- Want $G_{\mathbf{SM}} = \mathbf{SU}(3) \times \mathbf{SU}(2) \times \mathbf{U}(1)$ gauge group
  $\Rightarrow$ Choose holomorphic, stable **vector bundle** V to break $\mathrm{E}_8 \times \mathrm{E}_8 \to G_{\mathrm{SM}}(\times G_{\mathrm{hidden}})$
- Study resulting string theory

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Problem

- Resulting string theory described by **complicated NLSM**

Motivation

Solution ideas

- **Study** special CYs that allow for a free CFT description (**Orbifolds**) [Dixon,Harvey,Vafa,Witten],[Blaszczyk,Buchmüller,Groot Nibbelink,Nilles, Raby,Ramos Sanchez,Ratz,FR,Vaudrevange,. . .]
- **Study** generic CYs in **supergravity approximation** [Anderson,Bouchard,Braun,Donagi,Gray,He,Lukas,Ovrut,Palti,Pantev,Waldram,. . .]
- **Study** easy non-conformal **GLSM** that flows in the IR to conformal NLSM [Witten],[Adams,Blaszczyk,Blumenhagen,Carlevahro,Distler,Groot Nibbelink,Israel,Rahn,FR,. . .]

Our procedure

- Start with **orbifold** where string theory under control
  - Find consistency requirements from exact free CFT calculations [Blaszczyk,Groot Nibbelink,Ratz,FR,Trapletti,Vaudrevange]
  - Unknown how to get them in other phases like SUGRA approximation

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  - Study more general points in moduli space
  - Phenomenology (SUSY+exotics+GG) requires departure from orbifold point

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- Go to **GLSM** description of the **orbifold** [Blaszczyk,Groot Nibbelink,FR]

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- Depart from orbifold geometry
  - Study more general points in moduli space
  - Phenomenology (SUSY+exotics+GG) requires departure from orbifold point
- Go to GLSM description of the orbifold [Blaszczyk,Groot Nibbelink,FR]
- Study other GLSM phases like smooth CY
  - Phenomenological models require torsion and often NS5 branes

Outline

1. $\mathcal{N} = (2, 2)$ GLSMs
   - Introduction
   - GLSM description of compactification spaces
   - Study of moduli space
2. $\mathcal{N} = (0, 2)$ GLSMs & Anomalies
   - Introduction
   - Anomalies + Cancelation mechanism
   - Examples
3. $\mathcal{N} = (0, 2)$ GLSMs & Discrete Symmetries
   - Introduction
   - Study of discrete symmetries
4. Conclusion

Part I

$\mathcal{N} = (2, 2)$ GLSMs

Field content

Think of 2D $\mathcal{N} = (2,2)$ **superspace** as a dim. reduction of 4D $\mathcal{N} = 1$ superspace with Lorentz group $SO(1,1)$ [Witten]

Superspace coordinates

- Two **WS coordinates** σ_1, σ_2
- Two **complex Grassmann** variables θ^+, θ^-

Multiplets

- **Chiral** multiplet ($\overline{D}_\pm \mathcal{Z} = 0$): $\mathcal{Z} = (z, \psi_+, \psi_-, F)$
- **Vector** multiplet ($V = V^\dagger$): $V = (A, A_\sigma, A_{\bar{\sigma}}, \lambda_+, \lambda_-, D)$
- **Twisted chiral** multiplet ($\overline{D}_+ \Sigma = D_- \Sigma = 0$)

GLSM Action

GLSM Action

$S_{\text{GLSM}} = S_{\text{kin}} + S_{\text{FI}} + S_{\text{W}}$

- $S_{\text{kin}} = \int d^2\sigma d^4\theta \overline{\mathcal{Z}} e^{2qV} \mathcal{Z} + \frac{1}{e^2} \overline{\Sigma}\Sigma$
- $S_{\text{FI}} = \int d^2\sigma d\bar{\theta}^- d\theta^+ \rho\Sigma + \text{h.c.}$ with $\rho = b + i\beta$
- $S_{\text{W}} = \int d^2\sigma d^2\theta \mathcal{C} P(\mathcal{Z}) + \text{h.c.}$ with $q(\mathcal{C}) < 0, q(\mathcal{Z}) > 0$, $q(P(\mathcal{Z})) = q(\mathcal{C}) = \sum_i q(\mathcal{Z}_i)$

Algebraic EOMs

- D **term**: $\sum_i q_i |z_i|^2 + q_c |c|^2 = b$
- F **term**: $P(z) = 0, \; c \frac{\partial P(z)}{\partial z_i} = 0$

(2, 2) GLSMs describe heterotic string in standard embedding

Relations

- **GLSM fields** $\mathcal{Z}_i$ $\Leftrightarrow$ **Toric ambient space** coordinates
- **GLSM gaugings** $U(1)^N$ $\Leftrightarrow$ **Weights** of **toric spaces**
- F terms $\Leftrightarrow$ **hypersurface** constraints
- $q(\mathcal{C}) + \sum_i q(P(\mathcal{Z}_i)) = 0 \Leftrightarrow c_1(TX) = 0 = c_1(V)$
- **FI** terms b $\Leftrightarrow$ **Kähler** parameters
- D terms $\Leftrightarrow$ Kähler cone, **SR** ideal
- X is Kähler and Ricci-flat
- X has $SU(N)$ holonomy
- **Spin connection** is identified with **gauge connection**

Example: T^2

GLSM Fields	$\mathcal{Z}_1$	$\mathcal{Z}_2$	$\mathcal{Z}_3$	$\mathcal{C}$
U(1) charges	1	1	1	-3

Superpotential

$W = \mathcal{C}(\mathcal{Z}_1^3 + \mathcal{Z}_2^3 + \mathcal{Z}_3^3 + \kappa \mathcal{Z}_1 \mathcal{Z}_2 \mathcal{Z}_3)$

κ encodes CS of T^2 $\Rightarrow$ relate to τ via Weierstrass $\wp$ function

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D Term

$|z_1|^2 + |z_2|^2 + |z_3|^2 - 3|c|^2 = b$

- $b > 0$: SR=$\{z_1 z_2 z_3\}$, $c = 0$ (from F term) $\Rightarrow$ smooth T^2
- $b < 0$: SR=$\{c\}$, $z_1 = z_2 = z_3 = 0$ (from F term)
  $\Rightarrow$ $\mathbb{Z}_3$ LG orbifold

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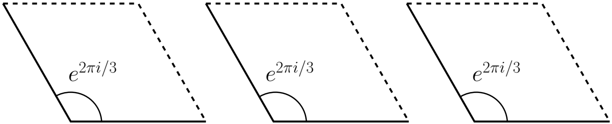
  $\Rightarrow$ $\mathbb{Z}_3$ LG orbifold

Note: We are **not** after the (non-geometric) **LG orbifold** but after the (geometric) **singular CY orbifold**

Example: $(T^2)^3/\mathbb{Z}_3$ orbifold



- Start with compactification space $(T^2)^3$:
  Choose complex coordinates z_1, z_2, z_3 and lattice Λ_T
- Remove parallelizable spinors by introducing non-trivial holonomy:
  Choose CS such that Λ_T has $\mathbb{Z}_3$ symmetry
  $\Rightarrow \Lambda_T$ is root lattice $A_2 \times A_2 \times A_2$

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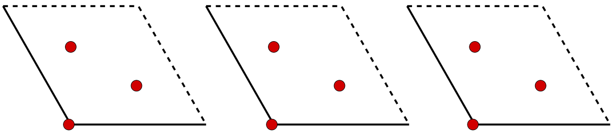

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  Choose complex coordinates z_1, z_2, z_3 and lattice Λ_T
- Remove parallelizable spinors by introducing non-trivial holonomy:
  Choose CS such that Λ_T has $\mathbb{Z}_3$ symmetry
  $\Rightarrow \Lambda_T$ is root lattice $A_2 \times A_2 \times A_2$
- Mod out $\mathbb{Z}_3$: Action described by orbifold twist $v = (v_1, v_2, v_3)$:
  $\theta : (z_1, z_2, z_3) \mapsto (e^{2\pi i v_1} z_1, e^{2\pi i v_2} z_2, e^{2\pi i v_3} z_3) \Rightarrow$ 27 fixed points
- CY constraint: $\Omega = dz_1 \wedge dz_2 \wedge dz_3$ invariant $\Rightarrow v_1 + v_2 + v_3 = 0 \mod 1$
- Modular invariance: Augment orbifold twist by shift V in gauge DOFs:
  $\theta : \Lambda_{E_8 \times E_8} \mapsto \Lambda_{E_8 \times E_8} + V$ with $V^2 - v^2 = 0 \mod 2$

Example: $(T^2)^3/\mathbb{Z}_3$ orbifold

$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3
R_1	1	1	1	0	0	0	0	0	0	-3	0	0
R_2	0	0	0	1	1	1	0	0	0	0	-3	0
R_3	0	0	0	0	0	0	1	1	1	0	0	-3

F–terms for c_i and $z_{i\rho}$:

$$z_{i1}{}^3 + z_{i2}{}^3 + z_{i3}{}^3 + \kappa_i z_{i1} z_{i2} z_{i3} = 0\,, \qquad i = 1,2,3\,,$$

$$c_i(z_{i\rho}{}^2 + \kappa_i z_{i\mu} z_{i\nu}) = 0\,, \qquad i,\rho = 1,2,3,\ \rho \neq \mu \neq \nu \neq \rho$$

D–terms:

$$|z_{i1}|^2 + |z_{i2}|^2 + |z_{i3}|^2 - 3|c_i|^2 = a_i\,, \qquad i = 1,2,3$$

Geometry:

$z_{i1},\ z_{i2},\ z_{i3}$: coordinate of i^{th} torus

Fix CS at $\tau_i = e^{2\pi \mathrm{i}/3} \Rightarrow \kappa_i = 0$ from Weierstrass map

Example: $(T^2)^3/\mathbb{Z}_3$ orbifold

$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3
R_1	1	1	1	0	0	0	0	0	0	-3	0	0
R_2	0	0	0	1	1	1	0	0	0	0	-3	0
R_3	0	0	0	0	0	0	1	1	1	0	0	-3

F–terms for c_i and $z_{i\rho}$:

$$z_{i1}{}^3 + z_{i2}{}^3 + z_{i3}{}^3 = 0\,, \qquad i = 1,2,3\,,$$

$$c_i z_{i\rho}{}^2 = 0\,, \qquad i,\rho = 1,2,3$$

D–terms:

$$|z_{i1}|^2 + |z_{i2}|^2 + |z_{i3}|^2 - 3|c_i|^2 = a_i\,, \qquad i = 1,2,3$$

Geometry:

a_i: sizes of tori

$a_i > 0 \Rightarrow$ at least 3 $z_{i,\rho} \neq 0 \Rightarrow c_i = 0$ (assume this case for now)

Example: $(T^2)^3/\mathbb{Z}_3$ orbifold

$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3	x_1
R_1	1	1	1	0	0	0	0	0	0	-3	0	0	**0**
R_2	0	0	0	1	1	1	0	0	0	0	-3	0	**0**
R_3	0	0	0	0	0	0	1	1	1	0	0	-3	**0**
E_1	**1**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**-3**

F–terms for c_i:

$$z_{i1}^3 x_1 + z_{i2}^3 + z_{i3}^3 = 0\,, \qquad i = 1, 2, 3$$

D–terms:

$$|z_{i1}|^2 + |z_{i2}|^2 + |z_{i3}|^2 = a_i\,, \qquad i = 1, 2, 3$$

$$|z_{11}|^2 + |z_{21}|^2 + |z_{31}|^2 - 3|x_1|^2 = b_1$$

Geometry: [Aspinwall, Plesser]

b_1: sizes of exceptional cycles

$b_1 < 0 \Rightarrow x_1 \neq 0 \Rightarrow \langle x_1 \rangle$ breaks E_1 to $\mathbb{Z}_3$ with 27 **FP** $z_{11} = z_{21} = z_{31} = 0$

$b_1 > 0 \Rightarrow$ **FP** $z_{11} = z_{21} = z_{31} = 0$ **forbidden** $\Rightarrow$ **smooth**

Example: $(T^2)^3/\mathbb{Z}_3$ orbifold

$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3	x_1	x_2
R_1	1	1	1	0	0	0	0	0	0	-3	0	0	**0**	**0**
R_2	0	0	0	1	1	1	0	0	0	0	-3	0	**0**	**0**
R_3	0	0	0	0	0	0	1	1	1	0	0	-3	**0**	**0**
E_1	**1**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**-3**	**0**
E_2	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**-3**

F–terms for c_i:

$$z_{i1}^3 x_1 + z_{i2}^3 x_2 + z_{i3}^3 = 0\,, \qquad i = 1, 2, 3$$

D–terms:

$$|z_{i1}|^2 + |z_{i2}|^2 + |z_{i3}|^2 = a_i\,, \qquad i = 1, 2, 3$$

$$|z_{11}|^2 + |z_{21}|^2 + |z_{31}|^2 - 3|x_1|^2 = b_1$$

$$|z_{12}|^2 + |z_{22}|^2 + |z_{32}|^2 - 3|x_2|^2 = b_2$$

Geometry:

$b_1 < 0 \Rightarrow x_1 \neq 0 \Rightarrow \langle x_1 \rangle$ breaks E_1 to $\mathbb{Z}_3$ with 9 **FP** $z_{11} = z_{21} = z_{31} = 0$
$b_2 < 0 \Rightarrow x_2 \neq 0 \Rightarrow \langle x_2 \rangle$ breaks E_2 to $\mathbb{Z}_3$ with 9 **FP** $z_{12} = z_{22} = z_{32} = 0$

Example: $(T^2)^3/\mathbb{Z}_3$ orbifold

$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3	x_1	x_2
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R_3	0	0	0	0	0	0	1	1	1	0	0	-3	0	0
E_1	1	0	0	1	0	0	1	0	0	0	0	0	-3	0
E_2	0	1	0	0	1	0	0	1	0	0	0	0	0	-3
E'	0	0	1	0	0	1	0	0	1	-3	-3	-3	3	3

$$|z_{i1}|^2 + |z_{i2}|^2 + |z_{i3}|^2 = a_i\,, \qquad i = 1,2,3$$

$$|z_{11}|^2 + |z_{21}|^2 + |z_{31}|^2 - 3|x_1|^2 = b_1$$

$$|z_{12}|^2 + |z_{22}|^2 + |z_{32}|^2 - 3|x_2|^2 = b_2$$

Where are last **9 fixed points**? $\Rightarrow$ Combine U(1)'s or D terms:

$$|z_{13}|^2 + |z_{23}|^2 + |z_{33}|^2 + 3|x_1|^2 + 3|x_2|^2 = \sum_i a_i + b_1 + b_2$$

Geometry:

$x_1 \neq 0 \Rightarrow 9$ **FP** $\quad z_{11} = z_{21} = z_{31} = 0$

$x_2 \neq 0 \Rightarrow 9$ **FP** $\quad z_{12} = z_{22} = z_{32} = 0$

Last 9 **FP** $\quad z_{13} = z_{23} = z_{33} = 0$. Note: Not smooth for $b_{1,2} > 0$!

$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3	x_1	x_2	x_3
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R_3	0	0	0	0	0	0	1	1	1	0	0	-3	0	0	0
E_1	1	0	0	1	0	0	1	0	0	0	0	0	-3	0	0
E_2	0	1	0	0	1	0	0	1	0	0	0	0	0	-3	0
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$$z_{i1}^3 x_1 + z_{i2}^3 x_2 + z_{i3}^3 x_3 = 0\,, \qquad i = 1,2,3$$

D–terms:

$$|z_{i1}|^2 + |z_{i2}|^2 + |z_{i3}|^2 = a_i\,, \qquad i = 1,2,3$$

$$|z_{1\rho}|^2 + |z_{2\rho}|^2 + |z_{3\rho}|^2 - 3|x_\rho|^2 = b_\rho\,, \qquad \rho = 1,2,3$$

Geometry:

$$\left.\begin{array}{l} \langle x_1 \rangle \neq 0 \text{ generates } \mathbb{Z}_3 \text{ with FP } z_{11}=z_{21}=z_{31}=0 \\ \langle x_2 \rangle \neq 0 \text{ generates } \mathbb{Z}_3 \text{ with FP } z_{12}=z_{22}=z_{32}=0 \\ \langle x_3 \rangle \neq 0 \text{ generates } \mathbb{Z}_3 \text{ with FP } z_{13}=z_{23}=z_{33}=0 \end{array}\right\}$$

3×9 fixed points

Smooth for $b_\rho > 0$!

General procedure:

To build an orbifold/resolution **GLSM** model

1. Choose toric description appropriate for orbifold action
2. Introduce exceptional divisors to smoothen the singularities
3. Set FI parameters $a \gg 0 > b$ to study orbifold or $a \gg b > 0$ to study blowup
4. Construct inherited divisors and linear equivalences
5. Read off intersection numbers

In this way, the resolution phase can be studied using a GLSM which can be smoothly connected to the orbifold.

The procedure yields GLSM description of previous approaches which proceeded via polytopes and gluings.
[Lust,Reffert,Scheidegger,Stieberger]

Study of Moduli Space

But we can **do more**! Having a **GLSM** realization, we can **probe** whole **moduli space** by simply varying **FI** parameter.

$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3	x_1	x_2	x_3
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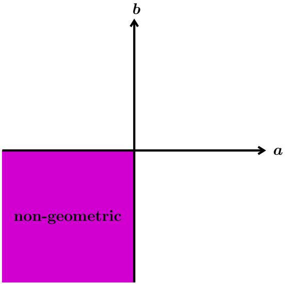
R_3	0	0	0	0	0	0	1	1	1	0	0	-3	**0**	**0**	**0**
E_1	**1**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**-3**	**0**	**0**
E_2	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**-3**	**0**
E_3	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**-3**

Superpotential:
$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$

D–terms:
$\sum_{\rho=1}^3 |z_{i\rho}|^2 - 3|c_i|^2 = a_i \,, \qquad i = 1,2,3$
$\sum_{i=1}^3 |z_{i\rho}|^2 - 3|x_\rho|^2 = b_\rho \,, \qquad \rho = 1,2,3$

For simplicity: Set $a_i = a$ and $b_\rho = b$.



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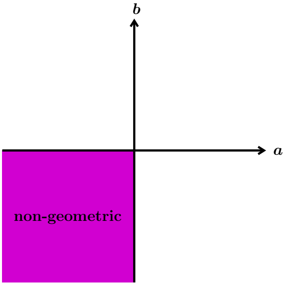
$$\sum_{\rho=1}^3 |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^3 |z_{i\rho}|^2 - 3|x_\rho|^2 = b$$

$a < 0, \; b < 0$:

$$\langle c_i \rangle = \frac{\sqrt{a}}{3}, \qquad \langle x_\rho \rangle = \frac{\sqrt{b}}{3}, \qquad z_{i\rho} = 0$$

Phase I: Non-Geometric Regime



Superpotential:

$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$$

D–terms:

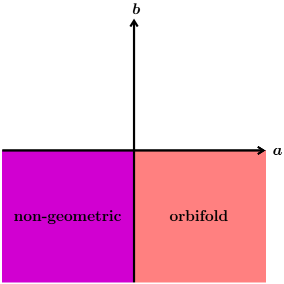
$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_\rho|^2 = b$$

$a < 0,\ b < 0$:

$$\langle c_i \rangle = \frac{\sqrt{a}}{3}, \qquad \langle x_\rho \rangle = \frac{\sqrt{b}}{3}, \qquad z_{i\rho} = 0$$

Target space is a **point**.

Phase II: Orbifold



Superpotential:

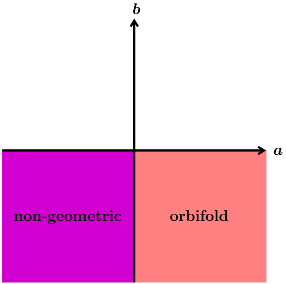
$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$$

D–terms:

$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_\rho|^2 = b$$

$a > 0,\ b < 0$:

$c_i = 0, \qquad \langle x_\rho \rangle > 0, \qquad \langle z_{i\rho} \rangle \geq 0$



Superpotential:

$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$$

D–terms:

$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|c_i|^2 = a$$
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$a > 0,\ b < 0$:

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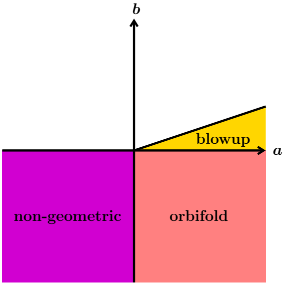
Target space is the $T^6/\mathbb{Z}_3$ **orbifold**.

Phase III: Blowup



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$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$$

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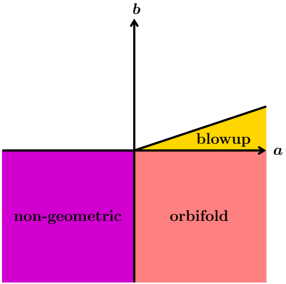
$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_\rho|^2 = b$$

$a > 3b > 0$:

$$c_i = 0, \qquad \langle x_\rho \rangle \geq 0, \qquad \langle z_{i\rho} \rangle \geq 0$$

Phase III: Blowup



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D–terms:

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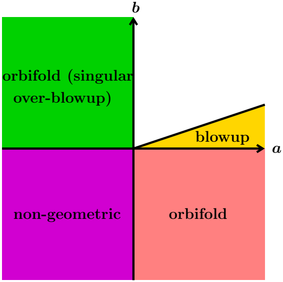
$$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_\rho|^2 = b$$

$a > 3b > 0$:

$$c_i = 0, \qquad \langle x_\rho \rangle \geq 0, \qquad \langle z_{i\rho} \rangle \geq 0$$

Target space is the **resolution CY** of the $T^6/\mathbb{Z}_3$ orbifold.



Superpotential:

$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$$

D–terms:

$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_\rho|^2 = b$$

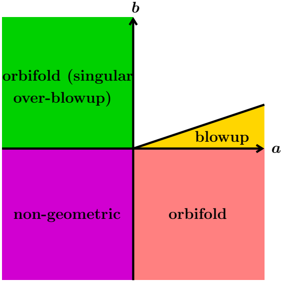
$a < 0,\ b > 0$:

Note complete symmetry of the model under

$x_\rho \leftrightarrow c_i,\ \ z_{i\rho} \leftrightarrow z_{\rho i},\ \ a \leftrightarrow b$

$\langle c_i \rangle > 0, \qquad x_\rho = 0, \qquad \langle z_{i\rho} \rangle \geq 0$

Phase IV: Singular Over-Blowup



Superpotential:

$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$$

D–terms:

$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_\rho|^2 = b$$

$a < 0,\ b > 0$:

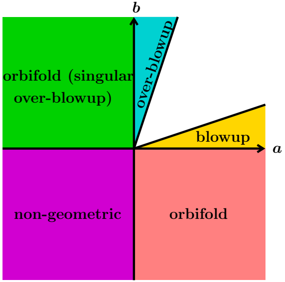
Note complete symmetry of the model under

$x_\rho \leftrightarrow c_i,\ z_{i\rho} \leftrightarrow z_{\rho i},\ a \leftrightarrow b$

$\langle c_i \rangle > 0, \qquad x_\rho = 0, \qquad \langle z_{i\rho} \rangle \geq 0$

Target space again $T^6/\mathbb{Z}_3$ **orbifold**, with x and c exchanged.

Phase V: Over-Blowup



Superpotential:

$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$$

D–terms:

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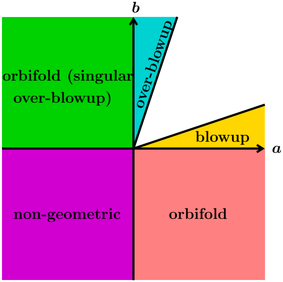
$$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_\rho|^2 = b$$

$b > 3a > 0$:

$\langle c_i \rangle \geq 0, \qquad x_\rho = 0, \qquad \langle z_{i\rho} \rangle \geq 0$

Phase V: Over-Blowup



Superpotential:

$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$

D–terms:

$\sum_{\rho=1}^3 |z_{i\rho}|^2 - 3|c_i|^2 = a$

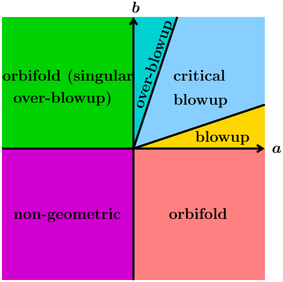
$\sum_{i=1}^3 |z_{i\rho}|^2 - 3|x_\rho|^2 = b$

$b > 3a > 0$:

$\langle c_i \rangle \geq 0, \qquad x_\rho = 0, \qquad \langle z_{i\rho} \rangle \geq 0$

Target space is the **resolution CY** of the "other" $T^6/\mathbb{Z}_3$ orbifold.

Phase VI: Critical Blowup



Superpotential:

$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$$

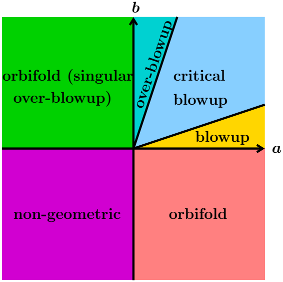
D–terms:

$$\sum_{\rho=1}^3 |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^3 |z_{i\rho}|^2 - 3|x_\rho|^2 = b$$

$a > 0,\ b \in \left[\frac{a}{3}, 3a\right]$:

$\langle c_{i\neq\rho}\rangle \geq 0, \qquad \langle x_\rho \rangle \geq 0, \qquad \langle z_{i\rho}\rangle \geq 0$

Phase VI: Critical Blowup



Superpotential:

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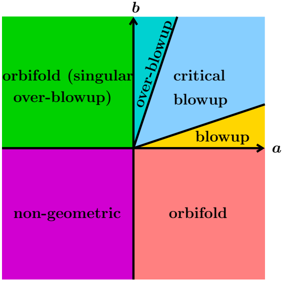
$$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_\rho|^2 = b$$

$a > 0,\ b \in \left[\frac{a}{3}, 3a\right]$:

$\langle c_{i \neq \rho} \rangle \geq 0, \qquad \langle x_\rho \rangle \geq 0, \qquad \langle z_{i\rho} \rangle \geq 0$

Target space is **hybrid phase** with blowup limits for $b \downarrow \frac{a}{3}$ & $b \uparrow 3a$.

Summary



Superpotential:

$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$$

D–terms:

$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_\rho|^2 = b$$

Note that in general

- The **dimension** of the **TS can jump** between the phases
- There can be **flop-transitions** also "outside" the **CY**
- There can be several **distinct singular phases** [Aspinwall, Greene, Morrison, Plesser, . . .]

There is a vast (moduli) space to be explored!

Part II

$\mathcal{N} = (0, 2)$ Models & Anomalies

Definition of GLSM

superfield type	superfield notation	charge	bosonic DOF	fermionic DOF
chiral	Ψ^a	$(q_I)^a$	z^a	ψ^a
chiral–Fermi	Λ^α	$(Q_I)^\alpha$	$\widetilde{F}^\alpha$	λ^α
gauge	$(V,A)^I$	0	$a^I_\sigma, a^I_{\overline{\sigma}}, \widetilde{D}^I$	Φ^I
Fermi–gauge	σ^i	0	s^i	φ^i
chiral	Φ^m	$(q_I)^m$	x^m	ψ^m
chiral–Fermi	Γ^μ	$(Q_I)^\mu$	$\widetilde{F}^\mu$	γ^μ

Think of 2D $\mathcal{N} = (0,2)$ **superspace** as $\mathcal{N} = (2,2)$ **superspace** and **dispense** of $\theta^+, \bar{\theta}^+$ [Dine, Seiberg]

Multiplets

- $(2,2)$ Chiral multiplet $\mathcal{Z}_{(2,2)} \Rightarrow (Z;\chi)$ =(chiral; chiral-Fermi)
- $(2,2)$ Vector multiplet $V_{2,2} \Rightarrow (V,A;\Sigma)$ =(gauge; Fermi-gauge)

Definition of GLSM

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chiral	Φ^m	$(q_I)^m$	x^m	ψ^m
chiral–Fermi	Γ^μ	$(Q_I)^\mu$	$\widetilde{F}^\mu$	γ^μ

Geometry:

$\widetilde{D}$ **Term**:

$$(q_I)^a|z^a|^2 + (q_I)^m|x^m|^2 - b_I = 0, \qquad b_I\text{: FI–parameter}$$

$\widetilde{F}$ **Term**:

$$W_{\text{geom}} = \Gamma^\mu P_\mu(Z) \Rightarrow P_\mu(Z) = 0$$

Geometry

$\widetilde{D}$ **Terms** give SR ideal $\Rightarrow$ Kähler cone, GLSM phase

$\widetilde{F}$ **Terms** give restriction to hypersurface

Definition of GLSM

superfield type	superfield notation	charge	bosonic DOF	fermionic DOF
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chiral–Fermi	Γ^μ	$(Q_I)^\mu$	$\widetilde{F}^\mu$	γ^μ

Gauge group:

Fermionic Transformation:

$$\delta_\Theta \Lambda^\alpha = M^\alpha{}_i(Z)\Theta_i$$

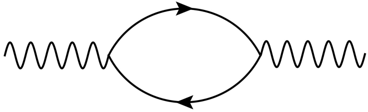
Superpotential:

$$W_{\text{bundle}} = \Phi^m N_{m\alpha}(\Psi)\Lambda^\alpha \Rightarrow \Phi^m N_{m\alpha}(Z) = 0$$

Gauge group

Gauge group and particle content given by (naturally arising) monad construction via $\ker(N)/\text{im}(M)$.

Anomalies



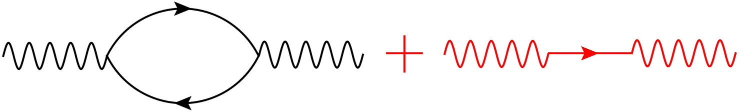
$$\mathcal{A}_{IJ} := \frac{1}{2}(Q_I \cdot Q_J - q_I \cdot q_J)\,,$$

$$Q_I \cdot Q_J := \sum_\alpha (Q_I)^\alpha (Q_J)^\alpha + \sum_\mu (Q_I)^\mu (Q_J)^\mu\,,$$

$$q_I \cdot q_J := \sum_a (q_I)^a (q_J)^a + \sum_m (q_I)^m (q_J)^m\,.$$

Problem

In general **many** U(1) gauge groups
$\Rightarrow$ **Huge amount** of stringent **anomaly conditions**.



$$\mathcal{A}_{IJ} := \frac{1}{2}(Q_I \cdot Q_J - q_I \cdot q_J) - \mathcal{T}_{IJ} \,,$$

$$Q_I \cdot Q_J := \sum_\alpha (Q_I)^\alpha (Q_J)^\alpha + \sum_\mu (Q_I)^\mu (Q_J)^\mu \,,$$

$$q_I \cdot q_J := \sum_a (q_I)^a (q_J)^a + \sum_m (q_I)^m (q_J)^m \,.$$

Idea

Introduce **new fields** to obtain **Green–Schwarz mechanism** on the world–sheet to **cancel gauge anomalies**. [Adams,Ernebjerg,Lapan]

Green–Schwarz mechanism needs fields that transform with shifts.

Our approach

$\Rightarrow$ Use **logarithm** of **coordinate fields** Ψ

$$W_{\mathrm{FI}} = \left[\rho_I^0 + T_{XI} \ln \left|R^X(Z)\right|\right] f^I \quad \Rightarrow \quad \mathcal{T}_{IJ} = r_I^X T_{XI}$$

$$\mathcal{A}_{IJ} = \frac{1}{2}(Q_I \cdot Q_J - q_I \cdot q_J) - \mathcal{T}_{IJ}$$

with

- ρ_I^0: constant FI parameter
- $R^X(Z)$: homogeneous polynomials w/ charges r_I^X
- T_{XI}: (quantized) coefficients: $T_{XI} \int F^I \in \mathbb{Z}$

Green–Schwarz mechanism needs fields that transform with shifts.

Our approach

⇒ Use **logarithm** of **coordinate fields** Ψ

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WS Anom: $I_4 = \mathcal{A}_{IJ} F^I F^J$

TS BI's: $\mathrm{d}H = \mathrm{ch}_2(V) - \mathrm{ch}_2(TX) - \mathrm{NS5}$

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TS BI's: $\mathrm{d}H = \mathrm{ch}_2(V) - \mathrm{ch}_2(TX) - \mathrm{NS5}$

Anom: $I_4 = \underbrace{[Q_I^\alpha Q_J^\alpha - q_I^m q_J^m] F^I F^J}_{Z^*(\mathrm{ch}_2(V))} - \underbrace{[q_I^a q_J^a - Q_I^\mu Q_J^\mu] F^I F^J}_{Z^*(\mathrm{ch}_2(TX))} - \underbrace{\mathcal{T}_{IJ} F^I F^J}_{Z^*(\mathrm{NS5})}$

1.) No anomalies

superfield	$Z^{a=1,\dots,8}$	$\Gamma^{\mu=1,\dots,4}$	$\Lambda^{\alpha=1,\dots,8}$	$\Phi^{m=1,\dots,4}$
gauge charge	1	-2	1	-2

$\mathbb{P}^7[2,2,2,2]$ with $SU(3)$ bundle

$$\mathcal{A}_{11} = \frac{1}{2}\left[Q_1^2 - q_1^2\right] - \mathcal{T}_{11}$$

$$W_{\text{FI}} = \rho^0 + T \ln |Z_1| \quad \Rightarrow \quad r_1^1 = 1\,, \quad \mathcal{T}_{11} = T\,, \quad T \in 2\mathbb{Z}$$

$$V_D = \frac{1}{2}\Big[\sum_a |z^a|^2 - 2\sum_m |x^m|^2 - b - T\ln|z^1|\Big]^2 \stackrel{!}{=} 0$$

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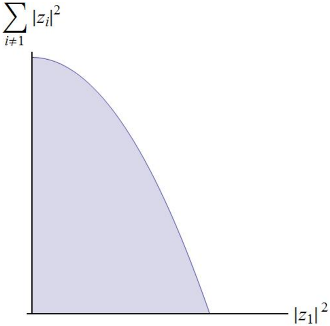
$$V_D = \frac{1}{2}\Big[\sum_a |z^a|^2 - 2\sum_m |x^m|^2 - b - T \ln |z^1|\Big]^2 \overset{!}{=} 0$$

$$\mathcal{A}_{11} = \frac{1}{2}\left[8\cdot(1)^2 + 4\cdot(-2)^2 - 8\cdot(1)^2 - 4\cdot(-2)^2\right] + \mathcal{T}_{11} \overset{!}{=} 0$$

$$\mathcal{T}_{11} = T = 0$$

1.) No anomalies



$b = 1$

$$b > 0 \;\Rightarrow\; |x^a| = 0\,, \quad V_D \overset{!}{=} 0 \;\Rightarrow\; \sum_{a=1}^{8} |z^a|^2 = b$$

Geometry **compact**, no anomalies.

superfield	$\Psi^{a=1,\dots,8}$	$\Gamma^{\mu=1,\dots,4}$	$\Lambda^{\alpha=1,\dots,4}$	$\Phi^{m=1,2}$
gauge charge	1	-2	1	-2

$\mathbb{P}^7[2,2,2,2]$ with $SU(2)$ bundle

$$\mathcal{A}_{11} = \frac{1}{2}\left[Q_1^2 - q_1^2\right] - \mathcal{T}_{11}$$

$$W_{\text{FI}} = \rho^0 + T\ln[Z^1] \quad \Rightarrow \quad r_1^1 = 1\,, \quad \mathcal{T}_{11} = T\,, \quad T \in 2\mathbb{Z}$$

$$V_D = \frac{1}{2}\Big[\sum_a |z^a|^2 - 2\sum_m |x^m|^2 - b - T\ln|z^1|\Big]^2 \stackrel{!}{=} 0$$

superfield	$\Psi^{a=1,\ldots,8}$	$\Gamma^{\mu=1,\ldots,4}$	$\Lambda^{\alpha=1,\ldots,4}$	$\Phi^{m=1,2}$
gauge charge	1	-2	1	-2

$\mathbb{P}^7[2,2,2,2]$ with $SU(2)$ bundle

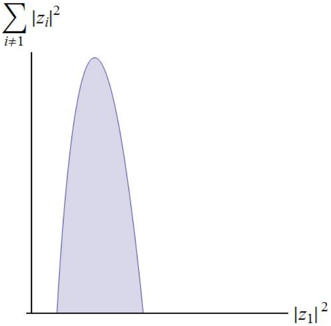
$$\mathcal{A}_{11} = \frac{1}{2}\left[Q_1^2 - q_1^2\right] - \mathcal{T}_{11}$$

$$W_{\mathrm{FI}} = \rho^0 + T\ln[Z^1] \quad \Rightarrow \quad r_1^1 = 1\,, \quad \mathcal{T}_{11} = T\,, \quad T \in 2\mathbb{Z}$$

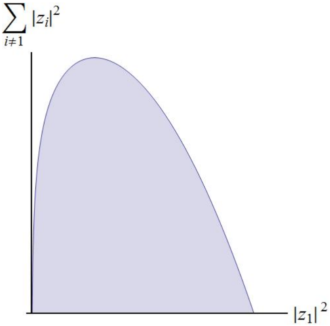
$$V_D = \frac{1}{2}\Big[\sum_a |z^a|^2 - 2\sum_m |x^m|^2 - b - T\ln|z^1|\Big]^2 \stackrel{!}{=} 0$$

$$\mathcal{A}_{11} = \frac{1}{2}\left[4\cdot(1)^2 + 4\cdot(-2)^2 - 8\cdot(1)^2 - 2\cdot(-2)^2\right] - \mathcal{T}_{11} \stackrel{!}{=} 0$$

$$\mathcal{T}_{11} = T = 2$$



$b = 1$



$b = 10$

$$b > 0 \;\Rightarrow\; |x^a| = 0\,, \qquad V_D \stackrel{!}{=} 0 \;\Rightarrow\; \sum_{a=2}^{8} |z^a|^2 = b + 2\ln|z_1| - |z_1|^2$$

Geometry still **compact**, anomalies canceled by **NS5 branes**.

3.) $T < 0$, decompactified geometry, non–effective curve

superfield	$Z^{a=1,\ldots,8}$	$\Gamma^{\mu=1,\ldots,4}$	$\Lambda^{\alpha=1,\ldots,8}$	$\Phi^{m=1,2}$
gauge charge	1	-2	1	-4

$\mathbb{P}^7[2,2,2,2]$ with $SU(6)$ bundle

$$\mathcal{A}_{11} = \frac{1}{2}\left[Q_1^2 - q_1^2\right] - \mathcal{T}_{11}$$

$$W_{\rm FI} = \rho^0 + T\ln[Z^1] \quad \Rightarrow \quad r_1^1 = 1\,, \quad \mathcal{T}_{11} = T\,, \quad T \in 4\mathbb{Z}$$

$$V_D = \frac{1}{2}\Big[\sum_a |z^a|^2 - 4\sum_m |x^m|^2 - b - T\ln|z^1|\Big]^2 \stackrel{!}{=} 0$$

superfield	$Z^{a=1,\dots,8}$	$\Gamma^{\mu=1,\dots,4}$	$\Lambda^{\alpha=1,\dots,8}$	$\Phi^{m=1,2}$
gauge charge	1	-2	1	-4

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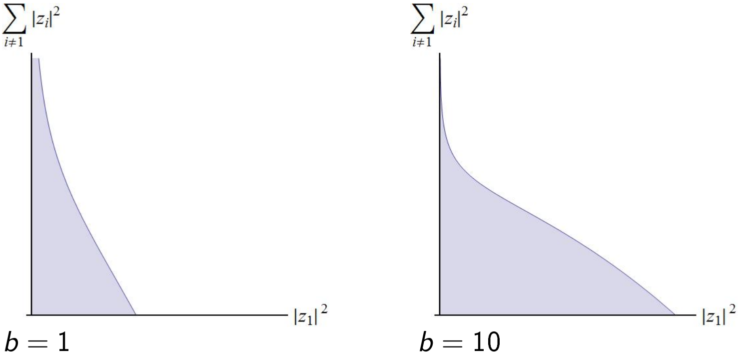
$$\mathcal{A}_{11} = \frac{1}{2}\left[8\cdot(1)^2 + 4\cdot(-2)^2 - 8\cdot(1)^2 - 2\cdot(-4)^2\right] - \mathcal{T}_{11} \overset{!}{=} 0$$

$$\mathcal{T}_{11} = T = -8$$

3.) $T < 0$, decompactified geometry, non–effective curve



$$b > 0 \;\Rightarrow\; |x^a| = 0\,, \qquad V_D \overset{!}{=} 0 \;\Rightarrow\; \sum_{a=2}^{8} |z^a|^2 = b - 8\ln|z_1| - |z_1|^2$$

Geometry **decompactified**, anomalies canceled with **anti–NS5 branes**.

Part III

$\mathcal{N} = (0, 2)$ Models & Discrete Symmetries

Introduction

Discrete Symmtries important for

- Absence of FCNCs
- Absence of μ–term [Dreiner,Lee,Luhn,Raby,Ratz,Ross,Schieren,Schmidt-Hoberg,Thormaier,Vaudrevange,. . .]
- Creation of hierarchies
- Proton stability

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- Creation of **hierarchies**
- **Proton stability**

Discrete symmetries on orbifold

- Non-R Symmetries from fixed point degeneracies
- R Symmetries from remnants of internal Lorentz group
  Lee at.al. found nice $\mathbb{Z}_4$ R-symmetry

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- Absence of **FCNCs**
- Absence of **μ–term** [Dreiner,Lee,Luhn,Raby,Ratz,Ross,Schieren,Schmidt-Hoberg,Thormaier,Vaudrevange,. . .]
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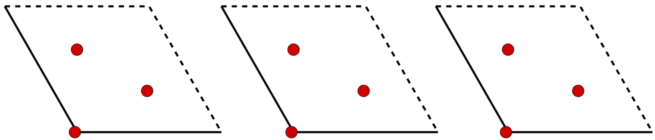
Discrete symmetries on orbifold

- Non-R Symmetries from fixed point degeneracies
- R Symmetries from remnants of internal Lorentz group
  Lee at.al. found nice $\mathbb{Z}_4$ R-symmetry

Discrete symmetries on CY

- Symmetries aries as isometries in GLSM
- Charges from graded cohomology

$\mathbb{Z}_3$ Orbifold



$$\theta : (z_1, z_2, z_3) \mapsto (e^{2\pi i/3} z_1, e^{2\pi i/3} z_2, e^{-2\pi i\, 2/3} z_3)$$

- **Orbifold action** given by **twist** vector $v = \frac{1}{3}(1, 1, -2)$
- **Gauge sector** given by **shift** V in the $\Lambda_{E_8 \times E_8}$
- Choose **Standard embedding** $V = \frac{1}{3}(1, 1, -2, 0^5)(0^8)$ "$= v$"

$$\begin{aligned} \text{Gauge group:} \quad & [E_6 \times SU(3)]_{\text{vis}} \times [E_8]_{\text{hidden}} \\ \text{Matter:} \quad & 3(\mathbf{27}, \overline{\mathbf{3}}; \mathbf{1}) + 27[(\mathbf{27}, \mathbf{1}; \mathbf{1}) + 3(\mathbf{1}, \mathbf{3}; \mathbf{1})] \end{aligned}$$

Consistency requirements

Want to construct **smooth CY** with line bundles from orbifold using **toric** (algebraic) **geometry**. Impose

- **Bianchi identity** (ensures anomaly cancelation):

$$H = dB + \omega_{YM} - \omega_L \to \int_D dH = \int_D \mathrm{tr}\mathcal{R}^2 - \mathrm{tr}\mathcal{F}^2 \overset{!}{=} 0$$

- **Donaldson–Uhlenbeck–Yau** (ensures 4d $\mathcal{N} = 1$ SUSY)

$$\int_X J \wedge J \wedge \mathcal{F} = 0$$

Resolution of $T^6/\mathbb{Z}^3$ with line bundles

Procedure: [Blaszczyk,Groot Nibbelink,Ha,Klevers,FR,Trapletti,Vaudrevange,Walter,...]

- Introduce **exceptional divisors** $E_{\alpha\beta\gamma}$ at $x_{\alpha\beta\gamma} = 0$
- Introduce **gauge flux** $\mathcal{F} = E_{\alpha\beta\gamma} V^I_{\alpha\beta\gamma} H_I$
  - The H_I are the 16 Cartan generators of $E_8 \times E_8$
  - The 16×27 matrix $V^I_{\alpha\beta\gamma}$ describes the gauge line bundle at the 27 fixed points
- Note that in the **orbifold limit** the $E_{\alpha\beta\gamma}$ are shrunk to a point $\Rightarrow$ **flux** is located at fixed points

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To make contact with the **orbifold** description:

- Choose the $V_{\alpha\beta\gamma}$ to coincide with the internal $E_8 \times E_8$ **momentum** of some **twisted orbifold state** located at (α, β, γ)
- **Vev** of **orbifold state** generates the **blowup** of the $E_{\alpha\beta\gamma}$

Resolution of $T^6/\mathbb{Z}^3$ with line bundles

Choose 3 different **bundle vectors** from $(\mathbf{27}, \mathbf{1})$ of $E_6 \times SU(3)$

- $V_1 = \frac{1}{3}(2, 2, 2, 0^5)(0^8)$ at 9 fixed points
- $V_2 = \frac{1}{3}(-1, -1, -1, 3, 0^4)(0^8)$ at 9 fixed points
- $V_3 = -(V_1 + V_2)$ at 9 fixed points

$\Rightarrow \mathcal{F} = \sum_{i=1}^{9} E_i V_1^I H_I + \sum_{j=10}^{18} E_j V_2^I H_I + \sum_{19}^{27} E_n V_3^I H_I$

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$[\mathrm{E}_6 \times \mathrm{SU}(3)] \times [\mathrm{E}_8] \to [\mathrm{SO}(8) \times (\mathrm{U}(1)_A \times \mathrm{U}(1)_B) \times \mathrm{SU}(3)] \times [\mathrm{E}_8]$, $\mathrm{U}(1)_{A,B}$ anomalous

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Bianchi identities

$$\int_{E_{\alpha\beta\gamma}} \mathrm{tr}\mathcal{F}^2 = \int_{E_{\alpha\beta\gamma}} \mathrm{tr}\mathcal{R}^2 \quad \Rightarrow \quad V_1^2 = V_2^2 = V_3^2 = \frac{4}{3}$$

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DUY equations

$$\int J \wedge J \wedge \mathcal{F} = 0 \ \Rightarrow \ \sum_{i=1}^{9} V_1^I \mathrm{vol}(E_i) + \sum_{j=10}^{18} V_2^I \mathrm{vol}(E_j) + \sum_{k=19}^{27} V_3^I \mathrm{vol}(E_k) = 0$$

Remnant non-R symmetries

Non-R symmetries arise as **discrete subgroups** of $U(1)_A$ and $U(1)_B$ which **leave vevs** of blowup modes **invariant**

$$\mathbf{27} \to \mathbf{8}_{s(1,-1)} + \mathbf{8}_{c(1,1)} + \mathbf{8}_{v(-2,0)} + \mathbf{1}_{(-2,-2)} + \mathbf{1}_{(-2,2)} + \mathbf{1}_{(4,0)}$$

Blowup modes:
$\mathbf{1}_{(-2,-2)}$, $\mathbf{1}_{(-2,2)}$, $\mathbf{1}_{(4,0)}$ corresponding to V_1, V_2, V_3

Leave **discrete $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry** generated by

$$T_\pm : \quad \phi_{(q_a,q_b)} \to e^{\frac{2\pi i}{2}(q_A \pm q_B)}\ \phi_{(q_A,q_B)}$$

Both **symmetries** are **non-anomalous**

Remnant R symmetries

Properties of R symmetries

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- Grassmann coordinate θ transforms under R symmetries
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R symmetries on orbifolds

R charge on orbifold defined via a combination of right-moving momenta q and oscillator numbers ΔN:
$R = q - \Delta N$ with $q = \frac{1}{3}(1,1,1)$ [Kobayashi,Raby,Zhang]

Remnant symmetry of internal space:
Sublattice rotations by $2\pi/3$ in each torus:

$$T_k^R : \phi \to e^{2\pi i/3 R_k} \phi \,, \qquad \text{k=1,2,3 labels tori}$$

Remnant R symmetries – Orbifold

Our orbifold **blowup modes** have

$$R = q - \Delta N = \frac{1}{3}(1,1,1)$$

To identify remnant **R-symmetries**, search for **invariant combinations** of T_k^R with $T_{U(1)_A}$ and $T_{U(1)_B}$:

$$\mathbf{1}_{(-2,-2)} \to (T_1^R)^a (T_2^R)^b (T_3^R)^c T_{U(1)_A} T_{U(1)_B} \mathbf{1}_{(-2,-2)} \overset{!}{=} \mathbf{1}_{(-2,-2)}$$
$$\mathbf{1}_{(-2,2)} \to (T_1^R)^a (T_2^R)^b (T_3^R)^c T_{U(1)_A} T_{U(1)_B} \mathbf{1}_{(-2,2)} \overset{!}{=} \mathbf{1}_{(-2,2)}$$
$$\mathbf{1}_{(4,0)} \to (T_1^R)^a (T_2^R)^b (T_3^R)^c T_{U(1)_A} T_{U(1)_B} \mathbf{1}_{(4,0)} \overset{!}{=} \mathbf{1}_{(4,0)}$$

Result

One finds that $a+b+c=3 \Rightarrow$ **only** a (trivial) $\mathbb{Z}_2$ **R-symmetry** remains in **blowup**.

Look at simplified model with 3 exceptional divisors:

$$
\begin{aligned}
0 &= z_{11}^3 x_1 + z_{12}^3 x_2 + z_{13}^3 x_3 \\
0 &= z_{21}^3 x_1 x_2 x_3 + z_{22}^3 + z_{23}^3 \\
0 &= z_{31}^3 x_1 x_2 x_3 + z_{32}^3 + z_{33}^3 \\
a_i &= |z_{i1}|^2 + |z_{i2}|^2 + |z_{i3}|^2 \\
b_\alpha &= |z_{1\alpha}|^2 + |z_{21}|^2 + |z_{31}|^2 - 3|x_\alpha|^2
\end{aligned}
$$

Symmetries:

- $z_{i\alpha} \to e^{2\pi i/3} z_{i\alpha}$
- $(x_1, x_2, x_3) \to e^{2\pi i/3}(x_1, x_2, x_3)$
- ...

Origin of Symmetries

Note that the symmetries are inherited from the special choice of complex structure on the orbifold (absence of $\kappa\, z_{i1} z_{i2} z_{i3}$ term)

Remnant R symmetries – GLSM

How to check which of these symmetries are R-symmetries?

R-**symmetries** will **transform** the holomorphic **$(3,0)$ form** Ω:

$\Omega \sim \eta\Gamma\eta dz^i dz^j dz^k \quad \Rightarrow \quad Q_R(\Omega) = Q_R(W)$ [Witten]

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How are the R-symmetries broken in blowup?

(Presumably) via marginal deformations in Kähler potential under the presence of the gauge bundle:

$$\int d^2\theta^+ \phi_{4D}(x^\mu) N(z,x) \Lambda\overline{\Lambda}$$

- ϕ_{4D}: 4D modes
- $N(z,x)$: Polynomial in the geometry fields $z_{i\alpha}, x_\alpha$
- Λ: WS fermions describing the gauge bundle

$N(z,x)$ might not be compatible with rotational symmetries
$\Rightarrow$ **R symmetry broken**

Remnant R symmetries – GLSM

To **check transformation** of bundle under **discrete symmetries**:

- **Find discrete transformations** of coordinate **fields** z, x under **symmetry** in question
- Write down **gauge bundle** in ambient space
- **Restrict bundle** to toric hypersurface via Koszul sequence
- Find **contributing monomials**
- **Check transformation** of **monomials** under discrete symmetry

Tools

The last three steps should be automatized using cohomcalg and the Koszul extension developed by Blumenhagen et.al.

Conclusion

$\mathcal{N} = (2, 2)$ GLSM

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Thank you for your attention!